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INTERMITTENCY OF BREMSSTRAHLUNG PHOTONS AND SPACE-TIME EVOLUTION OF HADRONIC COLLISIONS[1]

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The intermittent behaviour of final state hadrons in multiparticle production seems to be caused by the Hanbury-Brown and Twiss (HBT) correlations of identical hadrons. We follow this approach in a simple case of correlations of two hard bremsstrahlung photons. We point out that even rather conventional types of the assumed space-time behaviour of multiparticle production in hadronic collisions lead to the intermittent behaviour of photons bremsstrahled by charged hadrons. We present some conjectures concerning the applicability of this mechanism to correlations between two identical final state hadrons.

1. Introduction

The study of strong correlations (spikes) in rapidity distributions of multiparticle production in hadronic collisions at high energies has lead Bialas and Peschanski [1] to the conjecture of the existence of intermittency in multiparticle production. The concept of intermittency is well known in hydrodynamics [2] where it is connected with some features of turbulence.

Since pioneering papers by Bialas and Peschanski [1] the intermittent behaviour of multiparticle production has attracted a considerable attention both from the theoretical and experimental side; most of the work done is cited and described in recent reviews [3, 4, 5]. The origin of the intermittent behaviour has been sought at the beginning in the presence of cascades with self-similar features. Later on the experimental results [3, 4, 5, 6, 7] have shown that the short range correlations between identical particles in the final state are considerably stronger than between non-identical ones. This is consistent with the conjecture that the intermittent behaviour is caused – at least to a large extent – by the HBT correlations of identical particles [8, 9, 10, 11, 12, 13].

In Ref.[8] Bialas suggested that the intermittent behaviour of the two body correlation function is caused by the fluctuating size of the region emitting these particles.

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To describe this conjecture in more detail we have to introduce some standard notation [3]. The correlation function of two identical particles is defined as

$$C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2) \tag{1}$$

where, for the sake of simplicity, we consider the correlation in rapidity.The inclusive distributions ρ_1, ρ_2 are

$$\rho_1(y_1) = \frac{1}{\sigma_{\rm incl}}\frac{d\sigma}{dy_1}, \qquad \rho_2(y_1, y_2) = \frac{1}{\sigma_{\rm incl}}\frac{d^2\sigma}{dy_1\, dy_2} \tag{2}$$

are normalized by conditions

$$\begin{aligned} \int \rho_1(y_1)\, dy_1 &= \langle n\rangle \\ \iint \rho_2(y_1, y_2)\, dy_1\, dy_2 &= \langle n(n-1)\rangle \end{aligned} \tag{3}$$

what leads to the following normalization of the correlation function

$$\iint C_2(y_1, y_2)\, dy_1\, dy_2 = \langle n(n-1)\rangle - \langle n\rangle^2 \tag{4}$$

In Eqs.(1-4) the distributions and average values $\langle n\rangle$, $\langle n^2\rangle$ refer only to the selected identical particles, e.g. to π^+.

When studying the correlation between particle three-momenta $\vec{k}_1$, $\vec{k}_2$ one simply makes the replacement

$$y_1 \to \vec{k}_1, \qquad y_2 \to \vec{k}_2 \tag{5}$$

otherwise the Eqs.(1-4) remain unchanged.

The term intermittent behaviour of particle spectra has been defined by Bialas and Peschanski in the following way. Suppose we consider the rapidity interval $-Y/2 < y < Y/2$ of length Y and divide it into M bins each with the length $\delta = Y/M$. The scaled factorial moment in one of these bins is defined as average over events

$$F_q(\delta) = \frac{\langle n(n-1)\ldots(n-q+1)\rangle}{\langle n\rangle^q} \tag{6}$$

and for practical reasons one can average still over the bins. For the second factorial moment we have

$$F_2(\delta) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} = \frac{\int \rho_2(y_1, y_2)\, dy_1\, dy_2}{\left[\int \rho_1(y_1)\, dy_1\right]^2} \tag{7}$$

