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REVIEW OF EXPERIMENTAL RESULTS ON INTERMITTENCY[1]

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Experimental data on particle fluctuations in small phase space domains are presented for various high-energy multiparticle collisions. A clear evidence for the presence of dynamical fluctuations is shown. The application of the method of factorial moments and correlation integrals to study dynamical fluctuations in individual high multiplicity *nucleus-nucleus collisions as well as in the process of nuclear fragmentation is also discussed.

1. Introduction

An intermittency study in multiparticle production processes started almost a decade ago. It was prompted by a pioneering work of Białas and Peschanski published in 1986 [1], that was triggered by experimental observations of unusually large density fluctuations in small phase space domains, called 'spikes' [2, 3]. Białas and Peschanski suggested that spikes could be a reflection of intermittency, the concept which was originally developed in the field of fluid dynamics [4, 5, 6]. They proposed [1, 7] to identify intermittent behaviour in the physical process by analyzing the scaling properties of factorial moments, F_q (q is the order of the moment), as the resolution in the density distribution is varied. The intermittent behaviour, or self similarity of the production mechanism, should lead to a power law dependence of the factorial moments on the size of the phase space domain:

$$F_q(\delta) \propto (\Delta/\delta)^{\varphi_q}, \tag{1}$$

with $\varphi_q > 0$ for $\delta \to 0$. Δ and δ are sizes of the phase space cells and φ_q is the intermittency index measuring the strength of the intermittency signal.

This characteristic power law dependence was soon confirmed in different high energy processes, ranging from the most elementary e^+e^- annihilation to the most complex nucleus-nucleus collisions. More detailed analysis which followed the first discovery announcements, and particularly its extension to higher dimensions, revealed that a simple power law is only approximately obeyed. Thus, later on intermittency acquired

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a broader meaning, at least for experimentalists, and is now referred to the increase of factorial moments with increasing resolution which not necessarily follows a power behaviour. However that increase indicates the presence of dynamical fluctuations, while factorial moments independent of δ are expected in the case of purely statistical fluctuations.

During the last several years, other methods have been also utilized for studying nonstatistical fluctuations in small phase space domains in high energy particle collisions. Hwa et al. [8] proposed the analysis of frequency moments, the so called G-moments, to study fractal properties of density distributions. Recently, investigation of fluctuations by the method of correlation integrals, closely related to the techniques of statistical mechanics [9], has been suggested [10]. Factorial moments and correlation integrals are, however, not useful to reveal true higher order correlations because they contain combinatorial contributions from lower order correlations. A direct measure of genuine q-particle correlations is provided by the reduced moments [10, 11], known as cumulants, which vanish whenever one or more of q-particles becomes statistically independent of the others.

The aim of this paper is to offer an intelligible picture of the experimental status of intermittency studies. In this short review it is not possible to cover a wealth of data that has been accumulated during the last ten years. For more details see the comprehensive theoretical and experimental review on scaling laws in mutltiparticle dynamics by De Wolf, Dremin and Kittel [11] and also proceedings of series of workshops dedicated to fluctuation studies [12] or proceedings of the International Symposia on Multiparticle Dynamics [13]. Several topics, like higher order cumulants, factorial correlators, multifractal analysis or the analysis of multiplicity distributions in small phase space domains, are not discussed in this paper. The reader may consult the above mentioned papers in order to learn about these topics.

The paper is organized as follows. In the next two sections formalism of the factorial moments and correlation integrals is introduced. In Sect.4 results of the one dimensional analysis are presented, while higher dimensional factorial moments are discussed in Sect.5. Dependences of the observed intermittency effects on various parameters are considered in Sect.6. The analysis of fluctuations in individual high multiplicity events is presented in Sect.7. In Sect.8 the application of the factorial moment method to processes of nuclear fragmentation is discussed and finally Sect.9 contains concluding remarks and prospects.

2. Normalized factorial moments

The main advantage of the factorial moments method is that these moments are free of statistical noise and therefore measure the dynamical fluctuations. To determine the factorial moment of the order q one has to divide an overall phase space Δ, accessible in a given experiment, into M non-overlapping cells of size $\delta = \Delta/M$ and for each event calculate [1, 7]:

$$F_q(\delta) = \frac{1}{M}\sum_{m=1}^{M} n_m(n_m-1)...(n_m-q+1)/(\overline{n})^q, \tag{2}$$

where n_m is the number of particles emitted in the $m-th$ cell of size δ, $m = 1, ...M$ and $\overline{n} = \overline{N}/M$, $\overline{N}$ is the mean multiplicity measured in the full phase space, Δ.

