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INTERACTION OF TWO-LEVEL ATOMS WITH A SQUEEZED COHERENT FIELD. HERALD REVIVALS[1]

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The Jaynes-Cummings model of a single two-level atom coupled to an initially squeezed coherent field reveals ringing revivals following the main ones as a series of echoes. In the many-atom case a regime exists in which the beginnings of the revivals may have oscillatory envelopes. We refer to this phenomenon as herald revivals.

1. Introduction

The Dicke model (DM) deals with a system of A two-level atoms coupled in a high-Q cavity to a single-mode radiation field. In the rotating wave approximation the total number of excitations N (the number of photons n plus the number of excited atoms) is an integral of motion. If the field is initially in a Fock state with the photon number n and the atomic system in its ground state, then a single subspace with $N=n$ contributes to the evolution. The simplest case, $A=1$ is known as the Jaynes-Cummings model (JCM). The JCM dynamics in a single subspace with a given N is equivalent to spin-1/2 rotation with a quantum Rabi frequency ($\Omega_N \sim \sqrt{N}$ in the on-resonant case). The nonlinearity of the JCM and all the richness of its dynamics comes entirely from this square root dependence of the quantum Rabi frequency on the excitation number. For $A=2$ (N arbitrary) or $N=2$ (A arbitrary) the dynamics of the system is still periodic [1,2]. The case $n=A=3$ is the first in the hierarchy of those having unequidistant spectra inside a subspace with given values of N and A [3-6]. Hence, there is an additional (cooperative) source of the nonlinearity in the Dicke model, leading to

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distinct collective collapses and revivals of the oscillations of the system if $3 \leq n < A$ or $3 \leq A < n$, even if the initial field is in a Fock state. Walls and Barakat [7] showed numerically that there are two limits when the eigenvalue spectrum of such systems may still be treated as approximately equidistant and, consequently, their evolution considered as periodic, at least for not extremely long times. The one case occurs if the number of photons is very much smaller than the number of atoms ($n \ll A$) whereas the other is the opposite of the former ($n \gg A$). For convenience we shall call them "weak" and "strong" field limits, respectively. Obviously, they have nothing in common with the absolute intensity of the field; only the relation between the photon number and the number of atoms of the sample is taken into account in the above definition. If weaker inequalities $n < A$ and $A > n$ are satisfied we may speak of "weak" and "strong-field" domains, correspondingly. Here collective collapses and revivals are very well noticeable [5,6]. The revival (collapse) time of the pure collective revivals (collapses) is strongly related with the parity of n in the weak-field domain, and with the parity of A in the strong-field domain. For instance, for a given A and $n < A$ the revival (collapse) time is almost twice shorter for even n than for the nearest neighbouring odd n [6,8]. In the region of the greatest cooperative nonlinearity, $n \sim A$, collective collapses and revivals are not so well pronounced [9].

If the initial field state consists of many photon number states, then subspaces with different n's contribute to the evolution and the temporal behaviour of the atomic inversion is the sum of the atomic responses to each photon number state weighted with a corresponding photon number distribution P_n. Then, moreover, the nonlinearity of the system related with the quantum discreteness of the field starts to play a role. This is the only source of revivals in the JCM [10]. The most interesting, regular dynamics of the JCM in the form of collapses and revivals of the oscillations occurs when the nonlinearity of the Rabi frequency is slight. In the resonant case, this takes place for large mean numbers $\bar{n}$ of initially coherent photons [10,11]. Due to the nature of the Poissonian distribution, effective summation over the photon number n then begins beyond the interval of small photon numbers, in which the greatest nonlinearity of the Rabi frequency is apparent. If the initial field is in a thermal or squeezed vacuum state, the small photon numbers always contribute to the evolution with greatest weights and the temporal behaviour of the thermal [12] or squeezed vacuum resonant JCM [13] is irregular.

In the vicinities of the limits mentioned earlier, the time behaviour of the DM coupled to the initial field being a superposition of photon number states is very regular and governed by the interference of the various Rabi frequencies of the spread caused solely by the photon number distribution. Contrary to the strong-field limit, the weak-field one offers us regular dynamics for different initial fields [14]. Moreover, in the weak-field limit and domain the oscillation frequency is a decreasing function of n which is a necessary condition for appearance of herald revivals.

2. Time evolution

For an initially squeezed coherent field the JCM reveals ringing revivals following the main ones as a series of echoes [15]. In the model in question, the beginnings of the revivals may have oscillatory envelopes. We will call these revivals, preceding the main ones, herald revivals. We use the approximate analytical solution from Refs. [5] and [6], obtained by means of perturbation theory with the inverse square of the Rabi frequency as a small parameter. To start with, let us consider the weak-field limit. The time evolution of the model is then well described by the zeroth-order approximation for both eigenvalues and eigenvectors. In particular, the time behaviour of the expectation value of the photon number of the system reads [6]

$$\langle n(t) \rangle_{00} = \frac{1}{2}\Big(\bar{n} + \sum_{n=1}^{n \ll A} n\, P_n \cos \Omega_n t\Big)\,, \tag{1}$$

where the Rabi frequency of the oscillations is

$$\Omega_n = 2g\sqrt{A - n/2 + 1/2}\,. \tag{2}$$

The coefficient g describes the atom-field coupling. Every term in Eq. (1) represents the contribution from a subspace with a given photon number. The dynamic response to a single Fock state is completely periodic in this approximation.