The distribution of final state particles is called intermittent if for a fixed Δ and for $\delta \to 0$ we have

$$F_2(\delta) = \left(\frac{\Delta}{\delta}\right)^{f_2} F_2(\Delta) \tag{8}$$

The constant f_2, $0 < f_2 < 1$ is called the intermittency exponent. The behaviour corresponding to the Eq.(8) is obtained for

$$\rho_2(y_1 - y_2) \sim |y_1 - y_2|^{-f_2} \tag{9}$$

what is easily seen when the integrals in Eq.(7) are calculated for a small interval of length δ, when $\rho_1(y_1)$ can be taken as a constant. Assuming, what is most natural, that the product $\rho_1(y_1)\rho_1(y_2)$ is not responsible for this behaviour, we can claim that the intermittency of the second factorial moment is equivalent to the requirement

$$C_2(y_1, y_2) \sim |y_1 - y_2|^{-f_2} \tag{10}$$

for $(y_1 - y_2) \to 0$.

In studying of the HBT correlations, for reviews see e.g. [14, 15, 16], we introduce the spatial distribution of sources and the amplitude for the emission of the particle, e.g. positive pion, with momentum $\vec{k}$ from the point $\vec{r}$. The amplitude is written as

$$A(\vec{k}, \vec{r}) = \sqrt{\tilde{\rho}(\vec{r}, \vec{k})}\, e^{i\phi(\vec{k}, \vec{r})}\, e^{i\vec{k}.\vec{r}} \tag{11}$$

where the phase of the amplitude is assumed to be strongly dependent on $\vec{r}$.

The probability to emit a pion with momentum $\vec{k}$ is then given as

$$P(\vec{k}) = \left\langle\left\langle \left| \int A(\vec{k}, \vec{r})\, d^3\vec{r} \right|^2 \right\rangle\right\rangle \tag{12}$$

The internal bracket $\langle\ \rangle$ stands for the averaging over phases and the external one over the fluctuating size of the source. Assuming, for averaging over phases, that phases are completely chaotic

$$\left\langle e^{i\phi(\vec{k}, \vec{r}) - i\phi(\vec{k}, \vec{r}')} \right\rangle = \delta(\vec{r} - \vec{r}') \tag{13}$$

we find

$$P(\vec{k}) = \left\langle \int \tilde{\rho}(\vec{r}, \vec{k})\, d^3\vec{r} \right\rangle \tag{14}$$

The remaining averaging over fluctuations of the size of the source can be taken explicitly if the fluctuations depend on some specified parameters. Supposing e.g. that the source is always spherically symmetric, its radius R fluctuates,and the density of the source $\tilde{\rho}$ depends on R [8] we have

$$P(\vec{k}) = \int F(R)\, dR \int \tilde{\rho}(\vec{r}, \vec{k}; R)\, d^3\vec{r} \tag{15}$$

where $F(R)$ is the probability density that the radius of the source is equal to R.

The probability to emit two identical pions with momenta $\vec{k}_1$, $\vec{k}_2$ is given as

$$P(\vec{k}_1, \vec{k}_2) = \left\langle\left\langle \left| \iint A(\vec{k}_1, \vec{r}_1)\, A(\vec{k}_2, \vec{r}_2)\, d^3\vec{r}_1\, d^3\vec{r}_2 \right|^2 \right\rangle\right\rangle \tag{16}$$

When averaging over phases we assume again the complete chaoticity

$$\left\langle e^{i\{\phi(\vec{k}_1, \vec{r}_1) + \phi(\vec{k}_2, \vec{r}_2) - \phi(\vec{k}_1, \vec{r'}_1) - \phi(\vec{k}_2, \vec{r'}_2)\}} \right\rangle = \delta(\vec{r}_1 - \vec{r'}_1)\delta(\vec{r}_2 - \vec{r'}_2) + \delta(\vec{r}_1 - \vec{r'}_2)\delta(\vec{r}_2 - \vec{r'}_1) \tag{17}$$

Assuming further that $\vec{k}_1$ is close to $\vec{k}_2$ and $\tilde{\rho}(\vec{r},\vec{k}_1) \approx \tilde{\rho}(\vec{r},\vec{k}_2) \approx \tilde{\rho}(\vec{r},\vec{K})$, where $\vec{K} = (\vec{k}_1+\vec{k}_2)/2$ we find

$$P(\vec{k}_1,\vec{k}_2) = \left\langle \int \tilde{\rho}(\vec{r},\vec{k}_1)\, d^3\vec{r} \int \tilde{\rho}(\vec{r'},\vec{k}_2)\, d^3\vec{r'} \right\rangle + \left\langle \left| \int e^{i(\vec{k}_1-\vec{k}_2).\vec{r}} \tilde{\rho}(\vec{r},\vec{K})\, d^3\vec{r} \right|^2 \right\rangle \tag{18}$$

where the remaining averaging is over the fluctuations of the size of the region emitting identical particles.

