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LIFETIMES OF THE EXCITED STATES OF EXOTIC ATOMS

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In this work the lifetimes of the excited states of the exotic atoms are calculated using the Bethe-Salpeter equation. The level breadth is calculated to the first order, by considering innteractions of higher order than the ladder-type interaction. The effects of spin, nuclear interactions and the effect of the outer electrons in a mesic atom are neglected. The results are comparable with the experimental data.

1. Introduction

An exotic atom consists of a normal atom with one of its electrons replaced by a negatively charged particle, e.g. mesons or baryons. The formation of such atoms takes place when a negative meson, for example, is stopped in matter. First it looses energy by ionization, then it slows down by collisions with electrons of comparable velocities until it is captured by an atom into a higher Bohr orbit, from which it is rapidly deexcited to lower states, first by Auger transitions, until it reaches lower orbits when radiative transitions become predominant. The cascade of the particle from these higher levels can be calculated using the transition probabilities of the radiative and Auger processes [1,2]. For baryons, π-meson and K-meson the strong interactions will play a dominant role in the cascade process especially for the lower states.

The features common to all exotic atoms may be summarized as follows: In comparison with the electron all the particles captured are heavier in mass. Hence, the particles are much closer to the nucleus since the radius of the Bohr orbit is inversely proportional to the mass of the particle. Therefore, these heavy particles may be used to test the nuclear properties since the nuclear interactions will be more predominant. Also, the particle orbits with principal quantum number $n < \sqrt{m_h/m_e}$ (with m_h the mass of heavy particle and m_e the electron mass) lie within the innermost electron orbit. Therefore, screeninng by the electron cloud is negligible and the problem may be treated as a two-body problem as a hydrogen-like atom. The main features of the energy levels for these atoms can be obtained by solving the Schrödinger equation for the particle-nucleus system assuming only the electromagnetic interaction.

For spin 1/2 particles the energy levels are given by the solution of the Dirac equation

$$E_{nj} = -\frac{\mu c^2}{2n^2}(Z\alpha)^2\left[1 + \frac{(Z\alpha)^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right) - \ldots\right] \qquad (1)$$

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which is of the same form as the Klein-Gordon equation with j replacing l. Modifications to such formulas may be due to the following effects: finite size of nucleus, vacuum polarization, nuclear polarization (dispersion effect), electron screening and fine and hyperfine structures.

In a previous work [3] the level breadth of the excited states of the exotic atoms were calculated using a perturbative approach based upon the Green's function, by considering interactions of higher order than the ladder type interaction. The spin of the photon was neglected in such calculations. In the present work we elaborate on the previous work by taking the spin of photon into account. In the next section we shall outline the theory and in section 3 the results of the calculation are discussed.

2. Perturbation theory

The Bethe-Salpeter equation (BSE) has been applied previously to calculate the fine structure, Lamb split and the hyperfine structure of the hydrogen and exotic atoms. In this section we shall develop a method for calculating the level shift using a perturbation expansion of the Bethe-Salpeter equation. We shall start by taking a simple model in which we assume that a and b are two non-identical scalar particles exchanging scalar photons. The BSE for such system then reads

$$\left[\Delta_a(\frac{m_a}{m_a+m_b}P+p)\Delta_b(\frac{m_b}{m_a+m_b}P-p)\right]^{-1}\Psi_{P\alpha}(p,P)=\int I(p,p',P)\Psi_{P\alpha}(p',P)\mathrm{d}p' \tag{2}$$

where Δ_a and Δ_b are usual Feynman Green's functions for the scalar particles with masses m_a and m_b, respectively. The kernel $I(p,p',P)$ represents the sum over all possible Feynman diagrams and α represents all quantum numbers which define the state of the system. Equation (2) is very difficult to solve in the general case and most of the calculations were made using only the fiirst term, i.e. the ladder approximation. Proceeding as in Ref. 3[1] we obtain an expresiion to the first order for the energy shifts

$$\Delta E^{(1)}=-\frac{i}{T_0}\int\Psi^0_\alpha(x_1,x_2)I'(x_1,x_2;x_1',x_2')\Psi^0_\alpha(x_1',x_2')\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_1'\mathrm{d}x_2' \tag{3}$$

Now, if we are interested in the level shift of the excited states of the two particle system we may take I' as the self energy Σ of the whole system, since this allows for higher order processes, hence

$$\Delta E^{(1)}=-\frac{e^2}{(2\pi)^4}\sum_{\alpha'}\int\mathrm{d}^4k\frac{1}{[(P_{0\alpha}-k)^2+M^2_{\alpha'}-i\varepsilon]\,[k^2-i\varepsilon]} \tag{4}$$

Using known techniques for calculating such integral we get after the renormalization procedure

$$I=i\pi^2\frac{\rho}{1-\rho}\left(\log|\rho|-i\pi\right),\qquad \rho<0 \tag{5}$$

[1] see Ref. 3 for notations and detailed proof

where

$$\rho=\frac{M^2_{\alpha'}-M^2_\alpha}{M^2_{\alpha'}} \tag{6}$$

$M_\alpha=m_a+m_b-B_\alpha$ and B_α is the binding energy of the state α.

