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NON-MARKOVIAN MICROMASER FIELD DYNAMICS DUE TO NON-POISSONIAN PUMPING[1]

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We present a unified treatment of discrete and continuous non-Poissonian pumping of a micromaser and investigate the resulting non-Markovian dynamics of the cavity field. For a micromaser with discrete non-Poissonian pumping we show explicitly the equivalence of ensemble-averaging and time-averaging. Moreover, under trapping-state conditions we find exact analytical solutions for the time-dependent field-field correlation functions explicitly displaying the non-Markovian character of the cavity-field dynamics. For certain parameter choices these solutions exhibit an oscillatory decay to equilibrium which results in a splitting of the corresponding spectra into several equidistant peaks.

1. Introduction

The one-atom micromaser is pumped by a beam of excited Rydberg-atoms which interact with the radiation field in a microwave cavity in such a way that at most one atom at a time is present in the cavity [1]. Normally the atoms in the beam are statistically independent, and the pump statistics is therefore described by a Poissonian process. However, the effect of non-Poissonian pump fluctuations on the micromaser dynamics has attracted a great deal of interest recently. It has been investigated by two, seemingly unrelated, theoretical approaches. The first employs a discrete non-Poissonian pumping process where the pump atoms are allowed to arrive, with certain probability p, only at regularly spaced instants of time [2], while the second is based on continuous

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non-Poissonian pumping [3,4] . In this contribution we present a unified approach which is based on a generalization of the latter and can handle both situations on an equal footing [5]. The resulting field dynamics proves to be manifestly non-Markovian except for the case of a strictly Poissonian pumping process.

2. Unified treatment of discrete and continuous non-Poissonian pumping

We describe the change of the cavity-field density operator ϱ due to the transit of a single atom with the help of the superoperator M_{tr}, i. e. we write $\varrho(t+t_{int}) = M_{tr}\varrho(t)$, and we assume that cavity damping can be neglected over the transit time t_{int}. When we introduce the usual damping-superoperator L, the evolution of the cavity- field density operator ϱ from the initial time $t = 0$ to the final time $t = \tau$ can be described by the equation

$$\varrho(\tau) = V(\tau,0)\varrho(0) \tag{1}$$

where [4]

$$V(\tau,0) = \int_\tau^\infty dt_1 \int_{-\infty}^0 dt_0 Q_2(t_0,t_1)e^{L\tau} + \sum_{k=0}^{\infty}\int_\tau^\infty dt_{k+1}\int_0^\tau dt_k \int_0^{t_k} dt_{k-1}\ldots \tag{2}$$
$$\ldots\int_0^{t_2} dt_1 \int_{-\infty}^0 dt_0 Q_{k+2}(t_0,t_1\ldots t_{k+1})e^{L(\tau-t_k)}M_{tr}e^{L(t_k-t_{k-1})}\ldots M_{tr}e^{Lt_1}.$$

Here the quantities $Q_k(t_0,t_1,...,t_{k-1})$ are the exclusive probability densities for the injection of exactly k atoms at the time instants $t_0,t_1,...,t_{k-1}$. From inspection of Eq.(2) we conclude that in general the inequality $V(\tau,0) \neq V(\tau,t')V(t',0)$ holds true for $0 < t' < \tau$ which means that the evolution equation (1) is a non-Markovian one. The only exception consists in the case of strictly Poissonian pumping where $Q_{k+2}(t_0,t_1,\ldots t_{k+1}) = r^{k+2}e^{-r(t_{k+1}-t_0)}$ with r being the injection rate. In this case Eqs.(1) and (2) yield the well-known Markovian master equation $\dot{\varrho} = [r(M_{tr}-1)+L]\varrho$ [4].

In the following we assume that the atoms are injected into the cavity according to a stationary renewal process defined by the property

$$Q_{k+2}(t_0,t_1,\ldots,t_{k+1}) = r\prod_{i=1}^{k+1} f(t_i - t_{i-1}) \tag{3}$$

where $f(t)$ is the waiting-time distribution between consecutive injected atoms and r is the injection rate which can be calculated from the equation $r^{-1} = \int_0^\infty dt f(t)\, t$. By inserting Eq.(3) into Eq.(2) it can be shown that the steady-state density operator $\overline{\varrho} = \lim_{\tau\to\infty}\varrho(\tau)$ obeys the equation

$$r(M_{tr}-1)\overline{\varrho}^c + L\overline{\varrho} = 0 \tag{4}$$

where $\overline{\varrho}^c$ is the steady-state injection-time conditioned density operator which refers to the state of the field immediately before the injection of an atom [4]. It fulfills the mapping condition

$$\overline{\varrho}^c = \int_0^\infty dt\, f(t)e^{Lt}M_{tr}\overline{\varrho}^c \tag{5}$$

which expresses the fact that the state of the field is reproduced after the transit of an atom and after subsequent damping over the mean waiting time. From Eqs.(4) and (5) we obtain [5]

$$\left[(M_{tr}-1)\frac{L}{1-\left[\int_0^\infty dt f(t)e^{Lt}\right]^{-1}} + L\right]\overline{\varrho} = 0\,. \tag{6}$$

