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ABOUT ONE PHYSICAL MODEL OF SOLIDS RUPTURE[1]

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An improved statistical model of the rupture of solids is proposed. The model takes reproduction of defects in the deformation process and the non-linear properties of solids into account. The results of experiments, mainly of acoustic emission, are utilised to obtain information on reproduction kinetics. In the case of initially plastic solids the plausible stages of ageing process, strengthening, hardening and brittle rupture can be obtained with increasing of the concentration of defects.

1. Introduction

The authors of earlier papers [1,2] published the results of rupture probability calculations for solids with accidental defects distribution. The defects of the same dimensions were supposed[2]. The rupture occurs when the local defects concentration reaches the critical value. The rupture probability depends on the average defects concentration and external stress as

$$P(p) = \frac{Vn^*_{cr}}{\sqrt{2\pi}}\left(\frac{p_o}{p}\right)^6 \exp\left[-\beta_o\left(\frac{p_o}{p}\right)^4\right], \qquad (1)$$

where V is the sample volume,n^*_{cr} is the local concentration of critical defects,

$$p_o = k(hn^*_{cr})^{1/4}, \qquad (2)$$

$$\beta_o = \ln\frac{n^*_{cr}}{n_o} - 1, \qquad (3)$$

where k is the Griffit constant, h is the defect dimension and n_o is the average defects concentration.

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[2] The assumption of equal dimensions of all defects (elementary defects) is not so hard restriction, because the statistical treatment allows to consider the interaction of defects and the building up large defects of different forms. The model does not take the detailed mechanisms of the defects interaction into account.

The critical stress p^*_{cr} corresponds to the probability $P(p^*_{cr}) = 1$ and can be derived from (1) as

$$p^*_{cr} = p_o \left(\frac{\beta_o}{\mu}\right)^{1/4}, \tag{4}$$

where

$$\mu = \ln\left(\frac{V n^*_{cr}}{\sqrt{2\pi}}\right)$$

The critical stress depends significantly on the average concentration. It follows from the fact that the destruction processes accompanying the deformation were not considered. In this sense the model represented by [1,2] is a static one.

2. Theoretical analysis

The investigation of the initial latent phase at the very begining of the microdestruction process in the stress-strain linear region is very difficult because of different mechanisms of transport, tearing off and multiplying of dislocations. A lot of information on such processes can be obtained by means of the non-linear (amplitude dependent) absorption of sound.

In the steel the regular and intense acoustic emission occurs in the plastic flow region. It is caused by appearance of a large number of microcrushing centres. At the end of plastic flow region the acoustic emission decreases and sometimes disappears. After the hardening phase the appearance of the dangerous defects causes strong regular acoustic emission. In this region (before reaching the crushing phase) the rate of the emitted pulses per second increases often exponentially with the deformation [3-6]. The exponential rise of the number of defects indicates also X-ray scattering investigation at small angle.

The experimental results approve the assumption of exponential law of the defects creation. Another reason consists in the idea that the rate of increase of the defects number is proportional to the number of the existing defects $dn/d\varepsilon = \gamma n$, so that

$$n = n_o \exp[\gamma\varepsilon], \tag{5}$$

where n_o is the initial value of the defects concentration, see (3), ε is the relative deformation, γ is the multiplying coefficient depending on the mechanical properties of the material and the deformation process characteristics.

We will suppose very slow and quasi-equilibrium processes. It follows from the recent experiments represented by [8,9] that the non-linear parameters of solids depend of the defects concentration. It was shown for the quasi-linear region by means of the longitudinal [8] and surface [9] acoustic waves. In the plastic flow region before rupture the absolute value of nonlinear terms rise by two-three orders [10].

We can assume only quadratic approximation in the initial stress-strain dependence

$$p^o(\varepsilon) = a_o\varepsilon + b_o\varepsilon^2, \tag{6}$$

where a_o and b_o are the 2nd order and the 3rd order moduli. They contain information on the average defects concentration.

There exist two ways to determine the critical values of the rupture parameters, namely the critical stress p_{cr} , the critical strain ε_{cr} and the critical average concentration n_{cr}. The former of these ways consists in deriving $n(p)$ from (5) and (6) and then modification of (1) with respect of the fact that the average concentration is the function of stress. The latter simpler way is based on the following idea: the critical deformation ε_{cr} connected with critical defects concentration n_{cr} leads to the limit of local stress of the rupture $p^*_{cr} = 0$. We can obtain from (4) and (3)

$$n_{cr} = n^*_{cr}/e, \tag{7}$$

where e is the base of the natural logarithm.

Using $n = n_{cr}$, (7) and (3) the critical strain ε_{cr} can be expressed from (5)in the form

$$\varepsilon_{cr} = \frac{\beta_o}{\gamma} = \frac{\varepsilon_m}{d}, \tag{8}$$

where $\varepsilon_m = -a_o/(2b_o)$ corresponds to the extreme of (6) and

$$d = \gamma\varepsilon_m/\beta_o. \tag{9}$$

The parameters a_o and b_o correspond to the assumption that the number of defects does not change. In the real situation the generation of defects takes place. The modification of stress–strain dependence can be developed by means of energetic consideration. The defect creation is connected with a work W_o. The creation work corresponding to the increase of the deformation from ε to $\varepsilon + \delta\varepsilon$ is $\delta W_3 = W_o(dn/d\varepsilon)\delta\varepsilon$. The total deformation work is

$$W_2 = \int_0^{\varepsilon} p(\xi)d\xi = W_1(\varepsilon) - W_3(\varepsilon),$$

where $W_1(\varepsilon) = \int_0^{\varepsilon} p^o(\xi)d\xi$ is the deformation work made in the case of the deformation of a non-linear solid without creation of new defects. Taking (5) into account, the modified stress–strain relation can be expressed as

$$p(\varepsilon) = a_o\varepsilon + b_o\varepsilon^2 - W_o n_o\left(e^{\gamma\varepsilon} - 1\right). \tag{10}$$

