acta physica slovaca vol. 46 No. 3, 475 – 479 June 1996

THE PHOTON CHOPPING METHOD FOR QUANTUM STATE MEASUREMENTS[1]

H. Paul[2]
Arbeitsgruppe "Nichtklassische Strahlung" der Max–Planck–Gesellschaft an der Humboldt Universität zu Berlin

P. Törmä[3]
Research Institute for Theoretical Physics, University of Helsinki

T. Kiss[4]
Research Laboratory for Crystal Physics, Hungarian Academy of Sciences

I. Jex[5]
Institute of Physics, SAS and Department of Physics, FNSPE Technical University, Prague

Received 31 May 1996, accepted 7 June 1996

We propose a scheme in which coincidences, measured at the outputs of a passive multiport are used to reconstruct the quantum state of a single mode light field. We show that the coefficients of a finite superposition of Fock states can be extracted from a set of recorded data. Especially the measurement of the photon statistics seems to be feasible with realistic photodetectors.

1. Introduction

Quantum state measurement has become a topic of considerable interest in the last few years for different physical systems, such as molecular vibrations [1], trapped atoms [2] or electromagnetic fields [3-6] . In quantum optics the state reconstruction of both confined [3] and travelling [4-6] light fields have been studied. For propagating light waves the first experimental reconstruction of the Wigner function [4] was carried out by using optical homodyne tomography [5] . The tomographical method seems to be also appropriate for the indirect determination of the photon statistics [6] .

[1] Presented at the **4th central-european workshop on quantum optics**, Budmerice, Slovakia, May 31 - June 3, 1996

[2] E-mail address: paul@photon.fta-berlin.de
[3] E-mail address: Paivi.Torma@csc.fi
[4] E-mail address: ktamas@sparc.core.hu
[5] E-mail address: jex@br.fjfi.cvut.cz

The above mentioned efforts aim to reconstruct the full quantum state of a system from a set of measurements of appropriately chosen quantities. In case of the usual direct photodetection the phase information is lost, moreover, available photodetectors are not sensitive enough to resolve the fine details of the photon statistics. Coincidence detection enhances the amount of information of the measurement. For example the parameters characterising the squeezed light can be determined from direct coincidence measurements [7] .

In the photon chopping method [8] the coincidences are measured at the outputs of a symmetric multiport. A passive multiport [9] is a generalization of the usual beam splitter with N input and output ports built of beam splitters and phase shifters. The symmetric multiport applied in this paper can be effectively constructed from plate beam splitters [10] . We introduce two types of simple photodetectors: type I detectors indicate only the presence of photons while those of type II distinguish the cases of zero, one or more photons arriving.

2. Reconstruction of the full quantum state

Let us consider a signal, being in the superposition of Fock states

$$|\varphi\rangle = \sum_{n=0}^{n_{max}} c_n|n\rangle = \sum_{n=0}^{n_{max}} |c_n|\exp(i\phi_n)|n\rangle. \qquad (1)$$

We will use two setups, the first to measure the photon statistics $|c_n|$ and a second one with a reference beam to get phase information. In the first arrangement a 2N-port is simply fed with the signal through one of the input ports, all the other ports remain unused. The probability of detecting k_i photons at the ith output ($i = 1, 2, \ldots, N$) if the input was a Fock state with n photons follows the statistics of distinguishable particles

$$P_n(k_1, k_2, \ldots, k_N) = \frac{n!}{N^n}\frac{1}{k_1!k_2!\ldots k_N!}. \qquad (2)$$

The outputs are measured in coincidence with type II detectors and thus the probability w_n^N of getting n coincidence clicks is directly related to the photon statistics of the signal:

$$w_n^N = \binom{N}{n}\frac{n!}{N^n}|c_n|^2. \qquad (3)$$

Thus the photon statistics $|c_n|^2$ can be determined if $n_{max} \le N$.