Assuming the minor axis of the ellipse, representing the direction of squeezing, parallel to the coordinate of the field oscillator, the photon number distribution for a squeezed state reads (see for instance Refs. [16] and [17])

$$P_n = \frac{\nu^n}{2^n n! \mu^{n+1}} \left| H_n\left(\frac{\beta}{\sqrt{2\mu\nu}}\right)\right|^2 \exp[-|\beta|^2 + \frac{\nu}{\mu}\mathrm{Re}(\beta^2)],$$
$$\mu = \cosh r,\ \nu = \sinh r,\ \beta = \mu\alpha + \nu\alpha^*,\ \alpha = |\alpha| \exp(i\Theta)\,, \tag{3}$$

where H_n denotes the Hermite polynomial. The initial phase Θ of α is the angle between the direction of coherent excitation and the direction of squeezing. The mean photon number of this field is equal to $\bar{n} = |\alpha|^2 + \nu^2$. For $|\alpha| = 0$ the photon number distribution for an initial squeezed vacuum state is recovered. For the DM with the field initially in a squeezed vacuum state revivals appear twice faster than for coherent and thermal fields [14].

Let us expand the dispersion curve Ω_n around the point $\bar{n}$

$$\Omega_n = \Omega_{\bar{n}} + \Omega_{\bar{n}}^{(1)}(n - \bar{n}) + \Omega_{\bar{n}}^{(2)}(n - \bar{n})^2 + \ldots, \qquad \Omega_{\bar{n}}^{(r)} = \frac{1}{k!}\left.\frac{d^k \Omega_n}{dn^k}\right|_{n=\bar{n}}. \tag{4}$$

The first term represents rapid oscillations of the system while the remaining terms are responsible for their envelope. In general, if only the first-order derivative of the above expansion were different from zero, revivals of the oscillations would be perfectly periodic (linear or harmonic approximation) with the revival time

$$T_R = 2\pi/|\Omega_{\bar{n}}^{(1)}| = 2\pi\,\Omega_{\bar{n}}/g^2 = (4\pi/g)\sqrt{A - \bar{n}/2 + 1/2}\,. \tag{5}$$

If higher-order derivatives in Eq. (4) are significant they spread the revivals arising from the linear expansion. In the JCM with an initially squeezed coherent field such nonlinear terms contribute to the emergence of ringing revivals; this is attributed to the oscillatory character of the photon number distribution [15]. As shown [15,18], there is a close correspondence between the number of oscillations in P_n and the number of echoes following the main revival. Every subsequent ringing revival is related with the subsequent peak of the photon number distribution. Since different peaks in P_n have slightly different local mean photon numbers, slightly different revival periods appear [15,18]. The situation is quite similar here. In our case, however, the Rabi frequency is a decreasing function of n contrary to that for the JCM. Hence, the revival concerned with the farthest (smallest) local peak of P_n has the shortest revival time and appears first. In this way one can expect that the herald revivals preceding the main ones are formed.

In Fig. 1 the time evolution of the mean photon number (1) is plotted for $A=1000$ and $\nu=2, |\alpha|^2=15, \Theta=0$. Although five revivals are presented, only a slight decrease in the revivals maxima is noted. In other words, the harmonic (linear) approximation for the frequency works very well within this time interval and the herald revivals are absent. The approximate analytical expression for the series (1) and its envelope was found by us in the linear approximation for an initially squeezed coherent field in Ref. [14]. The ratio of the second- and first-order terms in Eq. (4) $\rho \approx (n-\bar{n})/4A$ is small within the width of any photon number distribution with $\bar{n} \ll A$. For a squeezed state with $\Theta=0$ the width of the photon number distribution is characterized by $\Delta n = [\bar{n}\exp(2r) + \sinh^2 r\,(1+\sinh 2r)]^{1/2}$. For $|\alpha|^2 = 15$ and $\nu=2$ this width amounts to ≈ 19.51. Hence, at $A=1000$ the ratio ρ is approximately equal to $4.9 * 10^{-3}$. The herald revivals would appear if at least the quadratic term in the expansion (4) were important. One can expect that the effect of the nonlinearity of the Rabi frequency on the time behaviour of the model becomes noticeable in long times. However, since the eigenvalue spectrum is only approximately equidistant the validity of the formula (1) is restricted to not very long times. In long times even if the relation $\bar{n} \ll A$ is satisfied the anharmonicity of the system contributes to the evolution and instead of the formula (1) we have to use higher-order approximations for the eigenvalues and eigenvectors. In particular, in the second-order approximation for the eigenvalues and in the first-order approximation for the eigenvectors (subscript *21*), the mean photon number is found to evolve according to the formula