For ensemble of events, the averaging over events and averaging over cells of a given size can be performed differently leading to the so called horizontal, F_q^H, or vertical, F_q^V, moments:

$$F_q^H \equiv \overline{F_q(\delta)} = \frac{\overline{\langle n(n-1)...(n-q+1)\rangle}}{\langle \overline{n}\rangle^q}, \tag{3}$$

$$F_q^V \equiv \langle F_q(\delta)\rangle = \langle\ \frac{\overline{n(n-1)...(n-q+1)}}{\overline{n}^q}\ \rangle, \tag{4}$$

where brackets denote averaging over M cells of size δ and bars denote averaging over all events in the sample.

The horizontal moments, F_q^H, are sensitive to the shape of single particle density distributions. To compensate for this dependence several correcting procedures were proposed. In [14] it was suggested to introduce appropriate correction factors which modify the normalization in Eq.3:

$$F_q^{H,corr} = \frac{\overline{\langle n(n-1)...(n-q+1)\rangle}}{\langle\overline{n^q}\rangle}. \tag{5}$$

Slightly different modification of the normalization was proposed in [15] which takes into account also the bias due to the finite number of particles in small phase space cells:

$$F_q^{H,corr} = \frac{\overline{\langle n(n-1)...(n-q+1)\rangle}}{1/N_{ev}^q\langle N_m(N_m-1)...(N_m-q+1)\rangle}, \tag{6}$$

where N_{ev} denotes the number of events in the sample and $N_m = \sum_{j=1}^{N_{ev}} n_{jm}$, n_{jm} is the number of particles in the m-th cell in the j-th event. The definitions (5) and (6) differ only when number of particles in a cell is very small.

The another, elegant way to eliminate the dependence on the shape of single particle spectra, was proposed in [16, 17]. The effect due to the variation of single particle distributions is removed by an appropriate transformation of an arbitrary distribution into a uniform one. The method works very well in a study of a single variable (one-dimensional analysis) but its application in multi- dimensional analyses (many, usually correlated variables) is not simple and requires large number of particles even at very small cell sizes, thus is restricted to rather large cells.

The effect of the non-flat single particle distribution is suppressed in the vertical factorial moments (Eq.4), however, it is practically impossible to use vertical moments in a multi-dimensional analysis and with limited statistics, because they suffer from large errors and thus lead to unstable results.

Another technical difficulty in the analysis of factorial moments lies in fitting power law dependences. The procedure used to calculate the moments is based on a successive rebinning of the same data and therefore it introduces the strong positive correlations between the moments measured for various cell sizes. In fitting the power law dependence these cell-to-cell correlations are usually neglected. Obvious way to include the correlations is to compute the whole covariance matrix of various measurements [18]. In practice, however, the low statistics of the data does not allow for sufficiently accurate determination of the covariance matrix to assure reliable fit parameters and its errors.

We would like to close this chapter with a warning statement about certain, quite common, experimental biases which may strongly influence the final results. The typically limited statistics (i.e. finite number of events and finite number of particles per event) causes the underestimation of factorial moments, especially those of higher orders, measured at small cell sizes. This is the so called empty bin effect [19] leading to the flattening of moments at small cells. A special care should be also taken by experimentalists to avoid double measurements of the same track, which can strongly enhanced the measured correlations [20]. In addition, for experiments without particle identification, corrections should be made for the contamination by non-primary vertex tracks, among these the most important are e^+e^- pairs from photon conversions. Usually the appropriate corrections have been done either by applying an offline statistical procedure based on the experimental information [21] or by using a special weighting procedure based on Monte Carlo simulations [20].

3. Correlation integrals

The analysis of factorial moments is not suitable for studying fluctuations in variables which cannot be defined for a single particle, like a Lorentz invariant four momentum transfer or an invariant mass. The method of correlation integrals offers this possibility. In addition it allows the maximum information to be extracted about the correlations in a given experiment, making it superior to the standard analysis of factorial moments.

Here, instead of dividing the overall phase space into fixed non-overlapping cells, one simply counts the total number of q-tuples of particles contained within a cell which freely moves in the overall phase space. In consequence the number of q-tuples counted for a given cell size is much larger than in the case of fixed phase space cells, leading to a reduction of statistical errors. Further more, the analysis of correlation integrals sets no restriction on the cell size.

