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THE EFFECT OF BEAM-SPLITTER IMPERFECTIONS AND LOSSES ON FRINGE VISIBILITY IN A MACH-ZEHNDER INTERFEROMETER[1]

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The fringe visibility in a Mach-Zehnder interferometer is adversely affected by beam-splitter imperfections and unequal losses in its arms. The contribution analyzes the effect of these factors on the visibility and shows how it may be improved by inserting additional losses in one of the arms of the interferometer. Relations expressing the dependence of fringe visibility on beam-splitters' reflectances and transmittances, and on the ratio of losses in the arms of a Mach-Zehnder interferometer are derived.

Since the late eighties, there have appeared many experiments testing the correctness of quantum mechanics. They are reviewed including many references, e.g., in [1]. Many of these experiments are based on measurements utilizing various forms of the Mach-Zehnder interferometer. The achievable precision of these measurements is governed by several dominating factors, such as detector dark-count rates, polarization mismatches of interfering photons, beam-splitter imperfections, or different amount of losses in each arm of the interferometer. Since the number of detector dark counts can be reduced only by developing lower-noise detectors, and polarization mismatches can be minimized by means of polarization controllers ([2]), we focused on the effects arising from beam-splitter imperfections and unequal losses in the arms of a Mach-Zehnder interferometer.

Let us first consider, e.g., beam splitter BS1 of the Mach-Zehnder interferometer outlined in Fig. 1. The relations between its input and output modes may then be written in the form ([3], [4], [5])

$$\begin{pmatrix} \hat{c}_1 \\ \hat{c}_3 \end{pmatrix} = \begin{pmatrix} t_1 & r_1 \\ r_1 & t_1 \end{pmatrix} \begin{pmatrix} \hat{a}_2 \\ \hat{a}_1 \end{pmatrix}, \tag{1}$$

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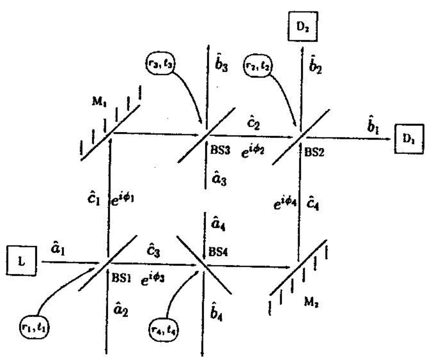

Fig. 1. L – laser, BSi – beam splitters, $\hat{a}_i$ – annihilation operators of input modes, $\hat{b}_i$ – annihilation operators of output modes, $\hat{c}_i$ – annihilation operators of intermediate modes, r_i – amplitude reflectance of BSi, t_i – amplitude transmittance of BSi, $e^{i\phi_i}$ – a phase shift experienced by mode $\hat{c}_i$, $i = 1, \ldots, 4$; D_1,D_2 – detectors; M_1,M_2 – lossless mirrors.

and analogically for BS2, BS3, and BS4. The requirement of energy conservation leads to the conditions

$$|r_i|^2 + |t_i|^2 = 1 \quad \text{and} \quad r_i t_i^* + r_i^* t_i = 0, \qquad i = 1, \ldots, 4, \tag{2}$$

where an asterisk denotes complex conjugate. It follows from Eq. (2) that

$$\arg(r_i) - \arg(t_i) = \frac{\pi}{2}. \tag{3}$$

Now we may turn to a treatment of the Mach-Zehnder interferometer. To include losses that occur during propagation through the arms of the interferometer and losses caused by imperfect beam splitters, we introduced two fictitious beam splitters BS3 and BS4 which are considered to be ideal and whose squared reflectances represent the percentage of photons lost in the particular arm of the interferometer. Of course in practice, losses are not confined to one point. For a realistic treatment of, e.g., a fiber-based Mach-Zehnder interferometer, a large number of such fictitious beam splitters would have to be employed. Fortunately, it is not necessary.

Consider now the two beam splitters on the left-hand side of Fig. 2. Provided that no light enters at inputs a_0 and a_0', and a_I and b_I are the only modes of interest, it is then possible to replace the two beam splitters by a single beam splitter of transmittance equal to the product of transmittances of the separate beam splitters. In this way, we can successively unite arbitrary two beam splitters in each arm, eventually replacing all of them by one. Incorporating losses by means of two extra beam splitters BS3 and BS4 is then usable for descriptions of both mirror-based Mach-Zehnder interferometers where most losses are concentrated at mirrors, and fibre-based ones where losses appear at connections, in bends, and within entire fibers due to absorption and scattering.

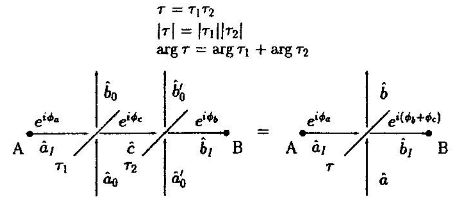

Fig. 2. Illustrating that two beam splitters may be replaced by one, provided that no light enters a_0 and a_0' and we are not interested in b_0 and b_0'.