Replacing in Eq.(1) rapidities by momenta we have

$$C_2(k_1,k_2) = \rho_2(k_1,k_2) - \rho_1(k_1)\rho_1(k_2) \tag{19}$$

Comparing Eqs.(18) and (19) we find that

$$C_2(\vec{k}_1,\vec{k}_2) = \left\langle \left| \int e^{i(\vec{k}_1-\vec{k}_2).\vec{r}} \tilde{\rho}(\vec{r},\vec{K})\, d^3\vec{r} \right|^2 \right\rangle \tag{20}$$

For a spherical source with fluctuating radius R described by the probability distribution function $F(R)$

$$C_2(\vec{k}_1,\vec{k}_2) = \int dR\, F(R) \left| \int e^{i(\vec{k}_1-\vec{k}_2).\vec{r}} \tilde{\rho}(\vec{r},\vec{K};R)\, d^3\vec{r} \right|^2 \tag{21}$$

The normalization of $\tilde{\rho}(\vec{r},\vec{k}_1)$ is given by the condition that $P(\vec{k}_1)$ as given by Eq.(12) corresponds to $\rho_1(y_1)$ in Eq.(2). In what follows we shall not be interested in the normalization, since our main concern is the presence of singularity in the correlation function corresponding to the intermittent behaviour, see Eq.(10).

2. Two ways to intermittent behaviour via HBT – fluctuating versus quiet sources

For a fluctuating source the size fluctuates from one event to another and the correlation function is given by Eq.(21). For a quiet source the density of sources is roughly the same for all the events, but the density may be a superposition of components with different sizes. To show the difference we shall use a simple example of Gaussian sources discussed by Bialas [8]. For a fluctuating source we take the Eq.(21), with

$$\tilde{\rho}(\vec{r},\vec{k};R) = \frac{1}{R^3\pi^{3/2}} \exp\left(-\frac{r^2}{R^2}\right) \tag{22}$$

as a normalized Gaussian distribution. Denoting $\vec{q} = \vec{k}_1 - \vec{k}_2$ we have as its Fourier transform

$$\int e^{i\vec{q}.\vec{r}} \tilde{\rho}(\vec{r},\vec{k};R)\, d^3\vec{r} = \exp\left(-\frac{R^2q^2}{2}\right) \tag{23}$$

and the correlation function $C_2(\vec{q})$ becomes

$$C_2(\vec{q}) = \int F(R)\, dR \exp(-R^2q^2) \tag{24}$$

Taking the probability distribution of the scaling form [8]

$$F(R) = \gamma L^{-\gamma} R^{\gamma-1}\Theta(L-R) \tag{25}$$

where L denotes the maximal size of the fluctuation one finds the scaling behaviour

$$C_2(q) \sim q^{-\gamma} \quad \text{for} \quad q \geq \frac{\sqrt{2}}{L} \tag{26}$$

what according to Eq.(19) corresponds to the intermittency.

For a quiet source the density $\tilde{\rho}(\vec{r},\vec{k})$ is the superposition of sources of different size. Using the same example of Gaussian distributions we have

$$\tilde{\rho}(\vec{r},\vec{k}) = \int f(R)\, dR \frac{1}{R^3\pi^{3/2}} \exp\left(-\frac{r^2}{R^2}\right) \tag{27}$$

The correlation function in this case becomes

$$C_2(q) = \left| \int e^{i\vec{q}.\vec{r}} \tilde{\rho}(\vec{r},\vec{k})\, d^3\vec{r} \right|^2 \tag{28}$$

Inserting (27) into (28) we find

$$C_2(q) = \left| d^3\vec{r} \int e^{i\vec{q}.\vec{r}} \int f(R)\, dR \frac{1}{R^3\pi^{3/2}} \exp\left(-\frac{r^2}{R^2}\right) \right|^2 \tag{29}$$

