

INTRODUCTION TO HANBURY-BROWN/TWISS INTERFEROMETRY IN HEAVY ION COLLISIONS¹

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We give an introduction to Hanbury-Brown/Twiss interferometry in heavy ion collisions, suitable for students at the graduate level. Its main focus is on the basic relation between the phase space emission function $S(x, K)$, the momentum space correlation function $C(q, K)$, and the HBT-radii $R_{ij}^2(K)$. We discuss in some detail how the latter can be calculated for a given phase space distribution $S(x, K)$ of boson emitting sources.

1. Introduction

Many of the open questions in our understanding of heavy ion collisions can be traced back to the fact that the space-time evolution of these collisions cannot be observed directly. Especially, while the energy involved in these collisions is measurable by particle calorimeters, a corresponding direct measurement of the locally reached energy density does not exist. Hence, a determination of the size and spatio-temporal evolution of the interaction region has to rely on indirect evidences, the most prominent of them being Hanbury-Brown/Twiss (HBT) interferometry.

This lecture provides a basic introduction to HBT interferometry. In a first section, we emphasize the pertinent physical arguments which allow to extract spatio-temporal information from the measured 2-particle correlation functions. To this aim, we discuss the connection between

- the emission function $S(x, K)$ which is a theoretical concept, specifying the space-time distribution of particle emitting sources.
- the 2-particle correlation function $C(K, q)$ which is an experimentally measurable quantity.
- the HBT-radii $R_{ij}^2(K)$ which specify a very popular parametrization of $C(K, q)$.

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As it will turn out, the correlation function $C(\mathbf{K}, \mathbf{q})$ does not determine $S(x, K)$ uniquely as long as no (model-dependent) physical assumptions restrict the class of emission functions under consideration. In the second section of this lecture, such a model-dependent investigation is presented in some detail.

2. Model-independent results

Hanbury-Brown and Twiss [1] have been the first to point out that the Bose-Einstein wave function symmetrization of identical bosons can be used for the determination of the size of boson emitting sources. The idea with which they determined the size of stars is very simple:

Consider the star as a static source, emitting photons with probability $\rho(\mathbf{x})$ from spatial points \mathbf{x} . The probability $P(\mathbf{p}_1, \mathbf{p}_2)$ of detecting two photons with momenta \mathbf{p}_1 and \mathbf{p}_2 is then given by the Born amplitude $|\psi_{12}|^2$ of the symmetrized 2-boson wave function $\psi_{12}(\mathbf{x}_1, \mathbf{x}_2)$. Choosing a plane wave for ψ_{12} , we obtain

$$\begin{aligned} P(\mathbf{p}_1, \mathbf{p}_2) &= \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \rho(\mathbf{x}_1)\rho(\mathbf{x}_2) |\psi_{12}(\mathbf{x}_1, \mathbf{x}_2)|^2 \\ &= 1 + |\bar{\rho}(\mathbf{p}_1 - \mathbf{p}_2)|^2 \quad ; \quad \int d^3\mathbf{x} \rho(\mathbf{x}) = 1. \end{aligned} \quad (1)$$

Hence, measuring the 2-particle correlation $P(\mathbf{p}_1, \mathbf{p}_2)$ gives direct access to the Fourier transform $\bar{\rho}(\mathbf{p})$ and, a fortiori, allows to specify the spatial distribution of the photon emitting source uniquely. The crucial point is of course, that $P(\mathbf{p}_1, \mathbf{p}_2)$ is directly measurable while a direct experimental determination of the size of a star is not feasible.

Along this line of argument, Goldhaber, Goldhaber, Lee and Pais [2] proposed to use HBT interferometry for determining the size of the $p\bar{p}$ collision region. More recently, the same idea has been applied to heavy ion collisions. [3, 4] The theoretical starting point is always the same abstraction of the collision region as an assembly of classical boson (here: pions or kaons) emitting sources in a certain space-time region. However, in contrast to the simple example of a static bosonic source of macroscopic size given above, the application of HBT interferometry to heavy ion collisions faces several technical and conceptual complications:

- *technical complications*: in heavy ion collisions, the bosonic source is strongly time dependent and cannot be assumed to have the same size for bosons of different momenta. Hence, instead of a spatial source density $\rho(\mathbf{x})$, the collision region is described by a space-time dependent emission function $S(x, K)$ which generally shows a correlation between the momenta of the emitted particles and their emission points (so-called $x - K$ correlations).
- *conceptual complications*: the emission function $S(x, K)$ is a classical concept well justified for macroscopic sources (e.g. stars). Clearly, the question of whether this classical concept is just an idealization or an overidealization of a given physical system can be asked in many different disguises: Is the explicit space-time dependence of $S(x, K)$ adequate for the microscopic distances involved in heavy ion

collisions? Is the description of the emitted bosons by plane waves or coherent states appropriate? etc. So far, these concerns have not allowed to replace the emission function $S(x, K)$ by a theoretically superior starting point. They should be kept in mind but this lecture does not pursue them further. The interested reader is referred to the literature, cf. [5] and references therein.

