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DECOHERENCE: PHASES WASHED OUT OR SMEARED RECOIL DRIFT[1]

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In an atomic interference experiment loss of coherence of the atomic center–of–mass wavefunction has been brought about by triggering the spontaneous emission of a single photon. This decoherence is analyzed in terms of relative phases being washed out and in terms of the recoil smearing being imposed by the emission. The two points of view are compared.

1. Introduction

When describing an excited atom which is spontaneously emitting a photon one usually considers the atom strongly localized, i.e. point–like compared to the wavelength of the emitted light. Though this assumption is usually adequate in regard to the extension of the electron cloud around the center–of–mass of the atom, it need not be so in regard to the atom's center–of–mass wave function $\psi(\boldsymbol{x}, t)$. Through modern experimental technology it has become possible to prepare atoms whose uncertainty in position considerably exceeds the wavelength of the emitted light [1]–[5]. In two recent experiments it was shown [4, 5] that in this case the emitted light decoheres the spatial correlations of the atomic density matrix $\varrho(\boldsymbol{x}, \boldsymbol{x}')$ describing the atomic center–of–mass wavefunction.

It was shown in a theoretical treatment [6] that the density matrix of an atom undergoing spontaneous emission ($\hat{\varrho}_{\text{emitt}}$) as compared to the density matrix ($\hat{\varrho}$) of an otherwise identically treated atom that has not suffered an emission obeys the following simple product formula[3]

$$\varrho_{\text{emitt}}(\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{r}, t) = \varrho(\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{r}, t) \cdot D(\boldsymbol{r}) . \qquad (1)$$

To assure the normalization condition $\mathrm{Tr}\{\hat{\varrho}\} = 1$ the decoherence function $D(\boldsymbol{r})$ has to obey $D(0) = 1$. To illustrate the physical meaning of this decoherence function in an

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[3] The same product form was derived by Cohen–Tannoudji *et al.* [8] for light scattered on an atom.

atomic interference experiment let us consider an experiment of Young's type. We assume the impinging wavepacket to be "quasimonochromatic", i.e. to have a well-defined de Broglie wavelength. Then, in perfect analogy to classical optics, the "intensity", that is the probability to detect an atom, at a given position in the observation plane, can be written as

$$I \sim \varrho(\boldsymbol{x},\boldsymbol{x}) + \varrho(\boldsymbol{x}+\boldsymbol{r},\boldsymbol{x}+\boldsymbol{r}) + 2\,\mathrm{Re}\left\{\varrho(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r})e^{\mathrm{i}\phi(\tau)}\right\}, \quad (2)$$

where the argument τ in the phase factor denotes the difference of the propagation times from the locations of the two Young's holes at $\boldsymbol{x}$ and $\boldsymbol{x}+\boldsymbol{r}$ in the interference screen to the observation point. It follows from Eq. (2) that the phase of the decoherence factor $D(\boldsymbol{r})$ gives rise to a shift of the interference pattern, whereas its modulus describes a reduction of the visibility [6].

2. Phases Washed Out

Since any decoherence effect and the corresponding wash out of interference patterns can be described in terms of an averaging over random phases [9] it is natural to seek such a description in the case considered here. One finds that the the decoherence function provides just this picture [6] since it is given by

$$D_{[\Delta\Omega]}(\boldsymbol{r}) = \mathrm{const}\int_{[\Delta\Omega]} d^3k\,|\lambda_{\boldsymbol{k}}|^2\frac{e^{\mathrm{i}\boldsymbol{kr}}}{(\omega-\omega_0)^2+\gamma_0^2}\,. \quad (3)$$

It is the Fourier transform of the momentum distribution of the emitted photons. Here $\lambda_{\boldsymbol{k}}$ describes the coupling of the atom to the plane-wave field modes and the denominator $(\omega-\omega_0)^2+\gamma_0^2$ reflects the standard Lorenz lineshape for exponential decay with a time constant γ_0 and angular frequency ω_0. The subscript $\Delta\Omega$ means to restrict the integration of emitted photons to those that are *registered* by a detector covering the solid angle $\Delta\Omega$ (as seen from the atom) thereby selecting a particular subensemble of atoms [6].

The validity of this representation has not only been shown theoretically [6] but also in experiment [4, 5].