The correlation integral of the order q is defined as [10]:

$$F_q^{CI}(\delta) = \frac{\overline{N_q(\delta)}}{\overline{N_q^{uncorr}(\delta)}}, \qquad (7)$$

where N_q is the number of q-tuples in an event contained within a cell of size $\delta = \Delta/M$, N_q^{uncorr} is the number of q-tuples of uncorrelated particles. The N_q^{uncorr} can be counted in the sample of fake events obtained by mixing particles from different real events in such a way that each particle in a fake event is taken from a different real event. The averaging (denoted by bars) is performed over the whole real (fake) event sample. The above definition refers to the symmetrized integral for which all q particles should be contained within a cell of size δ [9]. The other correlation integrals, with less restrictive criteria for identifying a q-tuple, have been also considered [22].

4. One dimensional analysis

At the beginning the method of factorial moments has been applied to study fluctuations in the rapidity or pseudorapidity distributions of particles produced in various high energy collisions. Later on this analysis was extended to the azimuthal angle distributions. The first evidence for the power law increase of the factorial moments with decreasing width of the pseudorapidity bin was announced by the KLM Collaboration for high energy nuclear collisions [23]. Soon, the intermittency phenomenon was also observed in other high energy collision processes. Presently, the results of the analysis of factorial moments are available for the e^+e^- annihilation at low energies ($\sqrt{s} \approx 30$ GeV) [24, 25] as wel as from all four LEP experiments at $\sqrt{s} = 91$ GeV [26, 27, 28, 29]. For μ-hadron collisions the data were reported by the EMC and E665 Collaborations [30, 31], while ν-nucleon collisions have been studied by the WA21, WA25 and WA59 Collaborations [32]. Hadron-hadron collisions have been extensively investigated by the NA22 Collaboration at $\sqrt{s} \approx 20$ GeV [33] and at $\sqrt{s} = 630$ GeV by the UA1 Collaboration [34]. The results for the interactions of hadrons and nuclei with nuclear targets were reported by several groups [20, 21, 23, 33, 35, 36] mainly at $\sqrt{s} \approx 20$ GeV/n, but also some data on nucleus-nucleus collisions are available at the lower AGS energies [36, 37].

For illustration we show in Fig.1 the dependence of the factorial moments on the width, $\delta\phi$, of the azimuthal angle bin for e^+e^- annihilation at $\sqrt{s} = 91$ GeV from the L3 experiment. It can be seen that at very large $\delta\phi$ bins the correlations are suppressed, probably due to the momentum conservation. Then, an approximately power law increase of the moments is seen and at small bin widths the moments flatten off. The data are compared to the Monte Carlo simulations of different models based on the QCD partonic cascade. The models describe the data reasonably well. Similar behaviour of the factorial moments is observed in the rapidity space. Other LEP experiments showed the results consistent with the L3 data and also satisfactorily well described by Monte Carlo models.

For other high energy collisions, the moments calculated in the one dimensional analysis of rapidity or azimuth satisfy a power law dependence over a large range of bin widths. In Figs. 2, 3 and 4 the fitted intermittency indices, φ_q, are plotted as a function of the order of the moment and compared to the calculations from MC models for μ-hadron [30], hadron-hadron [33] and nucleus-nucleus [38] collisions respectively. For all these reactions the intermittency indices increase with increasing order of the moment and in all cases they are larger than those obtained from Monte Carlo simulations. Thus, we can conclude that currently available models are unable to describe quantitatively experimental data on intermittency for all high energy collisions except the e^+e^- annihilation.

5. Higher dimensions

It is believed that the intermittency is a three dimensional effect and the projection onto lower dimensional subspace reduces the signal [39]. The extension of the one dimensional

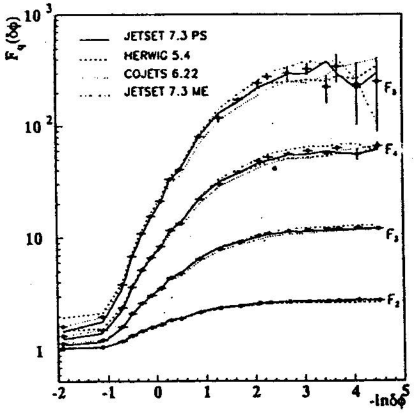

Fig. 1. Factorial moments in azimuthal angle, ϕ, for e^+e^- L3 data [29] compared to the predictions from Monte Carlo models.

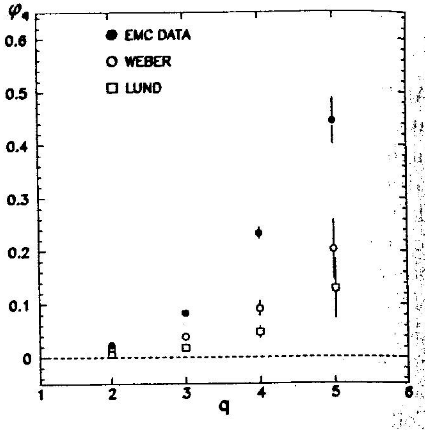

Fig. 2. Intermittency indices, φ_q, as a function of the order of the moments for $\mu - h$ EMC data [30] compared to the predictions from Monte Carlo models.