2.1. The connection between $C(\mathbf{K}, \mathbf{q})$, $S(x, K)$, and the HBT-radii

The emission function $S(x, K)$ determines the single particle momentum spectrum via $P_1(\mathbf{p}) = E_p dN/d^3p = \int d^4x S(x, p) = E_p \langle a^*(\mathbf{p}) a(\mathbf{p}) \rangle$, as well as the HBT two-particle correlation function $C(\mathbf{K}, \mathbf{q})$ where $\mathbf{K} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2)$ and $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ their relative momentum. The latter is given in terms of the 1- and 2-particle distributions $P_1(\mathbf{p})$, $P_2(\mathbf{p}_1, \mathbf{p}_2) = \langle a^*(\mathbf{p}_1) a^*(\mathbf{p}_2) a(\mathbf{p}_1) a(\mathbf{p}_2) \rangle$ and the average number $\langle N \rangle$ ($\langle N(N-1) \rangle$) of particles (particle pairs) produced in the reaction as [3, 6, 7, 8]

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{\langle N \rangle^2}{\langle N(N-1) \rangle} \frac{P_2(\mathbf{p}_1, \mathbf{p}_2)}{P_1(\mathbf{p}_1) P_1(\mathbf{p}_2)} \simeq 1 + \frac{|\int d^4x S(x, K) e^{iqx}|^2}{|\int d^4x S(x, K)|^2}. \quad (2)$$

In the numerator of the last expression we have introduced off-shell momentum 4-vectors K and q for the total and relative momentum of the particles in the pair by defining $K^0 = \frac{1}{2}(E_1 + E_2)$ and $q^0 = E_1 - E_2$ where $E_i = \sqrt{m^2 + \mathbf{p}_i^2}$. Here, the right hand side of (2) can be obtained e.g. by using the factorization $P_2(\mathbf{p}_1, \mathbf{p}_2) = \langle a^*(\mathbf{p}_1) a(\mathbf{p}_1) \rangle \langle a^*(\mathbf{p}_2) a(\mathbf{p}_2) \rangle + \langle a^*(\mathbf{p}_1) a(\mathbf{p}_2) \rangle \langle a^*(\mathbf{p}_2) a(\mathbf{p}_1) \rangle$ and the equation

$$\int d^4x e^{iqx} S(x, K) = \sqrt{E_{\mathbf{p}_1} E_{\mathbf{p}_2}} \langle a^*(\mathbf{p}_1) a(\mathbf{p}_2) \rangle$$

which links [8] the emission function (Wigner function) $S(x, K)$ to the ensemble expectation value $\langle \dots \rangle$. For azimuthally symmetric sources (see below), the connection between $S(x, K)$, $C(\mathbf{K}, \mathbf{q})$ and the HBT-radii $R_i(\mathbf{K})$ is given by: [11, 9]

$$C(\mathbf{K}, \mathbf{q}) \simeq 1 + \frac{|\int d^4x S(x, K) e^{iqx}|^2}{|\int d^4x S(x, K)|^2} \simeq 1 + e^{-R_2^2(\mathbf{K})q_x^2 - R_1^2(\mathbf{K})q_y^2 - R_1^2(\mathbf{K})q_z^2 - 2R_2^2(\mathbf{K})q_x q_y}. \quad (3)$$

Here, the subscript L denotes the longitudinal or z -direction parallel to the beam, the out or x -direction parallel to the transverse component of \mathbf{K} is denoted by the subscript o , and the remaining $side$ or y -direction carries the subscript s .