Whereas continuous non-Poissonian pumping has been treated with the help of various Ansatzes for a continuous function $f(t)$ characterizing the waiting-time distribution [3,4], we use the expression

$$f(t) = \delta(t-T) \tag{7}$$

which refers to the case that the atoms are injected regularly with the constant time interval T between neighbouring atoms [3]. To make contact with the model of discrete non-Poissonian pumping we suppose that not all of the injected atoms interact with the field because some of them are far from resonance. When p is the interaction probability, the operator M_{tr} occurring in Eq.(2) can be replaced by

$$M_{tr} = pM + (1-p)\underline{1} \tag{8}$$

where the operator M describes the atom-field interaction according to the Jaynes-Cummings-model and $\underline{1}$ is the unit operator.

3. Application to discrete pumping

When we insert Eqs.(7) and (8) into Eq.(2) and perform the integrations [3] we obtain [4]

$$V(\tau,0) = \frac{1}{T}\int_0^{xT} dt' e^{L(xT-t')}[1+p(M-1)]\left\{e^{Lt}[1+p(M-1)]\right\}^{[\frac{\tau}{T}]}e^{Lt'}$$
$$+\frac{1}{T}\int_{xT}^{T} dt' e^{L(xT-t')}\left\{e^{LT}[1+p(M-1)]\right\}^{[\frac{\tau}{T}]}e^{Lt'}. \tag{9}$$

Here $[\tau/T]$ denotes the largest integer that does not exceed τ/T, and $x = \tau/T - [\tau/T]$. The two parts of the sum in the above equation arise from the fact that either $[\tau/T]+1$ or $[\tau/T]$ atoms may be injected in an interval of length τ that is arbitrarily located with respect to the arrival times of the atoms. The steady-state density operator is found from Eqs.(6)–(8) to be determined by the equation

$$\left[p(M-1)\frac{L}{1-e^{-LT}} + L\right]\overline{\varrho} = 0\,. \tag{10}$$

To derive Eq.(10) we considered the whole quantum mechanical ensemble of cavities with regular injection of atoms where the arrival time of the first atom varies statistically in the members of the ensemble. The same equation could be also obtained by performing the time average in a single subensemble that belongs to a fixed value of the arrival time of the first atom, which, for simplicity, we denote by $\tau = 0$. In this subensemble one can define a steady-state injection-time conditioned density operator $\overline{\varrho}^{(x)}$ which refers to the state of the system at the time instants $\tau = mT + xT$ with $m = 0, 1 \ldots$ and $0 \leq x \leq 1$. It obeys the mapping condition [6]

$$\overline{\varrho}^{(x)} = e^{LxT}[1 + p(M-1)]e^{L(1-x)T}\overline{\varrho}^{(x)} . \quad (11)$$

The unconditioned steady-state density operator $\overline{\varrho}$ is found by time-averaging which can be expressed as $\overline{\varrho} = \int_0^1 dx \overline{\varrho}^{(x)}$ [6] and it can be shown that the resulting operator fulfills Eq.(10) [7]. Thus the equivalence of ensemble averaging and time-averaging is explicitly demonstrated for the special example of a micromaser with discrete pumping.

When the micromaser has reached the steady state, all two-time expectation values of the field variables can be obtained with the help of the evolution operator V [5]. In particular, we get

$$\langle a^{\dagger k}(\tau)a^k(0)\rangle_{ss} = \mathrm{Tr}\left[V(\tau,0)a^k\overline{\varrho}a^{\dagger k}\right] \quad (12)$$

where a and $a^\dagger$ denote the photon annihilation and creation operator of the considered cavity mode of frequency ν. We introduce the Fourier transform

$$S_k(\omega) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty d\tau \frac{\langle a^{\dagger k}(\tau)a^k(0)\rangle_{ss}}{\langle a^{\dagger k}a^k\rangle_{ss}} e^{-i(\omega-k\nu)\tau} \quad (13)$$

which could be measured with the help of a detector based on k-photon-absorption.