$$\langle n(t)\rangle_{21} = \frac{\bar{n}}{2} - \sum_{n=1} P_n \left\{ \frac{\epsilon n(n-1)}{16} - \sum_{p=1}^{n} C_p^n \cos \Omega^{(p)} t \right. \\ \left. - \frac{\epsilon}{8} \sum_{p=1}^{n} C_p^n \left\{ [(n-2p)^2 - 2p + 1] \cos \Omega^{(p)} t + 2(p-1) \cos \tilde{\Omega}^{(p)} t \right\} \right\}, \quad (6)$$

where $\epsilon = 4g^2/\Omega_n^2$. The principal term corresponds to the zeroth-order approximation for the eigenvectors but contains the second-order Dicke frequency

$$\Omega^{(p)} = \Omega_n \left\{ 1 + \frac{3\epsilon^2}{16} [5(p-1)(p-n) + (n-1)(n-2)] \right\}. \quad (7)$$

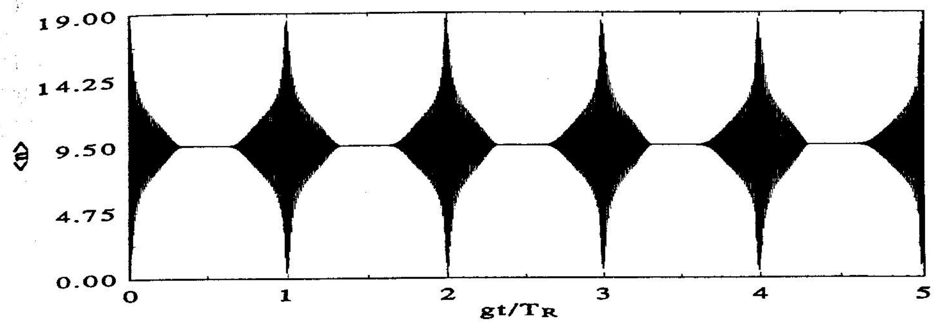

Fig .1. Time evolution of the expectation value of the photon number for the DM with $A=1000$, $|\alpha|^2 = 15$, $\nu = 2$, and $\Theta = 0$ (approximation *00*). The time is scaled relative to the mean revival period calculated from Eq. (5) for the mean photon number $\bar{n} = 19$.

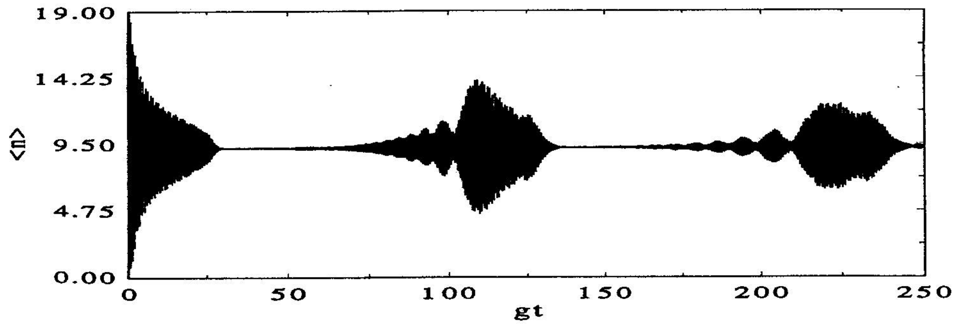

Fig .2. Expectation value of the photon number versus gt for the DM with $A=100$, $|\alpha|^2 = 15$, $\nu=2$, and $\Theta=0$ (approximation *21*).

In turn, the frequency $\tilde{\Omega}^{(p)}$ has the form

$$\tilde{\Omega}^{(p)} = 2\Omega_n \left\{ 1 + \frac{3\epsilon^2}{32} [10(p-1)(p-n-1) + 2n^2 - n + 4)] \right\}. \quad (8)$$

C_p^n is the binomial distribution multiplied by p.

To observe the herald revivals earlier, one should enhance the nonlinearity of the frequency (2) taking the ratio $\bar{n}/A$ not so small as that assumed in Fig. 1. Obviously, the anharmonic corrections to the spectrum are then important already in the early stage of the evolution.

In Fig. 2 the time evolution of the mean photon number (6) is presented. Already the first revival reveals a ringing structure. An exact analytical evaluation of the sums (6) is problematic. Nevertheless, this contribution is a first step to further studies of the Dicke model coupled to an initially squeezed coherent state.

The DM exhibits in the weak-field limit a well pronounced sequence of collapses and revivals for different quantum fields [14]. The origin of the herald revivals, similarly to the ringing revivals in the JCM [15,18], is related with the oscillatory nature of the photon number distribution. But they are also intimately related with the nonlinearity of the frequency. The herald revivals cannot be observed in the weak-field limit since the nonlinearity of the Rabi frequency is then too slight. However, they can appear in the weak-field domain. Obviously, nontrivial corrections to the motion inside each subspace with a given n then play a role and modify the shape of the revivals connected with the photon mechanism. To conclude, one has to apply the golden mean; the nonlinearity of the oscillation frequency should be slight (to observe revivals at all) but not too slight (to observe their ringing structure).

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