In order to measure the phases ϕ_n the same setup may be used with an added reference beam, which we choose to be in a coherent state $|\alpha\rangle$

$$|\alpha\rangle = \sum_{n=0}^{\infty}\alpha_n|n\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{(|\alpha|\exp(i\varphi))^n}{\sqrt{n!}}|n\rangle \equiv \sum_{n=0}^{\infty}\alpha_n|n\rangle. \qquad (4)$$

In this case the concrete realization of the multiport should also be specified. We will consider a $2N$-port defined with the following recursion

$$\mathrm{U}_N = \frac{1}{\sqrt{2}}\mathrm{U}_2\otimes\mathrm{U}_{N/2} = \begin{pmatrix} \mathrm{U}_{N/2} & \mathrm{U}_{N/2} \\ \mathrm{U}_{N/2} & -\mathrm{U}_{N/2} \end{pmatrix}, \qquad (5)$$

where

$$\mathrm{U}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \qquad (6)$$

Applying the recursion k times one arrives at a multiport with $N = 2^k$ outputs. If the signal enters the first port and the reference beam the port $N/2 + 1$ then the corresponding creation operators transform as

$$\hat{a}_1^\dagger \xrightarrow{\mathrm{U}_N} \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\hat{b}_i^\dagger \qquad (7)$$

$$\hat{a}_{N/2+1}^\dagger \xrightarrow{\mathrm{U}_N} \frac{1}{\sqrt{N}}\left(\sum_{i=1}^{N/2}\hat{b}_i^\dagger - \sum_{j=N/2+1}^{N}\hat{b}_j^\dagger\right). \qquad (8)$$

The probability of getting one-photon signals in coincidence at the detectors $d_1, d_2, \ldots, d_n$ reads

$$w_n^N(d_1, d_2, \ldots, d_n) = \frac{1}{N^n}\left|\sum_{k=0}^{n} f_{kn}(d_1, d_2, \ldots, d_n)\sqrt{(n-k)!n!}\binom{n}{k}c_{n-k}\,\alpha_k\right|^2. \qquad (9)$$

The coefficients $f_{kn}(d_1, d_2, \ldots, d_n)$ are combinatorical factors, depending on how many of the detectors with labels larger than $N/2$ click. For simplicity, keeping only the events, where all the detector labels are below $N/2$, all the coefficients are positive and independent of which detector cklicks. Since the probabilities w_n^N contain the unknown Fock coefficients only up to n, the unknown phases can be extracted systematically. Beginning with $n = 1$ and using that a global phase is free to choose (so e.g. ϕ_0 may be set to zero), there remains only one unknown parameter ϕ_1 to be determined. Since the phases occur in these equation in the form of $\cos(\phi_i - \varphi)$ the remaining ambiguity should be removed by performing a second measurement with a different phase. This process can be continued with $n = 2$ to determine ϕ_2 and so forth, extracting all the phases step by step.

3. Realistic scheme for measuring the photon statistics

The type of detectors discussed in the previous section is available in practice only with lower quantum efficiency (e.g. photomultipliers). Morè realistc are the detectors of type I. Such properties have avalanche photodiods with high quantum efficiency: they indicate only the presence of photons, because they get saturated. Applying such detectors for measuring the photon statistics, the probabilities of Eq. (2) should be summed up to get the probability of m coincidences for an input n-photon Fock state

$$P_{m,n}^N = \frac{n!}{N^n}\sum_{k_1+k_2+\ldots+k_N=n}^{(m)}\frac{1}{k_1!k_2!\ldots k_N!}, \qquad (10)$$

where (m) refers to the summation condition that exactly m of the indices $\{k_i\}$ are nonzero. The summation can be carried out, which yields

$$P_{m,n}^N = \frac{1}{N^n}\binom{N}{m}\sum_{i=0}^{m}(-1)^i\binom{m}{i}(m-i)^n\,, \quad n \geq m\,, \tag{11}$$

while for $n < m$, $P_{m,n}^N$ is zero. The signal photon statistics and the coincidence probabilities are now connected through a linear transform

