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SPONTANEOUS EMISSION IN FINITE PHOTONIC CRYSTAL

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We investigate basic features of a simplified model of a spontaneous emission from an excited level of a two-level atom inside a finite one-dimensional photonic band crystal bounded by a pair of perfectly conducting mirrors.

1. Introduction

Photonic band structures (PBS) make possible demonstration of quite unusual properties of an EM field and an interaction of such a field with matter. These properties are quite interesting itself and furthermore indicate to be very useful in potential applications. This is the reason of the strong theoretical and experimental interest in this field during last ten years [1,2,3,4,5].

The most striking feature of PBS is an expressive modification of the density of modes (DOM) of the EM field in the PBS in comparison with a field in a usual material. This density on the one hand tends to zero in the region of gaps and on the other hand it becomes very large near the edges of gaps. This feature (with purely classical origin) concerns of the macroscopic EM field and can be deduced from the dispersion relation connecting the wave vector and the angular frequency of the EM wave. The modification of the DOM entails unusual features in quantum optics of the light interacting with atoms in the PBS. The most instructive example is a spontaneous emission from a discrete atomic level. According to Fermi's golden rule

$$w_{fi} = \frac{2\pi}{\hbar^2}|\langle f|V|i\rangle|^2\rho(\omega_{fi}) \tag{1.1}$$

the spontaneous emission rate w_{fi} is directly proportional to the DOM. So by modifying of DOM the spontaneous emission can correspondingly be inhibited or enhanced [6].

The simplest model of a PBS was proposed by John and Wang [7]. It is an analogy with the one-dimensional Kronig-Penney model familiar in the solid-state physics. In its

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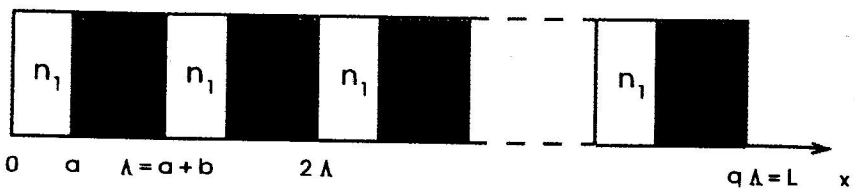

Fig. 1. The scheme of the considered PBC.

physical essence describes a device working on the same principle as photon band-pass filters used in optics. Although this one-dimensional simple model does not consider all aspects of the vector nature of the EM field, it is frequently used in theoretical analysis [8,9] because of its relative simplicity and possibility to obtain analytical expressions. The analysis based on this model makes possible qualitative understanding of the nature of propagating of an EM field in real PBS.

In the present paper we investigate several basic features of quantum optics of light inside the finite one-dimensional photonic band crystal (PBC) bounded by a pair of perfectly conducting mirrors. We investigate populations of levels of a single two-level atom enclosed in the PBC and spectral properties of a radiation emitted by the atom. Although this model does not expresses quantitatively real situations, it reveals the essence of physical processes under consideration.

In our quantum calculations we use the method of numerical diagonalization of the considered Hamiltonian of the system. We then develop a state-vector of the system in the basis obtained in this way.

2. Formulation of the problem

Let us consider the one-dimensional crystal according to the Kronig-Penney model except of the finite length. The crystal is composed of a certain number of identical periodically placed layers - lattice periods (see Fig. 1). Each period contains two sublayers with generally different thicknesses a and b and refractive indices n_1 and n_2, respectively. The refractive indices inside each sublayer are constant. We denote the lattice constant by Λ, i.e.

$$\Lambda = a + b \ . \tag{2.1}$$

The length of the crystal will be referred to as L, i.e.

$$L = q\Lambda \ , \tag{2.2}$$

where q is a positive integer number equal to the number of lattice periods of the crystal. We consider modes of a standing macroscopic EM field in the crystal. There is several possible branches allowed by the dispersion relation in the PBC. We choose the branches which correspond to a dispersion relation given by an increasing dependence $\omega = \omega(k)$. This dependence is a function except of points on the edges of the gaps. We

then formally quantize the field and investigate its interaction with a two-level atom enclosed in the structure. We investigate the simplest nontrivial case when the atom is initially in its excited level and the field is initially in a vacuum state. Numerically calculated probabilities of the atomic excitation and spectral properties of the radiated field are presented.