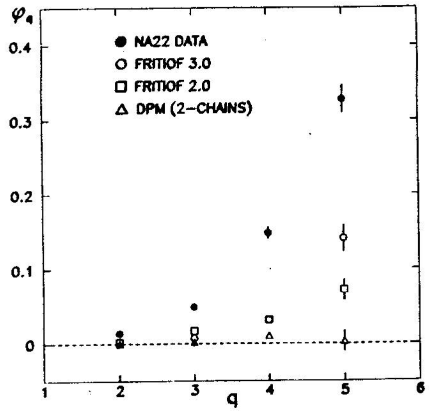

Fig. 3. The same as in Fig.2 but for h - h NA22 data [33].

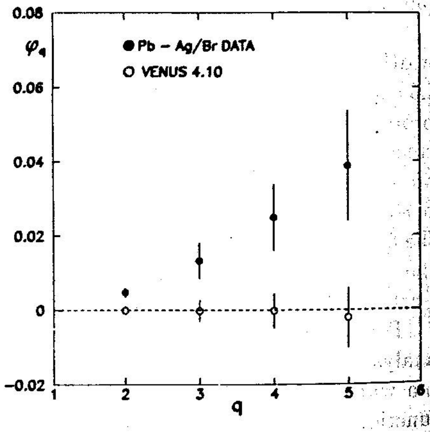

Fig. 4. The same as in Fig.3 but for Pb(158 GeV/n) - Ag/Br KLMM data [38].

analysis of the factorial moments is straightforward. The moments are calculated from the same formulae, but now Δ denotes the volume of a d-dimensional phase space, which is divided into M d-dimensional cells of size $\delta = \Delta/M$. The dependences of the F_2 moments on the size of the phase space cell, obtained from the one-, two- and three-dimensional analyses, are shown in Fig.5 for the e^+e^- annihilation and in Fig.6 for $p - \bar{p}$ collisions. Indeed, it can be seen that with increasing dimensionality of the analyzed phase space the rise of the moments becomes faster.

It was observed, in different experiments, that the three-dimensional moments ex-

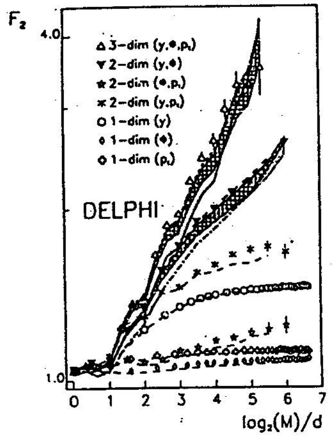

Fig. 5. Factorial moment,F_2, for 1, 2 and 3 dimensional analysis for e^+e^- DELPHI data [26] as a function of $log_2(M)/d$ (M denotes the total number of d-dimensional boxes).

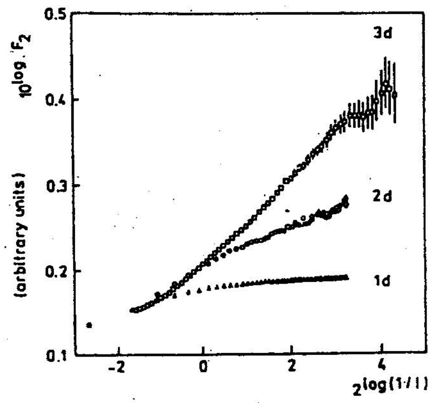

Fig. 6. The same as in Fig.5 but for $p - \bar{p}$ UA1 data [34].

hibit faster than a power law rise over a wide range of cell sizes. It was suggested [39] that the moments growing faster than a simple power law (Eq.1) can still obey a more general power law behaviour:

$$F_q \propto [g(\delta)]^{\varphi_q}, \tag{8}$$

where g is any function of the cell size. This modified power law implies the moment scaling i.e.:

$$\ln F_q = c_q + (\varphi_q/\varphi_2)\ln F_2. \tag{9}$$

This relation, with c_q coefficient equal zero, was found to be satisfied by factorial moments calculated in a large class of cascade models [39] and it was also confirmed by the experimental data (see Fig.7). Additionally, it was found that the slope ratios are independent of the dimensionality and the type of the collision process, as it is illustrated in Fig.7.