Changing the order of integration we have

$$\begin{aligned} C_2(q) &= \left| \int f(R)\, dR \int e^{i\vec{q}.\vec{r}} \frac{1}{R^3\pi^{3/2}} \exp\left(-\frac{r^2}{R^2}\right) d^3\vec{r} \right|^2 \\ &= \left| \int f(R)\, dR \exp\left(-\frac{R^2q^2}{2}\right) \right|^2 \end{aligned} \tag{30}$$

For $f(R)$ vanishing at $R > L$ and scaling as

$$f(R) \sim R^{\frac{\gamma}{2}-1}\Theta(L-R) \tag{31}$$

we obtain again the scaling behaviour of the type given in Eq.(26). Some technical details can be found in Ref.[8].

Previous elementary arguments lead to a point which we consider as rather important. The concept of intermittency has been introduced [1] on the basis on analogy with turbulence, fractals and fluctuations. The simple example discussed by Bialas [8] seems to indicate that the scaling behaviour Eq.(10) observed experimentally,even if explained by the HBT interference,is neccesarily connected with fluctuating size of the source.

The purpose of our discussion above has been to show that this is not the case and that mathematically the same results can be obtained both with fluctuating size of the source and with the source which is not fluctuating but its density is superposition of sources of different size. In the former case the size and shape of the source may be very different for different events, whereas in the latter case the density of sources is roughly the same for all the events.

In e^+e^- and hadronic collisions where strong intermittent behaviour of particle correlations is observed one does not expect that in standard events the transverse dimensions of sources will be very large. On the other hand the longitudinal dimensions may be, and probably are, large because of the space-time evolution of the collision. The large dimension is thus the longitudinal one (along the axis of the collision).

It is rather easy to construct simple examples of the longitudinal density of sources leading to the scaling behaviour of the correlation function. Denoting the longitudinal axis as z and the density of sources as $\tilde{\rho}(z)$ we find for the correlation function

$$C_2(q) = \left| \int_{-L}^{L} \tilde{\rho}(z)\, e^{iqz}\, dz \right|^2 \qquad (32)$$

we assume here that $\tilde{\rho}(z)$ is symmetric $\tilde{\rho}(z) = \tilde{\rho}(-z)$ and that $\tilde{\rho}(z)$ vanishes for $|z| > L$. Taking $\tilde{\rho}(z)$ of the scaling form

$$\tilde{\rho}(z) = A \frac{1}{|z|^{1-\frac{\gamma}{2}}}, \qquad 0 < \gamma < 1 \qquad (33)$$

we have

$$C_2(q) = \left| 4A \int_0^L \frac{1}{|z|^{1-\frac{\gamma}{2}}} \cos(q\,z)\, dz \right|^2 = q^{-\gamma} \left| 4A \int_0^{Lq} \frac{1}{|x|^{1-\frac{\gamma}{2}}} \cos x\, dx \right|^2 \qquad (34)$$

Similarly as in the case of the three–dimensional case considered by Bialas we find the approximate scaling of $C_2(q)$ for $q > 1/L$.

To show, how the integrand in Eq(34) looks like we present in Fig.1 the function $f(a)$ defined as

$$f(a) = \int_0^a \frac{1}{|x|^{1-\frac{\gamma}{2}}} \cos x\, dx \qquad (35)$$

where we set $\gamma = 1/2$.

The preceeding discussion seems to indicate that

a) the model of fluctuating source and the model of quiet source are equivalent, and

b) the density of sources as given by the usual picture of the space-time evolution of the e^+e^- collision might be able to explain, at least to same extend the observed intermittency patterns.

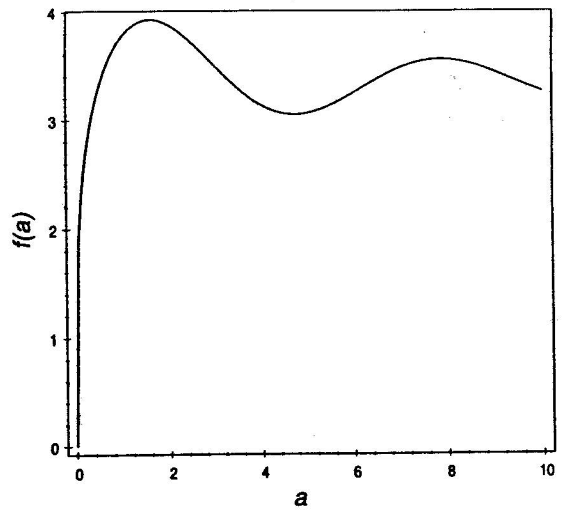

Fig. 1. The function $f(a)$ as given by Eq.(35).