This gives an energy shift which should be added to the energy shift due to finite size effects of the nucleus, vacuum polarization, etc. as was discussed previously.The imaginary part is the level width calculated to the first order which is related to the lifetime of the excited state. Thus, the probability of decay of the state α to the state α' would be given by

$$\mathrm{Im}\Delta E^{(1)}\propto\frac{\rho_{\alpha\alpha'}\log|\rho_{\alpha\alpha'}|}{(1-\rho_{\alpha\alpha'})}. \tag{7}$$

Results of the calculation using the above equation were reported in a previous work [3]. In the present work, the method of calculation of the energy shift is extended to include the spin of the photon. In this case the Klein-Gordon equation for the spin zero particle of mass M becomes

$$(\Box^2-M^2)\Psi=i\frac{\partial(A_\mu\Psi)}{\partial x_\mu}+iA_\mu\frac{\partial\Psi}{\partial x_\mu}-A_\mu A_\mu\Psi. \tag{8}$$

Hence, the integral given by equation(4) becomes

$$I=\int\mathrm{d}^4k\frac{(2P_{0\alpha}-k)_\mu(2P_{0\alpha}-k)_\mu}{[(P_{0\alpha}-k)^2+M^2_{\alpha'}-i\varepsilon]\,[k^2-i\varepsilon]}-\int\delta_{\mu\mu}\frac{\mathrm{d}^4k}{k^2-i\varepsilon} \tag{9}$$

where $\delta_{\mu\mu}=4$ comes from the $A_\mu A_\mu$ term and represents the possibility of simultaneous emission and absorption of the same virtual quantum. Following the same procedure as before the above integral gives the probability of the decay when spin of the photon is taken into account. Hence,

$$\mathrm{Im}\Delta E^{(1)}\propto 2\pi^2P^2_{0\alpha}\frac{\rho}{1-\rho}\left[1-\frac{2-3\rho}{1-\rho}\log|\rho|\right],\qquad \rho<0 \tag{10}$$

Equation (5) has been applied previously [3] to calculate the level width for the hydrogen and exotic atoms. In the present work we use equation(10) which includes the spin of the photon. The results are discussed in the next section.

3. Results and discussion

The results obtained in the previous section have been applied to calculate the lifetimes of the excited states of the following sestems: (e^-,p), (μ^-,p), (π^-,p), (e^-,e^+), (π^-,μ^+) and (μ^-,μ^+).

Transition $n \to n-1$	Spin 0 photon (e^-,p), 10^{-8}s	Spin 1 photon (e^-,p), 10^{-8}s	(μ^-,p) 10^{-10}s	(π^-,p) 10^{-10}s
$2 \to 1$	0.308	0.15	0.112	0.089
	0.21*		0.086*	0.067*
$3 \to 2$	1.52	0.74	0.537	0.426
$4 \to 3$	4.12	2.01	1.43	1.13
$5 \to 4$	8.58	4.19	2.95	2.33
$6 \to 5$	15.3	7.49	5.23	4.1
mean cascade time				2.5 [12]

* Dipole approximation

Table 1. The lifetimes of the excited states of the (x^-,p) system.

3.1 The hydrogen atom

The hydrogen atom is the fundamental two-body system and perhaps the most interesting problem in physics. The cascade of the electorn from higher excited states to the lower states can be calculated mainly by the dipole aproximation for the transition probability [1]. Relativistic corrections using the Dirac equation were calculated before [4] and the results agree with nonrelativistic calculation. Our aim here is to extend the calculation to include recoil corrections in a simple way where the spin of the proton and electron are neglected. The results are shown in table 1 in comparison with the previous calculations [3].

3.2. (x^-,p) systems

For the μ-meson the treatment is the same as for the hydrogen atom, but for the π-meson (spin 0 in this case) nuclear capture is expected to predominate for $Z > 2$. The experimental features of the cascade processes are available at the moment, but there is marked disagreement at low atomic numbers.

Cascade calculations for muonic atoms have been performed by Eisenberg and Kessler [5] and Hüfner [6] assuming a distribution over l subtates for an initial n ($n = 14$ in this case). For pion the situation is different as strong interactions takes place between pions and the nucleus in addition to the electromagnetic interactions.

For the K-mesons the situation is different as the nuclear interaction is stronger in this case which creates more complicated final states as a consequence of nuclear absorption. The cascade times of π^-, K^- and Σ^- in liquid hydrogen have been measured with cascade times in the range $(0-4) \times 10^{-12}$ s [7-9]. These results are consistent with the prediction of Day, Snow and Sucher [10] and the calculations of Leon and Bethe [11] that Stark effect mixing leads to rapid capture from states of high n value. However, in the case of hydrogen gas the cascade time is noticeably longer, 2.5×10^{-10}s [12]. This means that radiative processes are likely to be predominant in the gaseous case. Applying our model to the (μ^-,p) and (π^-,p) systems we obtained good results in comparison with the previous work as is shown in table 1.

3.3. Other exotic atoms

In this section we study the quasi-stable positronium, (μ^-, μ^+) and (π^-, π^+) systems. Using our model, we calculated the lifetime of the first excited state for positronium and found it to be 0.452×10^{-8}s to be compared with the value 0.32×10^{-8}s which is obtained using the dipole approximation. Also, the lifetime of a similar system (μ^-, μ^+) is 0.218×10^{-10}s. Here, we give also the lifetime of the first excited state of the (π^-, π^+) atom. The result is 0.192×10^{-10}s which is again comparable to the value obtained using the dipole approximation: 0.132×10^{-10}s (Notice the value calculated previously [3] for spin zero photons is 0.124×10^{-9}s).

To conclude, the lifetimes of the excited states of exotic atoms were calculated using a perturbative approach with the inclusion of the spin of the photon. The results are much better than those obtained previously [3] where the spin of the photon was ignored.

We believe that the inclusion of the spin of the two interacting particles will complicate the problem and will not affect the results significantly. The perturbation theory for the Bethe-Salpeter equation for a bound state comprised of two spin 1/2 particles has been developed in Refs. 13 and 14 where different contributions to the level shift were presented. Some recent calculations for spin 0 - spin 1/2 and spin 0 - spin 0 particles were made also by Own [15] and Halpern [16] for the level shift.

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