In order to enable an analytical treatment we assume that the micromaser is operated under the k-photon trapping condition [8]

$$gt_{\mathrm{int}} = l\frac{\pi}{\sqrt{k+1}} \quad (l = 1, 2 \ldots) . \quad (14)$$

with g being the atom-field coupling constant. In these cases the maximum photon number in the cavity is equal to k provided that thermal photons can be neglected. When we substitute Eq.(9) into Eq.(12) the time integration may be easily performed taking into account that, in the photon-number-representation, $\left[e^{Lt}\varrho\right]_{0k} = e^{-k\frac{\gamma}{2}}\varrho_{0k}$ and $[M\varrho]_{0k} = b_k(l)\varrho_{0k}$ where γ is the cavity damping constant and

$$b_k(l) = \cos(l\pi)\cos\left(l\frac{\pi}{\sqrt{k+1}}\right) . \quad (15)$$

We arrive at

$$g_k(\tau) \equiv \frac{\langle a^{\dagger k}(\tau)a^k(0)\rangle_{ss}}{\langle a^{\dagger k}a^k\rangle_{ss}} = e^{-\frac{k}{2}\gamma\tau}\left[1 + p(b_k-1)\right]^{\left[\frac{\tau}{T}\right]}\left\{1 + p(b_k-1)\left(\frac{\tau}{T} - \left[\frac{\tau}{T}\right]\right)\right\} \quad (16)$$

which yields

$$S_k(\omega) = \frac{1}{\pi}\mathrm{Re}\left[\frac{1}{\frac{k}{2}\gamma + i(\omega-k\nu)}\left\{1 + \frac{p(b_k-1)}{\left[k\frac{\gamma}{2} + i(\omega-k\nu)\right]T} \times \frac{1 - e^{-\left[k\frac{\gamma}{2}+i(\omega-k\nu)\right]T}}{1 - [1+p(b_k-1)]e^{-\left[k\frac{\gamma}{2}+i(\omega-k\nu)\right]T}}\right\}\right] . \quad (17)$$

Obviously the expressions (15) and (16) depend crucially on the value of the parameter b_k where $-1 \leq b_k \leq 1$ because of Eq.(15). The function $g_k(\tau)$ reveals a sawtooth-like behaviour when the quantity $1 + p(b_k - 1)$ takes on negative values which is possible for $p > \frac{1}{2}$. The corresponding spectrum is split into several equidistant peaks separated by $\Delta\omega = 2\pi/T$ (see Fig. 1).

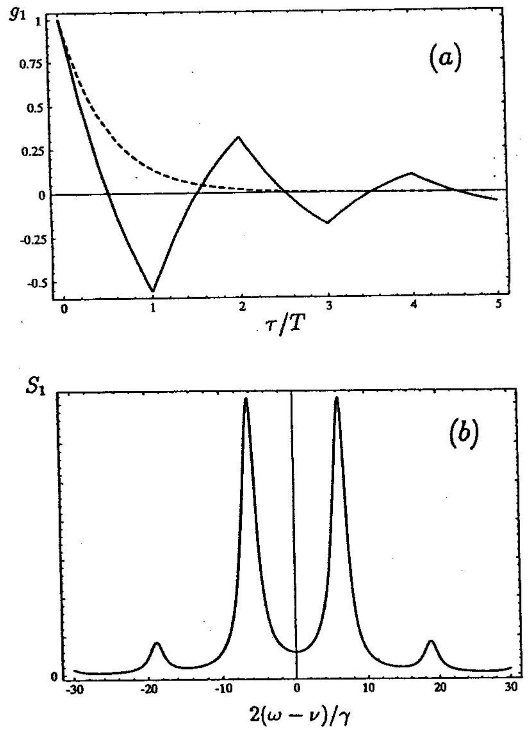

Fig. 1. Normalized field-field correlation function $g_1(\tau) = \langle a^\dagger(\tau)a(0)\rangle_{ss}/\langle a^\dagger a\rangle_{ss}$ (a) and the corresponding spectrum $S_1(\omega)$ (b) for a micromaser operated under the one-photon-trapping condition $gt_{\mathrm{int}} = 3\pi/\sqrt{2}$ with discrete pumping where $p = 1$ and $T = \gamma^{-1}$. The dashed curve in (a) corresponds to the Markovian approximation $\dot{\varrho} = p(M-1)L(1-e^{-LT})^{-1}\varrho + L\varrho$ [9] which is based on Eq.(10).

4. Discussion

We note that for $k = 1$ spectral splitting in a micromaser with discrete pumping has been already found previously for certain parameter regions under general operating conditions with the help of a numerical [9] and an approximate analytical [10] treatment both resting on a stroboscopic approach with subsequent time averaging. In contrast to this, we used the ensemble-average approach resulting from an unified treatment of the micromaser with continuous and discrete non-Poissonian pumping. Thus we were able to find, for special cases, exact analytical solutions explicitly displaying the non-Markovian character of the micromaser field dynamics for non-Poissonian pumping (see Eq.(16)).

For Poissonian pumping the micromaser field dynamics is a Markovian one, and in the most general case, the decay of the delayed field-field correlation function can be expressed as the sum of several exponential decaying functions [11]. When some of them enter with negative weight, the overall decay can become oscillatory giving rise to spectral splitting, too [12]. However, this effect has nothing in common with the splitting into several equidistant peaks occurring for discrete pumping and being due to the fact that a single atom can reverse the phase of the entire cavity field because of the interference of quantum Rabi oscillations with different frequencies [5].

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