3. Free classical macroscopic field in the finite photonic band crystal

To obtain a formula for the classical electric field, it is sufficient to use the results obtained for the infinite crystal with the same structure [3]. So when we have a formula for the electric field in the infinite crystal, we add to this formula another one with conjugate space-dependent amplitudes. The resulting field corresponds to the standing-wave field in the finite crystal bounded by the perfectly reflecting mirrors. For simplicity we firstly consider only a single-mode field.

According to [3], we can write for the field in the infinite crystal in the j-th layer and ρ-th sublayer ($j = 1, 2, ..., q$; $\rho = 1, 2$)

$$E_{j\rho}(x,t) = a_{j\rho}(x)e^{-i\omega t} + a^*_{j\rho}(x)e^{i\omega t} \ , \tag{3.1}$$

where

$$a_{j\rho}(x) = F_{j\rho}e^{i\kappa_\rho x} + G_{j\rho}e^{-i\kappa_\rho x} \ , \tag{3.2}$$

and

$$\kappa_\rho = \frac{n_\rho \omega}{c} \ . \tag{3.3}$$

The complex coefficients $F_{j\rho}$ and $G_{j\rho}$ are related by the relations

$$F_{j\rho} = e^{i(j-1)(k-\kappa_\rho)\Lambda}F_{1\rho} \ , \tag{3.4a}$$

$$G_{j\rho} = e^{i(j-1)(k+\kappa_\rho)\Lambda}G_{1\rho} \ , \tag{3.4b}$$

where k is the wave number. These relations are implied by the Floquet-Bloch theorem. The coefficients $F_{1\rho}$ and $G_{1\rho}$ are then obtained with the use of the boundary conditions. The results can be written (up to a complex constant) in the following form.

$$F_{11} = 2(1-\zeta)\delta^*(\beta^*\delta - \alpha\gamma) \ , \tag{3.5a}$$

$$F_{12} = \alpha(1-\zeta)\delta^*(1+\gamma)(\beta^* - \beta) \ , \tag{3.5b}$$

$$G_{11} = -2(1+\zeta)\delta^*(\beta\delta - \alpha\gamma) \ , \tag{3.5c}$$

$$G_{12} = -\alpha[\delta(1-\zeta)(1+\gamma)(\beta^* - \beta) + 2(1+\zeta)(\beta\delta - \alpha\gamma)] \ . \tag{3.5d}$$

Symbols used in the above formulas read

$$\alpha = e^{ik\Lambda} \ , \qquad \beta = e^{i\kappa_1 a} \ , \qquad \gamma = e^{i\kappa_2 a} \ , \qquad \delta = e^{i\kappa_2 \Lambda} \ , \qquad \zeta = \frac{\kappa_1}{\kappa_2} \ . \tag{3.6}$$

With the help of Eqs. (3.1) and (3.2) we obtain a formula for the standing electric field

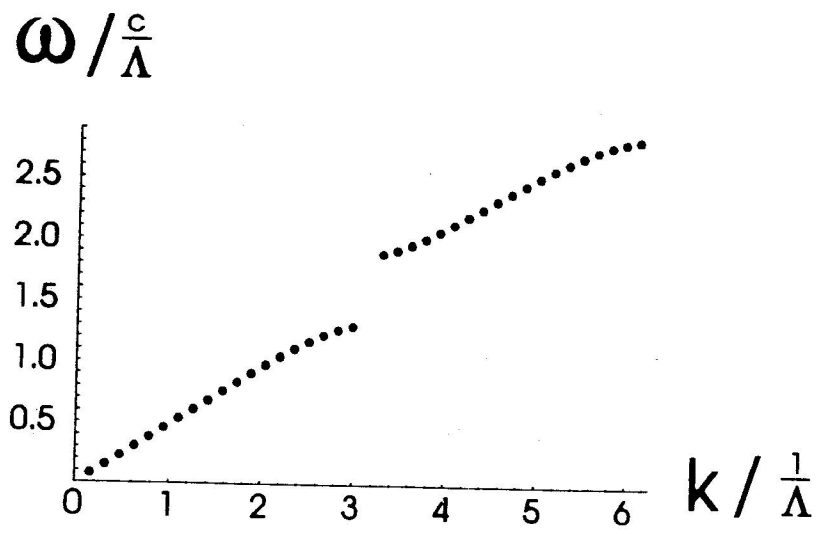

Fig. 2. The dispersion relation in the considered one-dimensional finite PBC. The crystal parameters are $a = 0.4$, $b = 0.6$, $q = 20$, $n_1 = 1$, $n_2 = 2.5$. The modes on the edges of the gaps are not depicted since the corresponding field is zero.

