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PHASE SPACE REPRESENTATION OF SCHRÖDINGER-CAT-LIKE STATES IN NEUTRON INTERFEROMETRY[1]

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Recently the Wigner function formalism has been applied to the superposition of two coherent beams in a three-plate perfect crystal neutron interferometer of the Laue type. Here we describe the beam superpositions in a four-plate neutron interferometer by means of Wigner's quasi-classical phase-space representation method. We consider the case of equal distances between the plates having equal thickness. Three independent phase shifts cause intensity oscillations at the rear surface of the interferometer as it was observed experimentally. We are dealing with the forward beam behind the interferometer which is a superposition of three partial beams each of them being twice transmitted and twice diffracted. The three partial beams are described by wave packets. The Wigner function of the superposition state is constructed, it consists of three single Wigner functions due to the three partial beams and of three interference terms. By integrating alternatively over the space and momentum coordinate the spectra of momentum and position are obtained respectively. The position spectra exhibit typical Schroedinger-cat-like states whereas the momentum spectra reflect a strong oscillatory behaviour. Using these spectra one can calculate the mean square deviation values of position and momentum which exhibit squeezing effects depending on various phase shifting. It can be shown that in a four-plate interferometer squeezing can be almost twice as big as in the three-plate interferometer. The smallest value of the mean square deviation of momentum can thus be nearly a quarter of the coherent state value of a single minimum wave packet. These highly non-classical states are made by the power of quantum mechanical superposition principle.

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1. Introduction

In recent papers [1-3] the Wigner function has been introduced in neutron interferometry in order to discuss the spectral distributions of momentum and position of the neutrons depending on the action of a phaseshifter. In such a three-plate Laue-type interferometer the quantum mechanical superposition of two wave packets has been considered. It could be shown that squeezed states are possible, leading to mean square deviation values of the momentum distribution which are considerably below the coherent state value of a single minimum wave packet. In 1988 the four-plate neutron interferometer has been discussed theoretically and experimentally [4]. Here we refer to that paper. We want to investigate the momentum and position spectra of this device using the phase-space representation of the Wigner formalism as well.

2. Four-plate neutron interferometer; notations

In Fig. 1 a sketch of the four-plate interferometer is shown. We consider the forward beam B0 which is the only one built up by the superposition of three wavefunctions of equal amplitudes in an empty interferometer. This is the most interesting case because a 100% contrast can be attained.

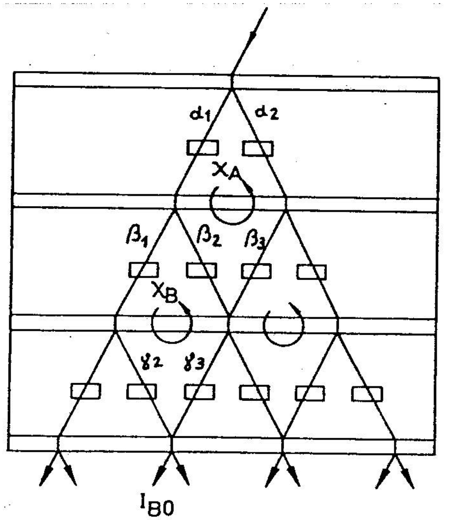

Fig. 1. Four-plate neutron interferometer; α_1 to γ_3 are phaseshifters; χ_A and χ_B are phaseshift loops (see text); I_{B0} is the intensity of the forward beam.

Two loops of phaseshifting can be distinguished: $\chi_A = \alpha_1 + \beta_2 - \beta_3 - \alpha_2$ and $\chi_B = \beta_1 + \gamma_2 - \gamma_3 - \beta_2$. Phaseshift α_1 for example can be expressed by $\alpha_1 = k_0\Delta_{\alpha_1}$. The quantity k_0 ($=2\pi/\lambda_0$, λ_0 = mean wavelength) denotes the mean wave number and Δ_{α_1} can be expressed by $\Delta_{\alpha_1} = (2\pi/k_0)(D_{P_{\alpha_1}}/D_{\lambda_0})$. $D_{P_{\alpha_1}}$ is the thickness of the phaseshifter α_1 and D_{λ_0} is the so called lambda thickness ($D_{\lambda_0} = k_0/(Nb_c)$, N = particle density and b_c = coherent neutron scattering length). Δ_{α_1} can be interpreted as the spatial shift of the wave train in relation to the reference beam.