6. Dependences of the intermittency effect

The vast amount of experimental data on the intermittency effect allows for detailed study of the dependence of the effect on various parameters. In this review, we limit the discussion to the multiplicity and charge dependences and to the dependence of the intermittency indices on the rank of the moments. The reader interested in the transverse momentum dependence or the dependence on the jet topology in the e^+e^- annihilation is advised to consult [11].

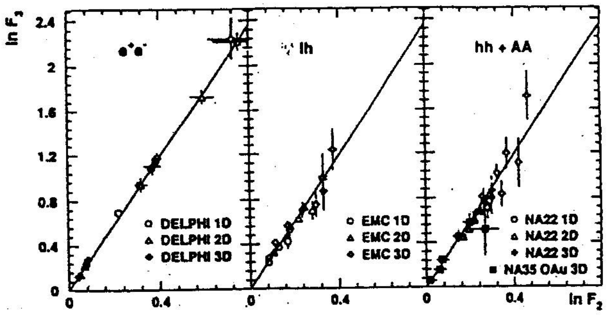

Fig. 7. Test of the universality of the generalized power law (Eq.9) in e^+e^-, l-h, h-h and A-A collisions. The straight line was fitted to the e^+e^- data and then compared to the other data sets.

6.1. Multiplicity dependence

All data available show that the intermittency indices decrease with increasing multiplicity or particle density.

The L3 Collaboration has studied the correlation integrals of various ranks as a function of the square of the four-momentum transfer, Q^2_{inv} for low and high multiplicity data [29]. They observed, in a large Q^2_{inv} region, a steeper rise of the correlation integrals for low multiplicity data than in high multiplicity events, whereas in the region of small Q^2_{inv} no multiplicity dependence was seen. The MC simulations showed the same trends as the data in the large Q^2_{inv} region.

For hadronic collisions the intermittency slopes decrease with increasing multiplicity, in clear contrast to MC simulations which show either a weaker decrease or even an increase [34]. For nuclear collisions the weakening of the intermittency signal with increasing event multiplicity or with increasing masses of the colliding nuclei (see e.g. [20, 21, 23, 36, 38]) was also observed.

Comparing different high energy processes, we see that intermittency indices decrease with increasing complexity of the collision process. This is illustrated in Fig.8 showing anomalous dimensions, $d_q = \varphi_q/(q-1)$, as a function of q for μ-p, h-p, ^{16}O-Ag/Br and ^{208}Pb-Ag/Br collisions. Significantly smaller values of d_q parameters are measured in the collisions between heavy nuclei than in a more elementary processes. The weak intermittency effect, observed in nuclear collisions, can be explained by assuming emission of particles from several independent sources like e.g. parton-parton or nucleon-nucleon collisions. The inherent averaging over particles originating from different sources would cause the fluctuations to decrease proportionally to the number of contributing subprocesses, N_{source} [40]. Thus one expects that

$$\varphi_q \propto \frac{1}{N_{source}} \propto \rho^{-1}, \qquad (10)$$

where ρ is the particle density measured in a given ensemble of events. The UA1 Collaboration has studied $p-\bar{p}$ collision events with different multiplicities and confirmed the expected (Eq.10) decrease of φ_q with increasing particle density, $\rho = dN/d\eta$. However, when we extrapolate $p-\bar{p}$ data to the densities measured in nucleus-nucleus collisions, we see that the slopes obtained in nuclear collisions do not fall on the extrapolated $p-\bar{p}$ dependence (Fig.9). At the same particle density, the intermittency slopes are significantly larger (almost by an order of magnitude) in nucleus-nucleus interactions than in extrapolated $p-\bar{p}$ data, suggesting occurrence of collective effects in nuclear collisions. Thus, although the overall intermittency signal is very weak in nuclear collisions as compared to hadronic collisions, it is significantly stronger when compared at the same particle density.

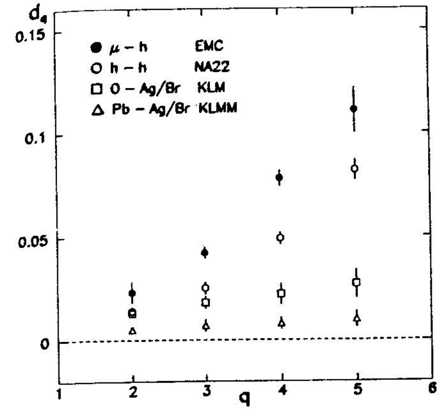

Fig. 8. Anomalous dimensions, d_q, measured in different collision processes at $\sqrt{s} \approx 20$ GeV/n as a function of the order of the moments.