2.2. The spatio-temporal information obtainable from $C(\mathbf{K}, \mathbf{q})$

Let us consider emission functions which are sufficiently well described by the gaussian approximation [9, 10]

$$S(x, K) \simeq S(\bar{x}, K) e^{-\frac{1}{2}(x-\bar{x})^T B_{\mu\nu} (x-\bar{x})}, \quad (4)$$

where $\bar{x} = \bar{x}(\mathbf{K})$ denotes the saddle point of $S(x, K)$. Doing the Fourier transform in (2), we find

$$\begin{aligned} C(\mathbf{K}, \mathbf{q}) &\simeq 1 + e^{-(B^{-1})_{\mu\nu} q^\mu q^\nu} \\ (B^{-1})_{\mu\nu} &= \langle x_\mu x_\nu \rangle - \langle x_\mu \rangle \langle x_\nu \rangle \\ \langle \xi \rangle &= \langle \xi \rangle(K) = \frac{\int d^4 x \xi S(x, K)}{\int d^4 x S(x, K)}. \end{aligned} \quad (5)$$

Note that the time component q^0 of \mathbf{q} is a function of \mathbf{q} and \mathbf{K} due to the on-shell constraint of \mathbf{p}_1 and \mathbf{p}_2 : if \mathbf{q} is sufficiently small, we have $q \cdot K \simeq 0$ and hence $q^0 \simeq \beta_L q_0 + \beta_L q_L$ where $\beta_L = K_L/E_K$.

In the case of an azimuthally symmetric source (i.e. $S(x, K)$ is invariant under $y \rightarrow -y$), inserting the on-shell constraint $q^0 \simeq \beta_L q_0 + \beta_L q_L$ in (5) combines the matrix elements $(B^{-1})_{\mu\nu}$ to the HBT-radii as follows: [11, 9, 10]

$$\begin{aligned} R_2^2 &= \langle y^2 \rangle, \\ R_0^2 &= \langle (x - \beta_L t)^2 \rangle - \langle (x - \beta_L t) \rangle^2, \\ R_L^2 &= \langle (z - \beta_L t)^2 \rangle - \langle (z - \beta_L t) \rangle^2, \\ R_{\perp 0}^2 &= \langle (x - \beta_L t)(z - \beta_L t) \rangle - \langle (x - \beta_L t) \rangle \langle (z - \beta_L t) \rangle. \end{aligned} \quad (6)$$

In general, $(B^{-1})_{\mu\nu}$ is a symmetric 4×4 matrix with 10 independent entries. The mass shell constraint combines them into 6 independent HBT-radii: $R_0^2(\mathbf{K})$, $R_L^2(\mathbf{K})$, $R_{\perp 0}^2(\mathbf{K})$, $R_{\perp 1}^2(\mathbf{K})$, $R_{\perp 2}^2(\mathbf{K})$. Further, azimuthal symmetry eliminates the side-out, side-longitudinal and side-temporal elements of $(B^{-1})_{\mu\nu}$, and the remaining 7 non-zero entries combine to the 4 different HBT-radii in (3,6).

The crucial point is: experiments measure the HBT-radii only, and hence they do not determine the $(B^{-1})_{\mu\nu}$'s uniquely. In practice, this forces us to make model-dependent assumptions about $S(x, K)$ if we want to reconstruct $S(x, K)$ from $C(\mathbf{K}, \mathbf{q})$. Only for simple situations like e.g. a static K -independent source, a unique reconstruction of $\rho(\mathbf{x})$ from $P(\mathbf{p}_1, \mathbf{p}_2)$ can be obtained according to equ. (1).

In heavy ion collisions, the model-dependent assumptions on $S(x, K)$ have to include dynamical information which is to a large extent encoded in the $x - K$ -correlation of $S(x, K)$. This in turn is experimentally accessible via the momentum dependence of the HBT-radii. Accordingly, it is the main task of the theorist to determine the momentum dependence of the HBT-radii for particular models $S(x, K)$.

2.3. The limited validity of approximation schemes

Given an emission function $S(x, K)$, how does one determine the HBT-radii in practice? The set-up presented above suggests three different approaches:

1. Evaluate numerically $C(\mathbf{K}, \mathbf{q})$ via the Fourier transform in (2). Then, the HBT-radii are determined by a numerical fit of the gaussian form (3) to $C(\mathbf{K}, \mathbf{q})$.
2. Evaluate numerically the model-independent expressions (6) for HBT-radii.

3. Evaluate analytically the HBT-radii via equ. (2) or (5,6).

Technically, the third possibility amounts to a 4-dimensional integration over $S(x, K)$. It should not be too surprising that most realistic emission functions $S(x, K)$ are too complicated to be analytically integrable. Even worse, there is no general approximation scheme which could be applied to the integration of arbitrary emission functions $S(x, K)$. This makes analytical calculations of HBT-radii a somewhat subtle issue, to say the least.