The four-plate interferometer produces three distinct beams I, II and III composed of the following spatial shifts: $\Delta_I = \Delta_{\alpha_1} + \Delta_{\beta_1} + \Delta_{\gamma_2}$, $\Delta_{II} = \Delta_{\alpha_1} + \Delta_{\beta_2} + \Delta_{\gamma_3}$ and $\Delta_{III} = \Delta_{\alpha_2} + \Delta_{\beta_3} + \Delta_{\gamma_3}$ or $(\Delta_I - \Delta_{II})k_0 = \chi_B$, $(\Delta_I - \Delta_{III})k_0 = (\chi_A + \chi_B) = \chi_{AB}$ and $(\Delta_{II} - \Delta_{III})k_0 = \chi_A$. Furthermore interference order parameters are defined through $m1 = \Delta_I/\lambda_0$, $m2 = \Delta_{II}/\lambda_0$ and $m3 = \Delta_{III}/\lambda_0$.

The wave function $\psi(x,t)$ is described by using the concept of the wave packet:

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \alpha(k) \exp[i(kx - \omega t)]dk \tag{1}$$

The frequency $\omega = \hbar k^2/(2m_N)$ and m_N is the mass of the neutron. The normalized Gaussian momentum spectrum $\alpha(k)$ (peaking at $k = k_0$) is given by (δk = mean square deviation)

$$\alpha(k) = \left[\frac{1}{2\pi(\delta k)^2}\right]^{1/4} \exp\left[-\left(\frac{k-k_0}{2(\delta k)}\right)^2\right] \tag{2}$$

3. Superposition and Wigner function

In direction B0 the superposition of the three wave packets, each of them being twice transmitted and twice diffracted, is given at the rear surface by

$$\psi_{B0}(x,t,\Delta_I,\Delta_{II},\Delta_{III}) = \psi(x+\Delta_I,t) + \psi(x+\Delta_{II},t) + \psi(x+\Delta_{III},t) \tag{3}$$

The momentum spectrum α_{B0} is obtained by the inverse Fourier transformation of Eq.(3) using Eqs.(1) and (2):

$$\alpha_{B0}(k,\Delta_I,\Delta_{II},\Delta_{III}) = \alpha(k)[\exp(ik\Delta_I) + \exp(ik\Delta_{II}) + \exp(ik\Delta_{III})] \tag{4}$$

The calculation of distribution functions of momentum and position (see section 4) is conveniently performed in a formalism using the Wigner function W, with specific emphasis on Gaussian wave packets [5]. The Wigner function itself depends both on x and k. In a four-plate interferometer (pure states) $W_{B0} = W_{B0}(k,x,t,\Delta_I,\Delta_{II},\Delta_{III})$ and can be written in the following manner:

$$W_{B0} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx'} \psi^*_{B0}(x+x'/2,\ldots)\psi_{B0}(x-x'/2,\ldots)dx' \tag{5}$$

After insertion of Eq.(3) the result is

$$W_{B0} = W_1 + W_2 + W_3 + 2[\cos(K\chi_B)W_{12} + \cos(K\chi_{AB})W_{13} + \cos(K\chi_A)W_{23}] \quad (6)$$