in the finite crystal of the length $L = q\Lambda$.

$$E_{j\rho}(x,t) = 2[(F_{j\rho} + G^*_{j\rho})e^{i\kappa_\rho x} + (F^*_{j\rho} + G_{j\rho})e^{-i\kappa_\rho x}] \cos\omega t$$
$$= 4|F_{j\rho} + G^*_{j\rho}| \sin(\kappa_\rho x + \phi_{j\rho}) \cos\omega t \ , \qquad (3.7)$$

where

$$\phi_{j\rho} = \arg(F_{j\rho} + G^*_{j\rho}) + \frac{\pi}{2} \ . \qquad (3.8)$$

The boundary condition

$$E(0,t) = 0 \qquad (3.9)$$

provides us with the appropriate phase factors of F and G. Eq. (3.7) is valid for the modes corresponding to the wavevectors

$$k_m = \frac{\pi}{L} m \ , \qquad (3.10)$$

where m is a positive integer number. The frequencies corresponding to the modes k_m will be referred by ω_m and are obtained from the dispersion relation

$$\cos k\Lambda = \cos\frac{an_1\omega}{c} \cos\frac{bn_2\omega}{c} - \frac{n_1^2 + n_2^2}{2n_1n_2} \sin\frac{an_1\omega}{c} \sin\frac{bn_2\omega}{c} \ . \qquad (3.11)$$

The existence of the PBG is given directly by this relation. Fig. 2 shows the part of the considered branch of the dispersion relation, i.e. also the mode structure in the depicted region. We see that the DOM is finite even near the band edges. The modes in Fig. 2 start with the first one having $k = \pi/L$. The modes exactly on the edges of the gap have equal wavenumbers $m\pi/\Lambda$ where m is in general any positive integer. [see the dispersion relation (3.11)]. We do not consider and do not plot these modes because the corresponding electric field has zero amplitude along all the crystal [see Eq. (3.7). The gaps are appearing quasiperiodically in the mode structure at $k = m\pi/\Lambda$ for generally each positive integer m.

4. Quantized field

In this section we write a quantum operator of the macroscopic classical electric field (3.8). Firstly we deal with a single-mode field to simplify notation. The classical electric field (3.8) can be written without subscripts j and ρ, providing that the amplitude $|F + G^*|$, κ, ρ and ϕ depend on x. Thus we have

$$E(x,t) = 4|F + G^*|(x) \sin[\kappa(x)x + \phi(x)] \cos\omega t \ . \qquad (4.1)$$

We define the corresponding "phenomenological" QM operator in the Schrödinger picture as

$$\hat{E}(x,t) = E_0 f(x)(\hat{a} + \hat{a}^\dagger) \ , \qquad (4.2)$$

where

$$E_0 = \sqrt{\frac{\hbar\omega}{\bar{\epsilon}L}} \ , \qquad (4.3)$$

and

$$f(x) = |F + G^*|(x) \sin[\kappa(x)x + \phi(x)] \ ; \qquad (4.4)$$

$\bar{\epsilon}$ is the averaged dielectric permitivity, i.e.

$$\bar{\epsilon} = \frac{1}{\Lambda}(an_1^2 + bn_2^2) \ . \qquad (4.5)$$

The operator (4.2) should be correctly normalized. The normalization (of F and G) must be chosen so that the condition

$$\langle 0| \int_0^L \epsilon(x)\hat{E}(x)^2 dx \ |0\rangle = \frac{\hbar\omega}{2} \qquad (4.6)$$

for the mean value of the vacuum-field energy is satisfied (for each field mode). In the l.h.s. of Eq. (4.6) a contribution of waves propagating in the unit-section part of the crystal is taken into account. The symbol $|0\rangle$ means the ground state of the harmonic oscillator (the field mode). For the case of a multimode field the electric field operator is generalized as

$$\hat{E}(x) = \sum_\omega E_\omega f_\omega(x)(\hat{a}_\omega + \hat{a}^\dagger_\omega) \ , \qquad (4.7)$$

where E_ω, $f_\omega(x)$ and $\hat{a}_\omega$ are analogies of E_0, $f(x)$ and $\hat{a}$.