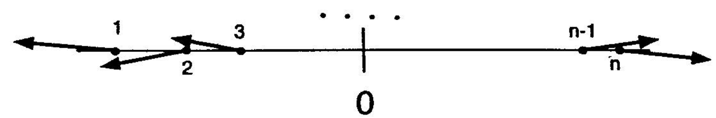

Fig. 2a Schematical illustration of production of charged particles in different positions on the z-axis. Particles with larger values of $|z| = |z_{CMS}|$ have larger velocities.

The statement a) is true in what concerns two–particle correlations. It is most likely not true when considering three– and more–particle correlations and in particular the relationships of these many–particle correlations to two particle ones.

The item b) is more complicated and requires detailed study. As a small step in this direction we shall discuss in the next section a simple model which has the standard space–time evolution and exhibits the intermittent behaviour.

3. Interference of hard bremsstrahlung photons in a model with longitudinal space–time evolution

We shall now study a simple model of HBT interference of hard bremsstrahlung photons. This choice has two reasons. First, the amplitudes for bremsstrahlung emission are well known; second, we have recently studied similar problems and the present model is a natural continuation of our preceeding studies [17, 18]. The term "hard bremsstrahlung" means simply that momenta of photons are large enough to justify the assumptions about the chaoticity of phases usually made in HBT studies. The point is explained in detail in [18]. In our model the photons are radiated by charged final state particles. For simplicity we shall neglect the transverse momenta of final state hadrons. In this

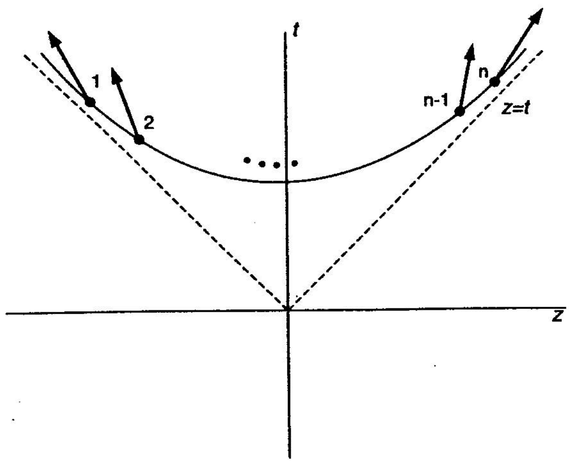

Fig. 2b The same situation as in Fig.2a in t–z diagram. The solid curve corresponds to $t^2 - z^2 = \tau_0^2$ where τ_0 is the formation time of the charged particle in its rest frame.

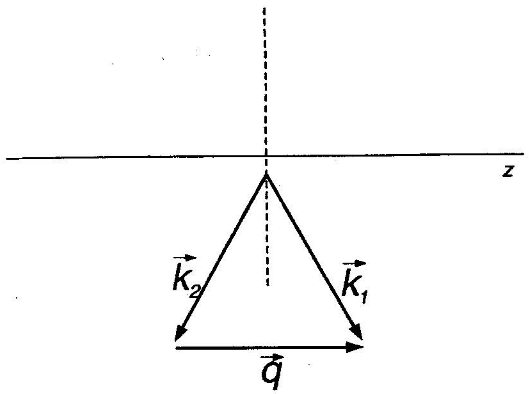

Fig. 3. Kinematical situation used in the calculations.

case each hadron is characterized by its momentum $p \equiv p_z$, energy E and rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \tag{36}$$