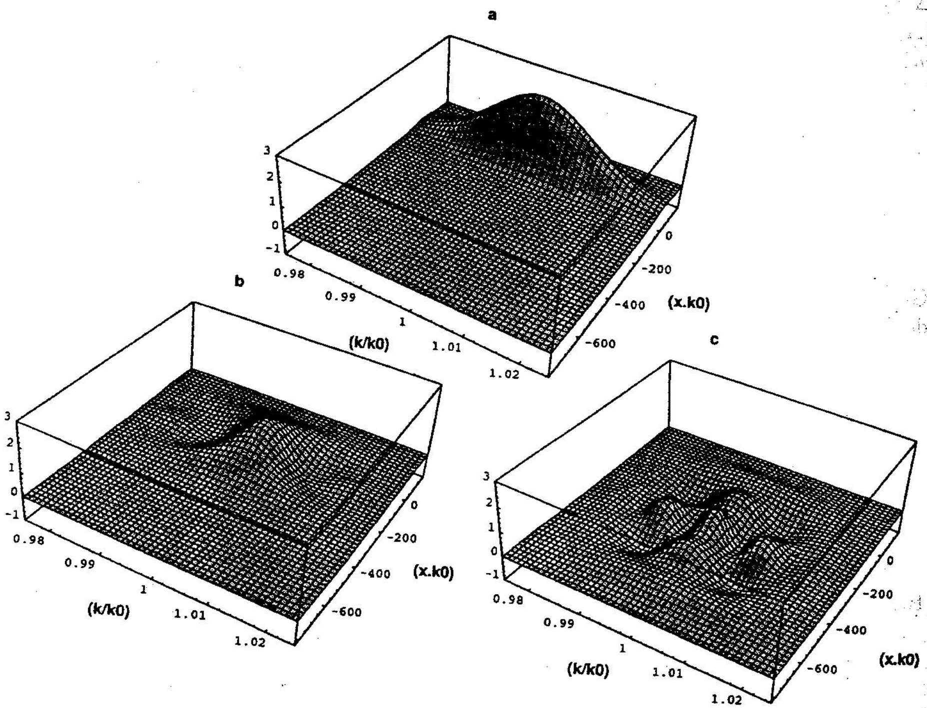

Fig. 2. Wigner functions for a) $m1 = m2 = m3 = 0$, b) $m1 = 0, m2 = 25, m3 = 50$, c) $m1 = 0, m2 = 50, m3 = 100$.

The following abbreviations have been used: $W_1 = W(k, x+\Delta_I, t)$, $W_2 = W(k, x+\Delta_{II}, t)$, $W_3 = W(k, x+\Delta_{III}, t)$, $K = k/k_0$, $W_{12} = W(k, x+(\Delta_I+\Delta_{II})/2, t]$, $W_{13} = W(k, x+(\Delta_I+\Delta_{III})/2, t]$ and $W_{23} = W(k, x+(\Delta_{II}+\Delta_{III})/2, t]$. The Wigner function of a single wave packet is defined by ($v = \hbar k/m_N$ is the group velocity of the neutrons)

$$W(k,x,t) = \frac{1}{\pi}\exp\left[-\frac{(k-k_0)^2}{2(\delta k)^2} - 2(\delta k)^2(x-vt)^2\right] \quad (7)$$

In Figs. 2a-2c Eq.(6) has been visualized for time $t = 0$ using different interference order parameters $m1$, $m2$, $m3$ and $\sigma = (\delta k/k_0) = 1\%$.

4. Distribution functions and squeezing

The distribution functions and the intensity can be expressed through the Wigner function. The calculation of the momentum distribution $\|\alpha_{B0}\|^2$ is easily performed by integrating W_{B0} over the space coordinate x. One obtains

$$\|\alpha_{B0}\|^2 = \alpha^2(k)\{3 + 2[\cos(K\chi_B) + \cos(K\chi_{AB}) + \cos(K\chi_A)]\} \quad (8)$$

Consequently the intensity I_{B0} is obtained by integrating W_{B0} over x and k:

$$I_{B0} = 3 + 2[e^{-(\sigma\chi_B)^2/2}\cos(\chi_B) + e^{-(\sigma\chi_{AB})^2/2}\cos(\chi_{AB}) + e^{-(\sigma\chi_A)^2/2}\cos(\chi_A)] \quad (9)$$

The position distribution $\|\psi_{B0}\|^2$ can be obtained by integrating W_{B0} over k. For a four-plate interferometer we are now able to determine the mean square deviation expression $\langle(\Delta k)^2\rangle_{B0} = \langle k^2\rangle_{B0} - (\langle k\rangle_{B0})^2$ in which case $\langle k^i\rangle_{B0} = \int\int k^i W_{B0}dxdk$ and $i = 1, 2$. Finally we have

$$\frac{\langle(\Delta k)^2\rangle_{B0}}{(\delta k)^2} = 1 - \frac{4}{I_{B0}^2}\left(\frac{I_{B0}}{2}\mathcal{A} + \mathcal{B} + 2\mathcal{C}\right) \quad (10)$$