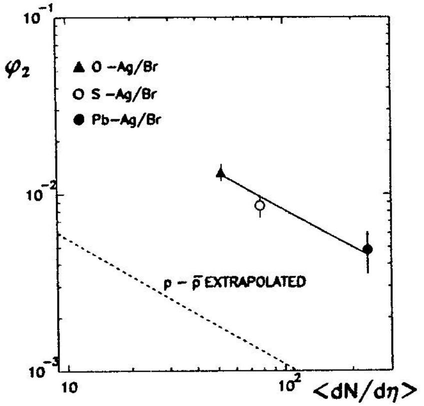

Fig. 9. Intermittency index, φ_2, as a function of particle pseudorapidity density. Points represent the KLMM nucleus-nucleus data. The dashed line is an extrapolation from the UA1 data to higher densities using Eq.10.

7. Charge dependence

The study of the dependence of the intermittency effect on the charge of analyzed particles can reveal whether the quantum mechanical Bose-Einstein (BE) interference between identical particles is responsible for the observed nonstatistical fluctuations. The best suitable for this study is the analysis of correlation integrals as a function of Q_{inv} for like-sign and unlike-sign particle pairs. In all high energy processes it was found that BE correlations play a significant role. The indices $\varphi_2(-,-)$ were found to be larger than $\varphi_2(+,-)$ in lepton-hadron [30, 31], hadron-hadron [41] and nucleus-nucleus [20] collisions. For illustration we show in Fig.10 the dependence of F_2^{CI} on $1/Q_{inv}$ for p-Au, S-S and O-Au collisions as reported by the NA35 Collaboration[20]. In the e^+e^- annihilation the L3 results showed [29] the stronger rise of correlation integrals for like-sign pairs than for $(+,-)$ pairs in the small Q^2_{inv} region. In the intermediate Q^2_{inv} region the opposite effect is seen which can be attributed to resonance contributions and

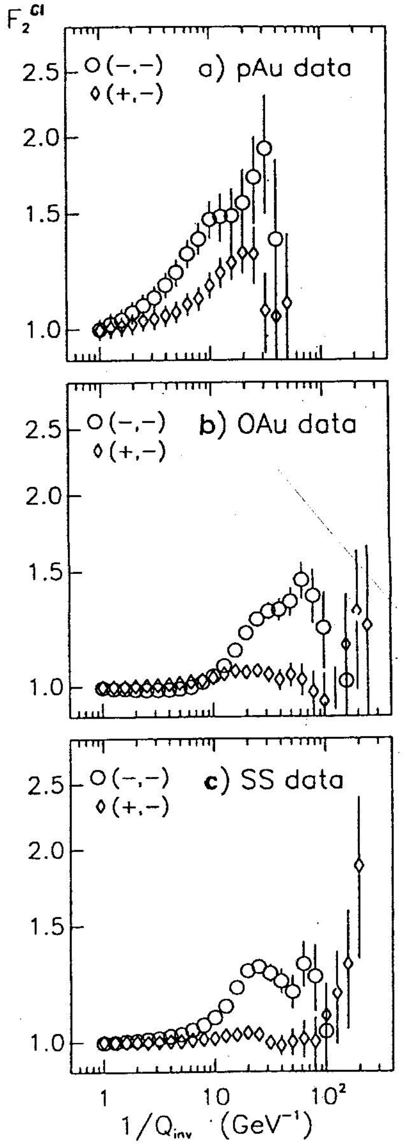

Fig. 10. Log-log plot of the correlation integrals, F_2^{CI}, vs $1/Q_{inv}$ for $(+,-)$ (diamonds) and $(-,-)$ (circles) pairs for the NA35 data [20].

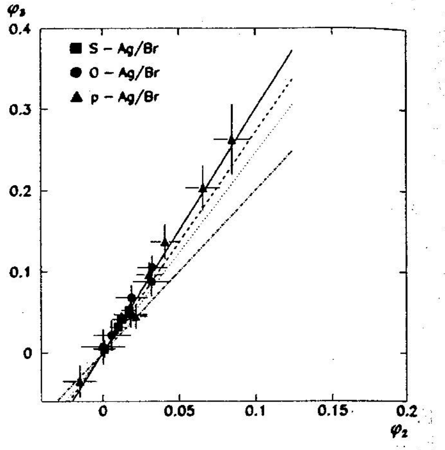

Fig. 11. Correlation between φ_3 and φ_2 obtained from one-dimensional analyses in η and ϕ variables for the KLM data [21]. See text for line descriptions.

finally in the region of large Q^2_{inv} there is no dependence on particle charges consistently with the QCD cascade evolution.