In the picture of the space–time evolution we are using, each hadron is emited at the space–time point $x = 0,\ y = 0,\ z,\ t$. The "space–time rapidity" η is defined as

$$\eta = \frac{1}{2} \ln \frac{t + z}{t - z} \tag{37}$$

and the strong correlation between η and y is assumed

$$\eta = y \tag{38}$$

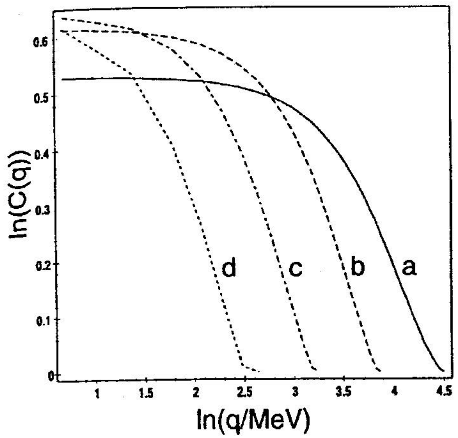

Fig. 4 The correlation function as given by Eq.(44) $C(k_1, k_2)$ plotted as $\ln C(q)$ versus $\ln q$ for

a) $\tau_0 = 1,\ \omega = 100$ MeV, $Y_0 = 1$;
b) $\tau_0 = 1,\ \omega = 100$ MeV, $Y_0 = 2$;
c) $\tau_0 = 1,\ \omega = 100$ MeV, $Y_0 = 3$;
d) $\tau_0 = 2,\ \omega = 100$ MeV, $Y_0 = 3$.

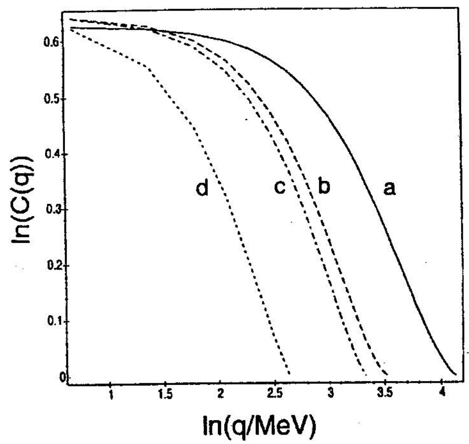

Fig. 5 $\ln C(q)$ given by Eqs.(44) and (46) vs $\ln q$. Notation of curves and values of parameters:

a) $Y_0 = 3,\ \tau_0 = 1,\ \sigma = 1$;
b) $Y_0 = 3,\ \tau_0 = 1,\ \sigma = 2$;
c) $Y_0 = 3,\ \tau_0 = 1,\ \sigma = 3$;
d) $Y_0 = 3,\ \tau_0 = 2,\ \sigma = 3$.

Momentum, energy and velocity of i-th final state hadron are given by standard expressions

$$E_i = m \cosh y_i, \quad p_i = m \sinh y_i, \quad v_i = \frac{p_i}{E_i} = \tanh y_i \tag{39}$$

where $m = m_T$ is the mass of the particle (a pion). The particle is emitted at the space–time point

$$t_i = \tau_0 \cosh \eta_i; \quad z_i = \tau_0 \sinh \eta_i, \quad \frac{z_i}{t_i} = \tanh \eta_i \tag{40}$$

where τ_0 is the parameter characterizing the space–time evolution of the process. The scheme of the calculation is rather simple. We take a set of rapidities of final state particles $(y_1, y_2, \ldots, y_n)$. According to Eq.(39) we calculate their momenta, energies and velocities. Making use of Eq.(38) we determine the space–time positions in which these particles were created.

By such an assumption the bremsstrahlung emitted by these charged particles is completly specified as well as its HBT interference pattern. We shall present here only a simplified version of the scheme of calculation, technical details can be found in Refs.[18, 19].

We consider the situation shown schematically in Figs.2a and 2b. The index i attached to each of produced particles symbolizes its momentum, energy and velocity as given by Eq.(39) as well as the coordinates (t_i, z_i) of its emission point.