To compare the representation of decoherence via the decoherence function with that by recoil drift, we now give a mathematical derivation of both. We implicitely use a recently found solution [10] to the problem of spontaneous emission from an extended atomic wavepacket. This is an extension of the treatment by Weisskopf and Wigner [11] taking the center–of–mass motion into account. All details can be found in [6], an intermediate and somewhat plausible result is

$$\varrho_{\mathrm{emitt}}(\boldsymbol{x},\boldsymbol{x}';t) = \mathrm{const}\int d^3p\int d^3p'\int_{[\Delta\Omega]} d^3k\,|\lambda_{\boldsymbol{k}}|^2 e^{\mathrm{i}(\boldsymbol{px}-\boldsymbol{p}'\boldsymbol{x}')/\hbar} \times\frac{\alpha_0(\boldsymbol{p}+\hbar\boldsymbol{k})\alpha_0^*(\boldsymbol{p}'+\hbar\boldsymbol{k})e^{-\mathrm{i}(p^2-p'^2)t/(2\hbar M)}}{(\omega_j-\omega_0)^2+\gamma_0^2}\,, \quad (4)$$

where $\alpha_t(\boldsymbol{p})$ stands for the Fourier transform of the atomic center–of–mass wavefunction $\psi_t(\boldsymbol{x})$, i.e. the wavefunction in momentum representation. After some more calculation [6] we arrive at the form

$$\varrho_{\mathrm{emitt}}(\boldsymbol{x},\boldsymbol{x}';t) = \mathrm{const}\int_{[\Delta\Omega]} d^3k\,|\lambda_{\boldsymbol{k}}|^2\psi_t(\boldsymbol{x}+\frac{\hbar\boldsymbol{k}}{M}t)\psi_t^*(\boldsymbol{x}'+\frac{\hbar\boldsymbol{k}}{M}t)\frac{e^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x})}}{(\omega-\omega_0)^2+\gamma_0^2}\,, \quad (5)$$

where $\omega = ck$.

We already had mentioned that the wavefunction $\psi_t(\boldsymbol{x})$ has to be quasimonochromatic in terms of the underlying de Broglie-waves. Only in this case a stationary interference pattern with strong contrast is found, clearly a prerequisite for an atomic interference experiment. In this case $\psi_t(\boldsymbol{x})$ can be split into a product of a part $\Xi_t(\boldsymbol{x})$ slowly varying in time and space multiplied with a plane wave $\exp[\mathrm{i}\boldsymbol{Kx}]$. Consequently we get

$$\psi_t(\boldsymbol{x}) = \Xi_t(\boldsymbol{x})\exp[\mathrm{i}\boldsymbol{Kx}]\exp[-\frac{\mathrm{i}t}{\hbar}\frac{\hbar^2K^2}{2M}]\,, \quad (6)$$

where $|\boldsymbol{K}| \equiv 2\pi/\Lambda$ and Λ is the dominant de Broglie-wave length of the atom. Using this quasimonochromaticity assumption and restricting ourselves to the short time interval when the spontaneous emission happens ($t \leq 10\gamma_0^{-1}$) we may set [6] $\Xi_t(\boldsymbol{x}+\hbar\boldsymbol{k}t/M) \approx \Xi_t(\boldsymbol{x})$ and so

$$\begin{aligned}\varrho_{\mathrm{emitt}}(\boldsymbol{x},\boldsymbol{x}';t) &\approx \mathrm{const}\int_{[\Delta\Omega]} d^3k\;\; \Xi_t(\boldsymbol{x})\;\exp[\mathrm{i}\boldsymbol{K}(\boldsymbol{x}+\frac{\hbar\boldsymbol{k}}{M}t)]\;\Xi_t^*(\boldsymbol{x}')\exp[-\mathrm{i}\boldsymbol{K}(\boldsymbol{x}'+\frac{\hbar\boldsymbol{k}}{M}t)] \\ &\quad\times|\lambda_{\boldsymbol{k}}|^2\frac{e^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x})}}{(\omega-\omega_0)^2+\gamma_0^2} &(7)\\ &= \psi_t(\boldsymbol{x})\psi_t^*(\boldsymbol{x}')\cdot\mathrm{const}\int_{[\Delta\Omega]} d^3k\,|\lambda_{\boldsymbol{k}}|^2\frac{e^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x})}}{(\omega-\omega_0)^2+\gamma_0^2} &(8)\\ &\equiv \psi_t(\boldsymbol{x})\psi_t^*(\boldsymbol{x}')\cdot D_{\Delta\Omega}(\boldsymbol{x}'-\boldsymbol{x})\,. &(9)\end{aligned}$$

Due to linearity this can be generalized to the case where we start with a mixed rather than a pure state of the atomic wavefunction. So we arrive at the product form (1) above as well as the result Eq. (3) for $D_{\Delta\Omega}(\boldsymbol{r})$.