To close this discussion, we would like to refer briefly to the ongoing debate whether the correlations between like-sign particles measured at small Q^2_{inv} follow a power law or, a conventionally used in the BE correlation study, gaussian or exponential behaviour. The data presented by the NA22 are rather inconclusive [33], while the UA1 [34] is more in favour of a power law increase of the correlation integrals with decreasing Q^2_{inv}. The possible evidence for a power law is quite appealing, due to the conjecture [42] that it would indicate a strong fluctuations of the size of the interaction region. It should be, however, taken into account that experimentally the region of small Q^2_{inv} is the most problematic due to the largest systematic and statistical errors.

7.1. Dependence of the intermittency indices on the order of the moments

It was suggested that measurements of φ_q or d_q dependence on q may reveal an underlying mechanism in particle production processes. Particularly, they may indicate whether the self-similar cascade mechanism is at work or critical phenomena occur. It was argued [43, 44] that an observation of d_q being independent of q (i.e. monofractal structure) would indicate a second order phase transition in the system. On the other hand, if a final state is formed as a result of the self-similar cascade process, one expects [17, 43, 45] d_q to increase with q. In this context, the study of d_q vs. q dependence is especially interesting for nucleus-nucleus collisions, since it could signal the occurence of a quark-gluon plasma.

In Fig.11 we show the correlation between φ_3 and φ_2 obtained from the fits to the correlation integrals measured in the azimuthal angle bins in the collisions of protons, ^{16}O and ^{32}S with the Ag/Br target by the KLMM Collaboration [21]. Different lines shown in this Figure are derived from a self-similar cascade process with either a gaussian approximation, $\varphi_q = \varphi_2 q(q-1)/2$ - solid line, or with the Levy stable law, $\varphi_q = \varphi_2(q^\mu - q)/(2^\mu - 2)$ - dashed line, or show the dependence expected in the case of a single fractal dimension, $\varphi_q = \varphi_2(q-1)$ - dashed-dotted line. It can be seen that the observed φ_3 vs. φ_2 dependence is consistent with the expectations from self-similar cascade processes, but disagrees with the single fractal structure which was associated with the second order phase transition. On the other hand, the application of the Ginzburg- Landau (GL) description of phase transition in statistical physics to multiparticle production [46], has led to a different prediction, namely $\varphi_q = \varphi_2(q-1)^{1.3}$, which is denoted by dotted line in Fig.11. We see that this prediction is close to the expectations from self-similar cascade processes in the range of φ_2 values accessible in this experiment. Thus, the above results indicate that a self-similar cascade mechanism is the origin of the fluctuations, even though the second order phase transition described in accordance with the GL theory cannot be definitely ruled out.

8. Fluctuations in individual events

Almost a decade ago Białas and Peschanski showed [1, 7] that factorial moments calculated for a single high multiplicity JACEE event [2] exhibit a power law rise with increasing resolution in pseudorapidity. Now, the new data on ^{208}Pb collisions at 158 GeV/n with heavy nuclear targets become available. In the central Pb-nucleus collisions the events with a multiplicity of the order of 1000 secondaries are just typical,

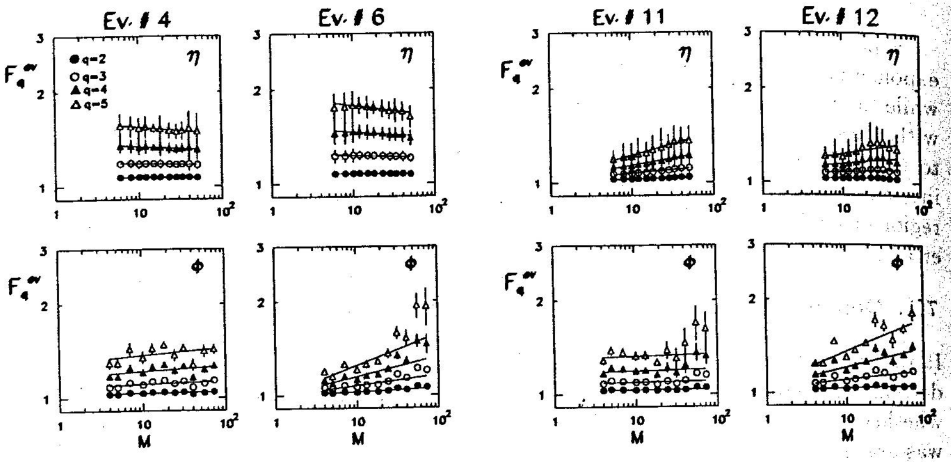

Fig. 12. Log-log plots of the factorial moments F_q^{ev} calculated for a four Pb(158 GeV/n)-Ag/Br high multiplicity events vs M in η (upper plots) and ϕ (lower plots) bins [38].

allowing for a more systematic analysis of individual events. The first data has been already reported by the KLMM Collaboration for Pb-Ag/Br collisions [38]. Examples of the dependences of the factorial moments on the number of subdivisions, M, of the pseudorapidity or azimuthal angle space are shown in Fig.12 for four randomly selected events.