We are interested in the correlation function for two hard bremsstrahlung photons

$$\rho(\vec{k}_1, \vec{k}_2) = \frac{P(\vec{k}_1, \vec{k}_2)}{P(\vec{k}_1)P(\vec{k}_2)} = 1 + C_2(\vec{k}_1, \vec{k}_2) \tag{41}$$

Here $P(\vec{k}_1)$ is the probability to detect a single photon with momentum $\vec{k}_1$ and $P(\vec{k}_1, \vec{k}_2)$ is the probability to detect two photons with momenta $\vec{k}_1$ and $\vec{k}_2$. We shall use the notation

$$\vec{q} = \vec{k}_1 - \vec{k}_2, \quad \vec{K} = \frac{\vec{k}_1 + \vec{k}_2}{2}, \quad C_2(\vec{k}_1, \vec{k}_2) = C_2(\vec{q}, \vec{K}) \tag{42}$$

The simplest kinematical situation is shown in Fig.3. The average momentum $\vec{K}$ is perpendicular to the z-axis and lies in the (z,x) plane. Momenta $\vec{k}_1$, $\vec{k}_2$ are also within this plane. The difference of the two momenta $\vec{q}$ is parallel to the z-axis. To make the problem as simple as possible we assume that we have just n charged particles with rapidities

$$y_i = -Y_0 + \frac{1}{2}(i-1), \qquad i = 1, 2, \ldots, n \tag{43}$$

The correlation function $C_2(\vec{q})$ is given by the folloving expression

$$C_2(q) = \frac{\sum'_{i,j} v_i^2 v_j^2 \left[1 - \left(\frac{qv_i}{\omega}\right)^2\right]^{-1} \left[1 - \left(\frac{qv_j}{\omega}\right)^2\right]^{-1} \cos[2q(z_i - z_j)]}{\left\{\sum_i v_i^2 \left(1 - \frac{qv_i}{\omega}\right)^{-2}\right\}\left\{\sum_j v_j^2 \left(1 + \frac{qv_j}{\omega}\right)^{-2}\right\}} \tag{44}$$

The prime in the sum over i, j in the numerator indicates that the term $i = j$ is omitted, $\omega = |\vec{k}_1| = |\vec{k}_2|$ is the common energy of the two photons and v_i, z_i are given by Eqs.(39,40).

The formula looks rather complicated but physics behind it is rather simple. Terms containing $(q\, v_i/\omega)$ or v_i are relativistic factors typical for the emission of bremsstrahlung. The velocities v_i are in fact v_i/c where c is the velocity of light. In the case when $v_i/c \ll 1$ the Eq.(44) simplifies to

$$C_2(q) = \frac{\sum'_{i,j} v_i^2 v_j^2 \cos[2q(z_i - z_j)]}{\{\sum_i v_i^2\}\left\{\sum_j v_j^2\right\}} \tag{45}$$

which corresponds to the HBT interference of identical particles emitted with intensities proportional to v_i^2 in points z_i.

In a more realistic model one should introduce probabilities $P(y_i)$ for the emission of a charged particle with rapidity y_i. The data on multiparticle production suggest parametrization of the form

$$P(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{y^2}{2\sigma^2}\right) \tag{46}$$

where σ increases with the c.m.s. energy of the collision. The corresponding factors $P_i = P(y_i)$ are then inserted into expressions Eq.(44) or Eq.(45).

In Figs.4 and 5 we give results of our calculations of the correlation function $C_2(q)$. The dependence of $\ln C_2(q)$ on $\ln(q/\mathrm{MeV})$ in both Figs.4 and 5. shows a similar qualitative shape. A slow decrease at low values of $\ln(q/\mathrm{MeV})$ is followed by linear decrease, what according to the Eq.(10), this time for the variable q, corresponds to the intermittent behaviour of the correlation function. The onset of this behaviour (the beginning of linear part) starts at value of q_0 which corresponds to the expectation based on the uncertainity principle

$$\frac{\hbar c}{q_0} \sim 2\,\tau_0 \sinh Y_0 \tag{47}$$

plus an effect due to the photon formation length. For instance in the case of curve c) in Fig.4 we have $2\tau_0 \sinh Y_0 \approx 20$ fm, what according to Eq.(47) leads to $q_0 \approx 10$ MeV and $\ln(q_0/\mathrm{MeV}) \approx 2.3$ and around this value the linear part really starts.

In this way our toy model indicates that the intermittency can be, via the HBT mechanism, connected with the "standard" picture of the space–time evolution of a hadronic or e^+e^- collisions, at least for the case of the correlation between hard bremsstrahlung photons.