$$w_m^N = \sum_{n=m}^{n_{max}} P_{m,n}^N |c_n|^2\,. \tag{12}$$

The transformation matrix can be inverted if $n_{max} < N$. In practice demanding a certain precisity also an infinite photon statistics can be measured if truncation at $n = N$ is justified. The inversion of the transformation can be computed using the following recursion

$$(P^N)^{-1}_{n,n+k} = -\frac{1}{P^N_{n+k,n+k}}\sum_{j=0}^{k-1}(P^N)^{-1}_{n,n+j}\,P^N_{n+j,n+k} \tag{13}$$

$$(P^N)^{-1}_{n,n} = 1/P^N_{n,n}\,. \tag{14}$$

The photon statistics is then obtained from the measured coincidence probabilities with the linear combination

$$|c_n|^2 = \sum_{m=n}^{N}(P^N)^{-1}_{n,m}w_m^N\,. \tag{15}$$

4. Discussion

We have shown that the photon chopping method in an ideal case and in the limit of large number of input and output ports allows the reconstruction of the quantum state of a single mode light field. Our simulations show that in case of realistically chosen parameters the measurement of the photon statistics of a weak squeezed coherent signal is possible and the nonclassical oscillations due to the photon pair correlations can be resolved. The considerations about the photon statistics are also valid for mixed states. The reconstruction formula Eq. (15) yields in this case the main diagonals of the density matrix. It is also possible the compensate loss-errors due to detector inefficiencies, applying a generalized inverse Bernoulli transformation [11] .

The presented method is in some sense just complementer to Optical Homodyne Tomography. In the homodyne technique the measured signal is mixed with a intensive, classical beam and a strong photocurrent is recorded, then an inherently continuous transformation is applied, which requires in principle to measure the quadrature phases at all the possible phase angles. In contrast, in the chopping scheme an array of detectors are employed and discrete coincidence events are detected. Neither of the methods can in practice yield a perfect reconstruction, even if no losses and no statistical errors are

assumed, due to the finiteness of the measurement: in the tomographical scheme the continuous variables have to be discretized, while the finite number of ports introduces a cut-off in the photon chopping scheme.

Acknowledgement This work was partly supported by the National Scientific Research Fund (OTKA) of Hungary under contract No. F017381.

References

[1] T.J. Dunn, I.A. Walmsley, S. Mukamel: *Phys. Rev. Lett.* **74** (1995) 884;

[2] S. Wallentowitz, W. Vogel: *Phys. Rev. Lett.* **75** (1995) 2933;

[3] P.J. Bardroff, E. Mayr, W.P. Schleich: *Phys. Rev. A* **51** (1995) 4963; P.J. Bardroff, E. Mayr, W.P. Schleich, P. Domokos, M. Brune, J.M.Raimond, S. Haroche: *Phys. Rev. A* **53** (1996) 2736;

[4] D.T. Smithey, M. Beck, J. Cooper, M.G. Raymer: *Phys. Rev. A* **48** (1993) 3159;

[5] see e.g. U. Leonhardt, M. Munroe, T. Kiss, M.G. Raymer, Th. Richter: *Opt. Commun.* **127** (1996) 144; and references therein;

[6] M.G. Raymer, J. Cooper, H.J. Carmichael, M. Beck, D.T. Smithey: *J. Opt. Soc. Am. B* **12** (1995) 1801; M. Munroe, D Boggavarapu, M.E. Anderson, M.G. Raymer: *Phys. Rev. A* **52** (1995) R924;

[7] J. Janszky, P. Adam, Y. Yushin: *Opt. Commun.* **93** (1992) 191;

[8] H. Paul, P. Törmä, T. Kiss, I. Jex: *Phys. Rev. Lett.* **76** (1996) 2464;

[9] I. Jex, S. Stenholm, A. Zeilinger: *Opt. Commun.* **117** (1995) 95;

[10] P. Törmä, I. Jex: to be published;

[11] T. Kiss, U. Herzog, U. Leonhardt: *Phys. Rev. A* **52** (1995) 2433; U. Herzog: *Phys. Rev. A* **53** (1996) 1245;