One can see that the events show very diverse behaviour. In the events analyzed by the KLMM Collaboration, no event with extremely large fluctuations was observed, but the feasibility of the method has been tested. In future, at RHIC or LHC experiments, such an analysis can be utilized as an effective triggering for interesting events, e.g. quark-gluon plasma candidates. Selected events with large fluctuations could be further investigated in more details to search for other QGP signatures.

9. Intermittency in nuclear fragmentation

Up to now we have been discussing the application of the factorial moments method to study dynamical fluctuations in density distributions of particles produced in high energy collisions. A similar analysis can be applied to other processes, such as e.g. nuclear fragmentation, hoping to obtain a better understanding of the physics involved.

Płoszajczak and Tucholski [47] were first to suggest searching for intermittency patterns in the mass and charge distributions of fragments emitted from the nuclei after their interaction. A nucleus- nucleus interaction proceeds via two stages. The initial stage of a collision involves the strong interactions between colliding nuclei, which leads to the creation of new particles, predominantly pions. This process was considered in previous sections. After this stage, a much slower process of de-excitation of the projectile and target nuclei occurs, accompanied by the break-up of nuclei into nuclear fragments. It is this later stage, and in particular the break-up of the projectile nucleus, that is of our interest now.

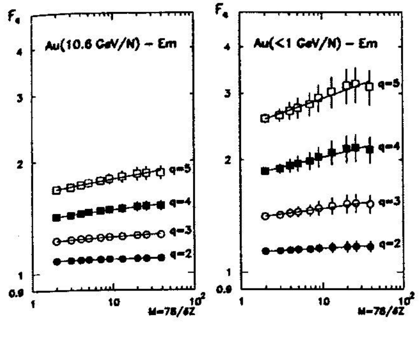

Fig. 13. Log-log plots of the factorial moments as a function of the width of charge bins, δZ, for Au-Em events with the number of heavy fragments greater than 2 [48].

Recently, the KLMM Collaboration has performed the study of the factorial moments calculated for charge distributions of fragments emitted from the projectile ^{197}Au nuclei after its interaction with an emulsion target [48]. In Fig.13 the dependences of the factorial moments on the size of the charge bin, δZ, are shown for 10.6 GeV/n and 0.1-1 GeV/n Au-Emulsion interactions. Clear evidence for intermittent behaviour can be seen, with a stronger effect observed at lower energies than at 10.6 GeV/n.

More detailed analysis, restricted to the multiply charged fragments and to the different classes of fragmentation processes has indicated, however, that the charge conservation effects and the finite size of decaying systems affect strongly the factorial moments and the strenght of the intermittency signal. These effects should be carefully investigated within the framework of fragmentation models, which presently are unable to describe the dynamical fluctuations observed in charge distributions of nuclear fragments.

10. Conclusions

To summarize the almost ten years of intense experimental and theoretical work on the intermittency phenomenon, we would like to stress only a few most important points.

A true and clear achievement of this work is that it provided powerful tools, factorial moments and correlation integrals, for a systematic study of dynamical correlations between many particles. We understand better all advantages, shortcomings and technical problems of different methods. Thus, the analysis of the two-particle correlations can be now effectively extended to multi-particle correlations.

All available data show a clear evidence for the presence of dynamical fluctuations in multiparticle production processes. As far as the main aim of intermittency studies is concerned, namely the search for scaling laws in density fluctuations, the present experimental observations confirmed only an approximate power law behaviour.

The study revealed that commonly used Monte Carlo models are unable to reproduce the results, particularly for hadronic and nuclear collisions. The only exception are processes of e^+e^- annihilation, for which the QCD based models provide a satisfactory description of the data. It has been shown that in all types of high energy collisions the quantum mechanical interference effects give significant contributions to the measured correlations. Thus, the proper theoretical description of these effects, which is still far from satisfactory, and its incorporation into Monte Carlo models should be a foremost task of future work. It should be noticed that this challenge has been already undertaken by several groups and the hope is that it will be soon accomplished.

From experimental side, in the nearest future, we forsee results from the HERA experiments, which are of great interest due to the access to very low Bjorken-x region. The study of fluctuations is also one of main objectives of future heavy ion experiments planned for the RHIC and LHC accelerators. Here, the analysis of individual events may appear to be a promising way in the quest for a quark-gluon plasma.

These few summary remarks show that the field of many-particle fluctuation study will be still active.

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