5. Hamiltonian

We choose an interaction of the electric dipole type in the rotating-wave approximation between the two-level atom and the EM field. The two level atom is characterized by the Hamiltonian $\hat{H}_A$ with two eigenstates $|g\rangle$ and $|e\rangle$ (ground, excited). The energies of the two levels are 0 and $\hbar\omega_A$. The system Hamiltonian in the Schrödinger picture reads

$$\hat{H} = \frac{\hbar\omega_A}{2}(1+\hat{S}_z) + \sum_\omega \hbar\omega \hat{a}^\dagger_\omega \hat{a}_\omega + \sum_\omega \hbar G^{(\mathrm{eff})}_\omega(x)(\hat{S}_+\hat{a}_\omega + \hat{S}_-\hat{a}^\dagger_\omega) \ , \tag{5.1}$$

where

$$\hat{S}_z = |e\rangle\langle e| - |g\rangle\langle g| \tag{5.2}$$

is the atomic inversion operator, $\hat{S}_+$ and $\hat{S}_-$ are the atomic raising and lowering operators, respectively. $G^{(\mathrm{eff})}_\omega(x)$ is an effective coupling constant between the two-level system and the field mode ω,

$$\hbar G^{(\mathrm{eff})}_\omega(x) = e\sqrt{\frac{\hbar\omega}{\bar{\epsilon}L}}|D_{ge}|\, f(x) \ , \tag{5.3}$$

where eD_{ge} is the matrix element of the electric-dipole operator.

Having the Hamiltonian (5.1) we can start to solve the given problem of the spontaneous emission of the two-level atom. The initial state of the system to be such that the atom is in its excited level and the field is in the vacuum, i.e.

$$|\Psi(0)\rangle = |e;0\rangle \ . \tag{5.4}$$

An important complete basis of states for the problem under consideration is the basis of the unperturbed Hamiltonian (first two terms of (5.1))

$$\{|e;0\rangle, |g;1_1\rangle, |g;1_2\rangle, ...\} \ . \tag{5.5}$$

In our calculations we write the perturbed Hamiltonian (5.1) in the basis (5.5). Then we numerically calculate eigenvalues and eigenvectors of this Hamiltonian. In this procedure we truncate the infinite Hilbert space (5.5) to a finite dimension by taking a finite number of the field eigenmodes.[2] Then we develop the time-dependent state of the system in the given basis taking into account the initial condition (5.4). Having evaluated the state vector of the complete system in a time t we can use it to calculate excitation probabilities of the atom and the radiation. In the remaining sections we present and discuss some of the obtained results.

6. Spontaneous emission of the two-level system in the finite PBC.

It is seen from Fig. 2 and from the formula (5.3) that the character of the investigated system yields two basic unusual properties of the system in comparison with the infinite

[2] We use 95 field eigenmodes, i.e. first 5 allowed bands, the last mode having $k_{99} = 99\pi/L$.