Indeed, various approximation schemes used in the literature have a very small range of validity. A very naive but popular calculational scheme e.g. is based on a gaussian saddle point approximation using

$$?? \quad B_{\mu\nu}(\mathbf{K}) = -\partial_\mu \partial_\nu \ln S(x, K)|_{x=\bar{x}(\mathbf{K})} \quad ?? \quad (7)$$

for the determination of equ. (4). Here, it is easy to play devils' advocat: choose $S_{devil}(x, K) = e^{-a \cosh x^2} g(\mathbf{K})$, and $B_{\mu\nu}$ in equ. (7) vanishes. Obviously, equ. (7) does not lead to a meaningful result for this type of emission function. As we shall see in the next section, S_{devil} is not an academic example but the limiting case of a relevant theory!

Technically, the crucial point is that $(B_{\mu\nu}^{-1}(\mathbf{K}))$ in equ. (5) should not be determined by inverting the curvature of the emission function at $\bar{x}(\mathbf{K})$, but from the half widths of $S(x, K)$ which are essentially given by the variance $(B^{-1})_{\mu\nu} = \langle x_\mu x_\nu \rangle - \langle x_\mu \rangle \langle x_\nu \rangle$. [10] For all emission functions investigated so far, using this variance in the approximation (4) of $S(x, K)$ leads to sufficiently accurate results. This is the very reason why we have coined the terminology *model-independent HBT-radii* for the set of equations (6).

3. A model for the emission function

In this section, we introduce and discuss a simple and reasonably realistic model in some detail. We use rapidity coordinates, i.e., $z = \tau \cosh \eta$, $t = \tau \sinh \eta$ and $d^4 x = \tau d\tau d\eta dx dy$ where τ denotes the proper time and η the space-time rapidity $\eta = \frac{1}{2} \ln \frac{z+t}{z-t}$. We consider the emission function

$$S(x, K) = \frac{m_\perp \cosh(\eta - Y)}{(2\pi)^3 \sqrt{2\pi(\Delta Y)^2}} e^{-\frac{K_\perp^2}{2\pi^2}} e^{-\frac{(Y-\eta)^2}{2(\Delta Y)^2}} e^{-\frac{x^2}{2(\Delta x)^2}}, \quad (8)$$

where $K_\perp = (m_\perp \cosh Y, K_\perp, 0, m_\perp \sinh Y)$, Y being the momentum rapidity and $m_\perp = \sqrt{m^2 + K_\perp^2}$ the transverse mass.

3.1. Physical assumptions entering $S(x, K)$

Two classical concepts are used in the description of a heavy ion collision region via (8). A velocity field $u_\nu(x)$ is associated with every space-time point of the collision region. Also, we assume that the many interactions taking place in the collision region equilibrate the system locally - this motivates the Boltzmann factor $e^{-\frac{K_\perp^2}{2\pi^2}}$ in (8). Furthermore, several parameters specify the extension of the (pion) source: R describes

the finite radial extension in the transverse coordinate $r = \sqrt{x^2 + y^2}$, the length of the rapidity interval containing the source is specified by $\Delta\eta$, and we have introduced a finite duration $\Delta\tau$ of the emission time centered around the proper emission time τ_0 . Finally, we have to specify the explicit form of the velocity field, e.g.,

$$u_r(x) = (\cosh \eta \cosh \eta_r, \frac{x}{r} \sinh \eta_r, \frac{y}{r} \sinh \eta_r, \sinh \eta \cosh \eta_r) \quad ; \quad \eta_r(r) = \eta_f \frac{r}{R}. \quad (9)$$

Clearly, there is a considerable freedom in the choice of $u_r(x)$. The main point is to use an expression which includes both longitudinal and transverse expansion, the latter being supported by the experimentally measured transverse momentum spectra. [12] In (9), the longitudinal velocity component shows Bjorken expansion, and we have used a linear transverse velocity profile $\eta_r(r)$.

The emission function (8) has five free parameters: R , $\Delta\tau$, $\Delta\eta$, τ_0 , and η_f . These have to be determined by a simultaneous fit to the measured normalized 1-particle distribution $P_1(p) = \int d^4x S(x, p)$ which depends on R and η_f only (the other parameters contribute to the normalization only), and to the HBT-radii (6) (which are four K -dependent functions!). Clearly, the 1- and 2-particle distributions provide sufficient information for either determining the set of parameters uniquely or excluding this type of model. A comparison with experimental data will be carried out in the near future.