4. Conjectures on the applicability of this mechanism to the correlations of pions

In models of space–time evolution based on hydrodynamics one usually assumes that the connection between rapidity y of the particle and the space–time rapidity η characterizing the point in which the particle has been emitted is given by the Boltzmann–type factor

$$\frac{dn}{dy} = \exp\left(-\frac{m_T \cosh(\eta - y)}{T}\right) \tag{48}$$

In models based on the leading logarithm approximations in perturbative QCD [20] the gluons are produced in the first stage of the collision by the QCD bremsstrahlung process. In the second stage the gluons hadronize by a process which, being non-perturbative, is less understood. But some bremsstrahlung features may be preserved during the hadronization.

The termal emission and bremsstrahlung amplitudes differ in one important aspect. To see that, consider the amplitude for the bremsstrahlung of the photon with four-momentum k and polarization ε, emitted by a charged particle which has the four-momentum p after the emission (and $p+k$ before that) and the mass M. The amplitude for bremsstrahlung is

$$A(k,\varepsilon;p) \sim \frac{\varepsilon.p}{(p+k)^2 - M^2} = \frac{\varepsilon.p}{E\omega - \vec{p}.\vec{k}} \qquad (49)$$

here E, $\vec{p}$ denote energy and momentum of the charged particle and ω, $\vec{k}$ the same quantities for the photon. Making use of $E = m_t \cosh y_h$, $p_z = m_T \sinh y_h$, $\omega = k_T \cosh y_\gamma$, $k_l = k_T \sinh y_\gamma$ and of $\cosh y_h \cosh y_\gamma - \sinh y_h \sinh y_\gamma = \cosh(y_h - y_\gamma)$ we get

$$A(k,\varepsilon;p) \sim \frac{\varepsilon.p}{m_T k_T \cosh(y_h - y_\gamma) - \vec{p}_T.\vec{k}_T} \qquad (50)$$

where the index h (γ) refers to hadron (photon). The amplitude decreases much more slowly with the difference $(y_h - y_\gamma)$ than the expression in Eq.(48) with $(y-\eta)$. This slow decrease permits to have sizeable interference of bremsstrahlung photons originated by charged particles with a rather large rapidity difference. So far as identical hadrons are originated by bremsstrahlung type mechnism via gluons in perturbative QCD [20] or if, for instance, a part of pions is bremsstrahled by nucleons, the mechanism described in the previous section can apply also to their HBT interference.

Let us note that the amplitude for a bremsstrahlung of a pion by a nucleon is given by the expression

$$\begin{aligned} B(k;p) &\sim \frac{g(p,k)}{(p+k)^2 - M^2} \sim \frac{g(p,k)}{E\omega - \vec{p}.\vec{k} + m_\pi^2} \\ &\sim \frac{g(p,k)}{m_T^N\, m_T^\pi \cosh(y_N - y_\pi) - \vec{p}_T.\vec{k}_T + m_\pi^2} \end{aligned} \qquad (51)$$

with the notation corresponding to the one used in Eq.(49). So far as the term m_π^2 is small with respect to other terms in the denominator the expressions Eq.(50) and (51) are rather similar. The factor $g(p,k)$ in the numerator is the formfactor for the vertex $N \to N + \pi$.

5. Comments and conclusions

In the first part of the present paper we have shown that in what concerns the intermittent behaviour of the two–particle correlation function there are two possible ways how to interpret it via HBT interference. The former is given by the model of the fluctuating size of the source [8] the latter assumes that the source is not fluctuating but is a superposition of densities corresponding to various sizes of the source. Although being equivalent in the case of the scaling of two–particle correlation function, the two interpretations will probably not be equivalent in the relationship of 3–particle to 2–particle correlations. If this conjecture turns out to be true, then the two interpretations might be disentangled by experimental data.

In the second part we have presented a simple model for the HBT interference of bremsstrahlung photons which exhibits the intermittency patterns for the case of a rather standard model of the space–time description of hadronic collisions.

Our model is really only a very simplified toy model and only further, more detailed study might lead to firmer conclusions. Finally we have made a conjecture that due to possible bremsstrahlung features of production of hadrons in e^+e^- and in hadronic collisions, the present toy model could also be relevant for the HBT interferometry of identical hadrons. If this conjecture would turn out to be true, the intermittency analysis of experimental data would be an extremely useful tool for gaining information on the space–time evolution of hadronic collisions.

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