The following abbreviations have been used:
$\mathcal{A} = E_B C_B + E_{AB} C_{AB} + E_A C_A$,
$\mathcal{B} = (E_B S_B)^2 + (E_{AB} S_{AB})^2 + (E_A S_A)^2$,
$\mathcal{C} = E_B E_{AB} S_B S_{AB} + E_B E_A S_B S_A + E_{AB} E_A S_{AB} S_A$,
$E_B = \exp[-(\sigma\chi_B)^2/2]$, $E_{AB} = \exp[-(\sigma\chi_{AB})^2/2]$, $E_A = \exp[-(\sigma\chi_A)^2/2]$,
$C_B = (\sigma\chi_B)^2\cos(\chi_B)$, $C_{AB} = (\sigma\chi_{AB})^2\cos(\chi_{AB})$, $C_A = (\sigma\chi_A)^2\cos(\chi_A)$,
$S_B = (\sigma\chi_B)\sin(\chi_B)$, $S_{AB} = (\sigma\chi_{AB})\sin(\chi_{AB})$, $S_A = (\sigma\chi_A)\sin(\chi_A)$.
In a similar manner the mean square deviation expression of the space coordinate can be determined. In Fig. 3 Eq.(10) is presented graphically (bottom) and compared to the case of the three-plate interferometer (top, [2]).

5. Discussion

The Wigner function Eq.(6) depends strongly on the interference parameters $m1$, $m2$ and $m3$ (Figs. 2a-2c). For $m1 = m2 = m3 = 0$ the three interfering beams have no mutual phaseshifts and consequently the Wigner function is merely a double Gaussian peak as shown in Fig. 2a. In Fig. 2b the three wave packets begin to separate ($m1 = 0, m2 = 25, m3 = 50$) and the Wigner function is considerably squeezed relative to the $K = k/k_0$ - axis. Fig. 2c ($m1 = 0, m2 = 50, m3 = 100$) shows that the three wave packets cause the Wigner function to exhibit a strong oscillatory behaviour and are separated distinctly in space (Schroedinger-cat-like states). Eq.(8), the momentum distribution function which results from the Wigner function, reflects this oscillatory behaviour as well. The squeezing of the Wigner function mentioned above manifests itself in the mean square deviation expression of Eq.(10). In Fig. 3 (bottom) this expression is visualized for the special case $\chi_A = \chi_B = X/\sigma$ and $\sigma = 1\%$. A minimum of about 0.28 at $X = \pi/2$ can be observed which is actually more strongly below the coherent state value of a single minimum wave packet (value 1.00) than that of a three-plate interferometer (minimum = 0.48, Fig. 3 top).

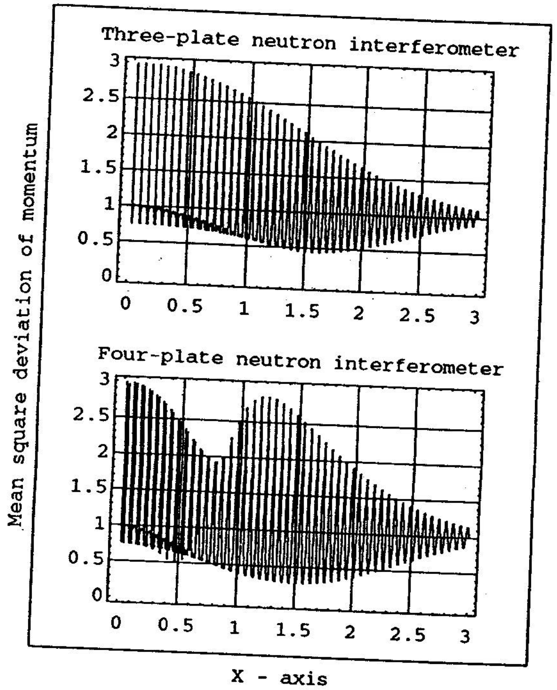

Fig. 3. Comparison of m. squ. dev. of momentum for three and four plates; $\sigma = \delta k/k_0 = 1\%$, $\chi_A = \chi_B = \chi$; maximum squeezing appears at $X = \chi\sigma = \pi/2$.

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