3. Smeared Recoil Drift

Alternatively we may drop this slowly varying envelope-approximation in (7) and the restriction to short times $t \leq 10\gamma_0^{-1}$ that goes with it. This allows us to consider the full evolution of the wave packet provided the evolution is *free* – see Eq. (4). This is a rather serious limitation, however, since in some interferometers more gratings follow the spontaneous emission further downstream, see e.g. [5]. In that case the atoms are redirected and our simple description breaks down. Yet, in cases where the atoms are allowed to travel freely from the location of the spontaneous emission to the screen [4], the following description can be used.

We are interested in the interference pattern the atoms form on the screen, that is, we want to know the atom's probability density or 'intensity' $I(\boldsymbol{s})$, where $\boldsymbol{s}$ stands for a screen coordinate. Using Eq. (5) and setting $\boldsymbol{x} = \boldsymbol{x}' = \boldsymbol{s}$ (and inserting the correct flight time T) yields

$$I(\boldsymbol{s},T) = \varrho_{\text{emitt}}(\boldsymbol{s},\boldsymbol{s};T) = \quad (10)$$

$$= \text{const} \int_{[\Delta\Omega]} d^3k \, \frac{|\lambda_{\boldsymbol{k}}|^2}{(\omega-\omega_0)^2+\gamma_0^2} \, \varrho(\boldsymbol{s}+\frac{\hbar \boldsymbol{k}}{M}T, \boldsymbol{s}+\frac{\hbar \boldsymbol{k}}{M}T;T) \, . \quad (11)$$

This is the main result of this paper. It shows that the resulting interference pattern on the screen for the case of spontaneous emission is given by averaging over the original patterns shifted by the recoil due to the emitted photon. It is a simplification and generalization of a recent treatment by Tan and Walls [12].

4. Comparison

The description of decoherence by the decoherence function is more universal [9, 6, 5] than the recoil drift representation, but due to the approximation in Eq. (7) it neglects some details of the atomic motion, namely the shift of the envelope of the interference pattern [5].

On the other hand the recoil formulation reflects all details of the motion but is limited to the case of free motion after the spontaneous emission. Yet it is conceptually so simple that it provides a very clear description and some insight into the decoherence effect.

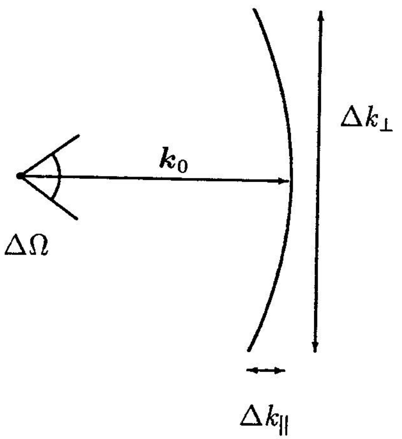

Fig. 1. The transversal uncertainty $\Delta k_\perp$ of the momenta of the emitted photons is much greater than the longitudinal uncertainty $\Delta k_\parallel$. It is assumed the solid angle $\Delta\Omega$ is not too large. The vector k_0 points along the main axis of the detector.

For illustration let us consider the decoherence of a subensemble selected by a detector covering a solid angle $\Delta\Omega$, considerably smaller than 4π. In this case the corresponding decoherence effect is strongly orientation dependent, namely with respect to the detector's main axis the transversal decoherence is much greater than the longitudinal one. In terms of a complementarity argument this is explained by the fact that an optical imaging system has a far better transversal than longitudinal optical resolution [6]. Alternatively the very simple explanation in terms of the recoil drift smearing reads as follows:

For simple geometrical reasons, see Fig. 1.,the momenta of photons emitted into a given solid angle are much more uncertain in the transversal than in the longitudinal direction; the same applies to the recoil drift. Therefore in accord with the decoherence function description the recoil drift smearing preferably washes transversal coherences out.

5. Conclusion

Decoherence of an atomic center–of–mass wavefunction can be described by phases washed out or by smeared recoil drift. The first description is general and more abstract the second one is specialized to free atoms, more precise and very intuitive.

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