The low deformation approximation (6) becomes

$$p(\varepsilon) = a\varepsilon + b\varepsilon^2, \tag{11}$$

where

$$a = a_o(1 - C) \tag{12}$$

$$b = b_o(1 + \gamma\varepsilon_m C) \tag{13}$$

$$C = W_o n_o \gamma/a_o.$$

It is seen that the non-linear parameter $X = |b|/a$ increases with increasing C. The change of ε_m caused by the defects generation is for the case of $C \ll 1$

$$\Delta\varepsilon_m = -\varepsilon_m C(1+\gamma\varepsilon_m). \qquad (14)$$

The critical rupture stress resulting from (10) is

$$p_{cr} = p(\varepsilon_{cr}) = p_m\left[\left(\frac{2d-1}{d^2}\right) - \frac{2c}{d}\left(\frac{e^{\beta_o}-1}{\beta_o}\right)\right], \qquad (15)$$

where $p_m = -a_o^2/(4b_o)$ is the extreme value of $p^o(\varepsilon)$, see (9).

The first term in the brackets does not exceed the value 1 for $d > 1/2$. It is known that the deformation coefficient of the relative sound velocity change is of the order of 10^{-5} to 10^{-4} in the elastic region. According to (12) it gives $C \approx 10^{-5} \div 10^{-4}$. The second term in the brackets (15) gives the average concentration increase from the begining of the process up to the rupture. Its value is proportional to n_{cr}/n_o and changes from 10^2 to 10^4 for different materials (see below). This second term is small for the brittle materials with large initial defects concentration and the rupture limit can be estimated only by the first one.

Obviously, the parameter d determines the initial brittle-plastic properties of solids. According to (8) $\varepsilon_{cr} \geq \varepsilon_m$ for $d \leq 1$ and it corresponds to the region of "plasticity". On the other hand $\varepsilon_{cr} < \varepsilon_m$ for $d > 1$, which represents the brittle rupture. The parameter d determines only the initial brittle or plastic mechanical properties. In all cases the model gives the brittle rupture. Indeed, it follows from (15) that the mechanical properties evolution due to the defects concentration rise-up is

$$\frac{\partial p_{cr}}{\partial n_o} = W_o\gamma\left(\frac{1-d}{Cd}+1\right). \qquad (16)$$

It means that $\partial p_{cr}/\partial n_o > 0$ for $d \leq 1$. For initially plastic solid, its strength rises first (so called hardening process), then the solid becomes a brittle one at $d \approx 1/(1-C)$, and at last the brittle rupture takes place. These are the well-known stages of the plastic solid ageing process.

4. Results and discussion

It is too complicated to compare the theory results with the experimental data. In the theory some new unknown parameters and values are introduced. At first it concerns the multiplying coefficient γ and in general the quantitative data about the destruction process kinetics. We have used the acoustic emission [4-6] and X-ray scattering [7] data for obtaining γ. The determined values of γ are given in the Table 1. The exponential extrapolation to $\varepsilon = 0$ gives the value of n_o. The value of n_{cr} is generally known. An order of the non-linear parameter X is known as well for all materials mentioned in Tab.1 . The exact value of X was obtained in [11] for the steel with near the same composition. The estimated values of W_o are given in the last column of the Tab.1. The value of W_o of capron is in agreement with the energy of dislocation creation

Material	a_0 [Pa]	p_{cr} [Pa]	ε_{cr}	γ	n_0 [m^{-3}]	n_{cr} [m^{-3}]	β_0	Estimated value ε_m	d	X	W_0
Kapron [7]	7.9×10^9	5.5×10^8	0.22	30	4.5×10^{19}	4×10^{22}	6.7	≥ 0.17	~ 0.8	≤ 3.6	$\sim 10^{-16}$
Steel 95X18 [5]	2×10^{11}	2×10^9	2.7×10^{-2}	3.6×10^2	15*)	2.7×10^{5}**)	9.9	2.5×10^{-2}	0.92	20	10^{-4}***)
Iron ore [4]	$(5-10) \times 10^{10}$	$(0.7-1.4) \times 10^8$	3.4×10^{-3}	1.1×10^3	5.5×10^{2}*)	2.4×10^{4}*)	3.8	$\sim 5 \times 10^{-3}$	~ 1.4	~ 100	$(2-10) \times 10^{-3}$
Concrete M200 [6]	2.65×10^{10}	1.43×10^7	1.18×10^{-3}	2.5×10^3	90*)	1.7×10^{3}*)	3.0	$\geq 1.7 \times 10^{-3}$	≥ 1.4	≤ 500	$1.18 \times 10-3$

Table 1. Summary of the determined parameters for some selected materials
*) The emissoin sources whole number in investigated sample; the volume is unknown.
**) Volume of the sample is taken $\sim 2 \times 10^{-3}m^3$.
***) The sample's volume is taken $\sim 2 \times 10^{-5}m^3$.

$W_o = Gb^2L$ at $b \sim L \approx 10^{-8}$m , where G is the shear modulus, $b = |\mathbf{b}|$ is the Burger vector and L is the length of microcrack. Microcracks of such dimensions were observed in the experiment [7]. Much more creation energy was obtained for acoustic emission sources.

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