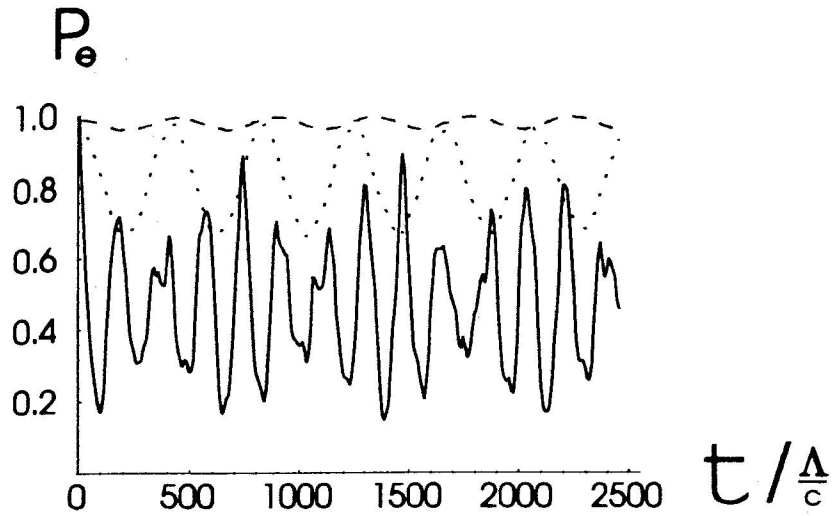

Fig. 3. The time-dependence of the atomic excited-state population for the crystal parameters as in Fig. 1, the atomic parameters are $x_A = \Lambda/10$, $\omega_A = \omega_{10}, \omega_{19}, \omega_{30}$ (solid, dashed and dotted, respectively), $D_{ge} = 10^8\Lambda$.

crystal case. The first feature is the discreetness of the field eigenmodes. The second feature is the position dependence of the modulus of the effective coupling "constant" $G^{(\mathrm{eff})}_\omega(x)$ given by (5.3). This means that by modifying the position of the atom and its transition eigenfrequency we can observe various situations with qualitatively different regimes of the atom-field interaction. We choose several regimes and study the atomic populations and the radiated spectra. Important parameters of the crystal are the number q of periods of the crystal, the refractive indices n_1 and n_2 of the crystal material and the thicknesses a and b of the sublayers. These parameters must be "tuned" to the atomic transition frequency ω_A so that interesting effects can be observed. This is achieved when the wavelength of the light corresponding to the atomic eigenfrequency is of the order of the lattice period. The number q of the lattice periods must not be very large if we want to study significantly discrette-mode cases. The course of the interaction is essentially influenced by the value of the matrix element D_{ge} in the coupling constant. We choose a fixed value $D_{ge} = 10^8\Lambda$. This value does not aspire to be physical; it is chosen with regard to the unit section of the considered part of the crystal and our requirement to investigate the case when the time-dependent state of the system has nonnegligible overlaps with several eigenstates of the Hamiltonian (5.1). This feature corresponds to the noncomplete decay of the atomic level.

Fig. 3 exhibits a typical behaviour of the atomic excited state population for three values of ω_A and the atomic position near the mirror ($x_A = \Lambda/10$). We can see oscillatory patterns around average values. A strongly oscillating pattern of the evolution

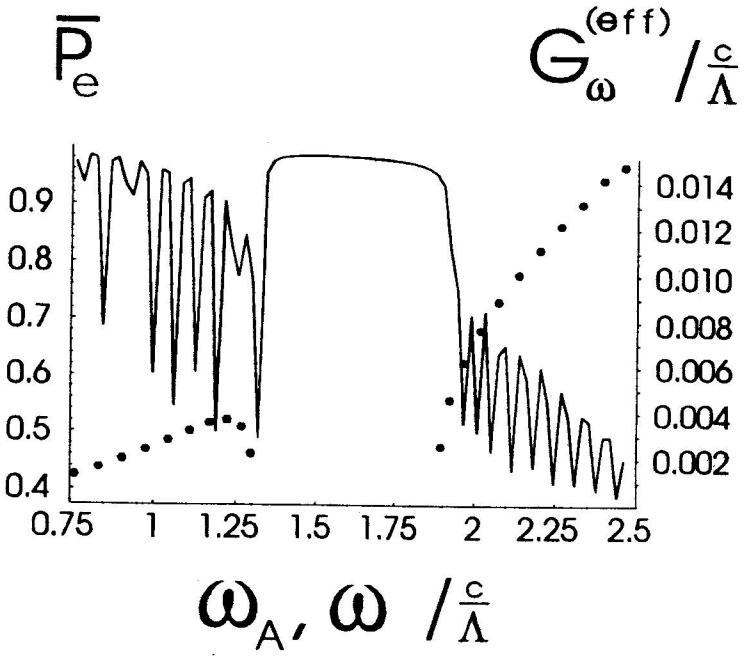

Fig. 4. The long-time average value of the atomic excited-state population (solid line) versus the atomic transition frequency varying around the first band gap of the crystal with parameters as in Fig. 2 and the other atomic parameters as in Fig. 3. The effective coupling constant $G_\omega^{(\mathrm{eff})}$ versus the mode frequency is depicted by points.