More detailed physical assumptions could be included in the model (8) in various ways. E.g. one might test how the predictions change if the temperature is taken to be a space-time dependent quantity $T(x)$. [13] In the more conservative approach we advocate here, one investigates the physical relevance of models with a minimal set of parameters first before introducing additional fit parameters.

3.2. Calculating HBT-radii for a concrete model

The study of simplified models allows to test technical tools and may reveal in great clarity important physical features which persist in more complicated models. A very interesting simplification of (8) [14] is obtained in the limit $\Delta\tau \rightarrow 0$, $\Delta\eta \rightarrow \infty$,³

$$S(x, K) = \frac{m_\perp \cosh(\eta - Y)}{(2\pi)^3} e^{-\frac{K_\perp x}{m_\perp}} e^{-\frac{1}{2\pi R^2} \delta(\tau - \tau_0)}. \quad (10)$$

Physically, this limiting case amounts to the highly idealized assumption that all particles are emitted from a sharp freeze-out hypersurface at τ_0 and that we are dealing with a longitudinally infinite boost-invariant system. This invariance property is known as *Bjorken scaling*. [15] For (10), HBT-radii have been calculated [10] with all three methods described in subsection 2.3. Especially, an analytical approximation scheme has been developed which allows for a convergent order by order expansion of the HBT-radii (6). The lowest and next-to-lowest order results reproduce various statements in the literature. However, higher order corrections turn out to be non-negligible (the lowest order is not necessarily the leading one!). In general, only at third order, the

numerical and analytical expressions for (6) agree within sufficiently small errors. One has to conclude that the previously published lowest order results obtained from naive approximation schemes as e.g. (7), are both qualitatively and quantitatively unreliable. Here, we emphasize two major technical points only:

For the model (10), a lowest order calculation of the HBT-radii (6) leads to [16, 11]

$$R_1^2 = \tau_0^2 \frac{T}{m_\perp}, \quad R_2^2 = \frac{R^2}{1 + \frac{m_\perp}{T} v^2} + \frac{1}{2} \left(\frac{T}{m_\perp} \right)^2 \beta_1^2 \sigma_0^2, \quad R_3^2 = \frac{R^2}{1 + \frac{m_\perp}{T} v^2}. \quad (11)$$

These results have been obtained for a particular Lorentz frame, the longitudinal co-moving system (LCMS), in which $Y = 0$, $\beta_L = 0$ for all pion pairs.

Higher order corrections change (11) for the longitudinal radius e.g. to

$$R_L^2 \simeq f_{corr} \tau_0^2 \frac{T}{m_\perp}, \quad f_{corr} \simeq 2, \quad (12)$$

where the correction factor f_{corr} depends on the size of the transverse flow. Hence, if one wants to extract the emission time τ_0 from the measured radius $R_L(K)$, an accurate higher order calculation is indispensable.

Also, important higher order corrections occur for R_2 in (11). Here, let us draw attention to the β_1^2 -term in $R_2^2(K)$ which can be traced back to the β_1^2 -dependent time variance $\langle t^2 \rangle - \langle t \rangle^2$ in (6). Typically, this leads to a quadratic increase of $R_2^2(K)$ for small momenta K . However, numerically the rise is given by a smaller coefficient than in (11), and the difference $R_2^2 - R_2^2$ saturates well below $\frac{1}{2} \tau_0^2 (T/m_\perp)^2$ as might be expected optimistically. For further discussion and technical details, we refer to the literature. [10]

3.3. Some final remarks

Clearly, the model (10) is too simple to be compared to experimental data. However, it provides a testing ground for technical and interpretational ideas. This lecture is not able to exhaust even the most important of them. The reader is invited to consult the literature or to investigate some of the important questions himself: e.g., is it possible to extract the time variance $\langle t^2 \rangle - \langle t \rangle^2$ from the measured HBT-radii? How far is this determination model-dependent? How could one include pions coming from resonance decays? Which physical features can be expected to survive in this case, which will be washed out? etc. These and many other issues had to be omitted in this short introduction, but people have begun to discuss them in the literature. For the experimental heavy ion program which has stimulated a large part of the theoretical investigations presented above, we refer to the lectures of C. Fabjan and E. Quercigh in this workshop.

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³The devils' advocate argument following Eq. (7) can be made e.g. with an emission function (10) and a quadratic transverse flow $\eta_r(r) = \eta_f \frac{r^2}{R^2}$ for small momenta K .

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