remains for all atomic frequencies in the allowed regions. The atomic population only slightly differs from unity if the atomic frequency lies near the center of the gap. Fig. 4 exhibits a long-time average value of the atomic population as a function of the atomic transition frequency. (The matrix element D_{ge} remains constant.) To provide us with a more complex view we plot the frequency dependence of the effective coupling constant $G_\omega^{(\mathrm{eff})}$ in the same graph. We can clearly see from Fig. 4 that the time averaged atomic population reaches the highest values inside the gap. The lowest values of the population are reached at the right side of the gap. This is naturally given by the fact that the effective coupling constant is relatively large in the right side of the gap. The oscillatory behaviour outside the gap is given by the discreetness of the field eigenmodes.[3] As we have already mentioned, the pattern of the atom-field interaction in the studied

[3] An important role is played by a spontaneous-emission shift of the atomic level. A very detailed discussion of this effect can be found in [4] for the continuum spectrum case. However, essential qualitative features of the mention discussion remains valid also for the example studied in the discrete case.

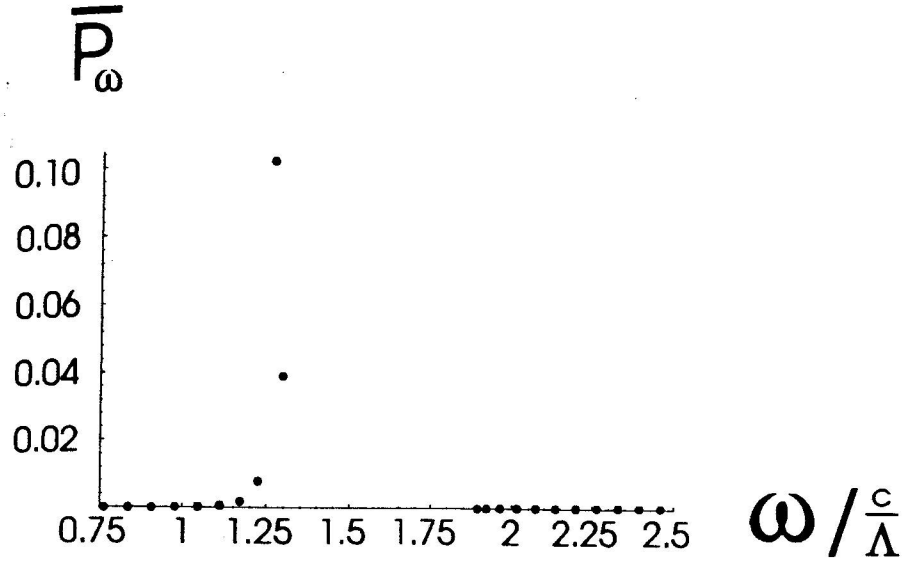

Fig. 5. The long-time average of the radiated spectrum. The crystal parameters are as in Fig. 2, $\omega_A = \omega_{19}$ and the other atomic parameters are as on the Figs. 3,4.

example depends sensitively on the position of the atom. The results presented in Fig. 4 are relatively interesting because of the position-dependence of the coupling constant. It corresponds to the atom near of the mirror ($x_A = \Lambda/10$). We can see relatively slowly varying behaviour of the effective coupling "constant" as a function of the mode frequency. The situation is different when the atom is placed far from the mirrors. The reason is obvious - the effective coupling constant varies with κ and x as goniometric functions of $\kappa\, x$. So if the atom is far from the mirrors the effective coupling constant varies rapidly as a function of ω.

Fig. 5 displays a long-time averaged photon spectrum (i.e. squares of the modulus of probability amplitudes) for the case when the atomic transition frequency coincidences with the 19th field eigenmode. We can see which modes contributes to the interaction most significantly.

In conclusions, we have studied spontaneous emission of a two-level atom in a relatively simple model of a photonic band structure, when a one-dimensional photonic band crystal is bounded by a pair of perfectly conducting mirrors. If the crystal contains a small number ($< 10^2$) of lattice periods and the atom is placed near of the mirror, significant effects due to the presence of the photonic band gaps, the space and frequency dependence of the effective coupling constant and due to the discrete spectrum of the field modes take place. Namely, the atomic population performs oscillations rather then a complete decay. The effect of the highest density of the modes at the edges of the

gaps is particularly compensated by decreasing of the effective coupling constant in this frequency region.

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