However, beam-splitter imperfections need not only show as losses. They can also result in deviations of the actual reflectance and transmittance of the beam splitter from the specified values. Our design allows for this by letting r_1 and r_2 be variable. We can then investigate how changes in reflectances, and thereby transmittances, of beam splitters BS1 and BS2 affect fringe visibility.

Returning to Fig. 1, we can write with the help of Eq. (1) and its analogs for BS2, BS3, and BS4

$$\begin{aligned} \hat{b}_1 &= \left(r_1 t_2 t_3 e^{i\phi_A} + r_2 t_1 t_4 e^{i\phi_B}\right)\hat{a}_1 + \left(t_1 t_2 t_3 e^{i\phi_A} + r_1 r_2 t_4 e^{i\phi_B}\right)\hat{a}_2 + r_3 t_2 e^{i\phi_2}\hat{a}_3 \\ &+ r_2 r_4 e^{i\phi_4}\hat{a}_4 \end{aligned} \tag{4}$$

and similarly for $\hat{b}_2$, where $e^{i\phi_1}e^{i\phi_2} = e^{i\phi_A}$ and $e^{i\phi_3}e^{i\phi_4} = e^{i\phi_B}$. In order to calculate the fringe visibilities at detectors D_1 and D_2, we need to know expressions for the rates of photodetection at these detectors in terms of $\hat{a}_i$'s and $\hat{a}_i^\dagger$'s, $i = 1, \ldots, 4$. If we assume quasi-monochromatic photons to be fed to the input of the interferometer, then the rate of photodetection, as defined, e.g., in [3], Chap. 12, or [6], is proportional to the mean number of photons, where the proportionality constant is the quantum efficiency of the detector.

Assuming now that the input field in mode a_1 is in the one-photon Fock state $|1\rangle$ and the other input modes a_2, a_3, and a_4 are in the vacuum state $|0\rangle$, we can directly find for the mean number of photons at D_1 that

$$\begin{aligned} \langle \hat{b}_1^\dagger \hat{b}_1 \rangle &= \langle 1_1, 0_2, 0_3, 0_4 | \hat{b}_1^\dagger \hat{b}_1 | 1_1, 0_2, 0_3, 0_4 \rangle = |r_1|^2 |t_2|^2 |t_3|^2 + |r_2|^2 |t_1|^2 |t_4|^2 \\ &+ 2|r_1||r_2||t_1||t_2||t_3||t_4| \cos(\phi_A - \phi_B), \end{aligned} \tag{5}$$

and similarly for detector D_2

$$\begin{aligned} \langle \hat{b}_2^\dagger \hat{b}_2 \rangle &= \langle 1_1, 0_2, 0_3, 0_4 | \hat{b}_2^\dagger \hat{b}_2 | 1_1, 0_2, 0_3, 0_4 \rangle = |r_1|^2 |r_2|^2 |t_3|^2 + |t_1|^2 |t_2|^2 |t_4|^2 \\ &+ 2|r_1||r_2||t_1||t_2||t_3||t_4| \cos(\phi_A - \phi_B + \pi), \end{aligned} \tag{6}$$

where we have made use of Eq. (4), its analog for $\hat{b}_2$, and Eq. (3), and have assumed that the phase shifts imparted by beam splitters BS3 and BS4 are equal.

To obtain the mean numbers of photons at D_1 and D_2 for a coherent state $|\alpha\rangle$ at the input in place of the Fock state $|1\rangle$, the right-hand sides of (5) and (6) must be multiplied by $|\alpha|^2$.

The fringe visibilities at detectors D_1 and D_2 are by definition

$$V_j = \frac{\langle \hat{b}_j^\dagger \hat{b}_j \rangle_{max} - \langle \hat{b}_j^\dagger \hat{b}_j \rangle_{min}}{\langle \hat{b}_j^\dagger \hat{b}_j \rangle_{max} + \langle \hat{b}_j^\dagger \hat{b}_j \rangle_{min}}, \quad j = 1, 2 \tag{7}$$

where $\langle\ \rangle_{max}$ and $\langle\ \rangle_{min}$ denote maximum and minimum values of the mean number of photons striking the j-th detector, respectively corresponding to the phase differences between the arms of the interferometer set to $\phi_A - \phi_B = 2k\pi$ and $\phi_A - \phi_B = (2k+1)\pi, k = 0, 1, 2, \ldots$ On using Eqs. (5) and (6), the relation (7) immediately yields

$$V_1 = 2\left(\frac{|r_1|}{|t_1|}\frac{|t_2|}{|r_2|}\frac{|t_3|}{|t_4|} + \frac{|t_1|}{|r_1|}\frac{|r_2|}{|t_2|}\frac{|t_4|}{|t_3|}\right)^{-1}, \tag{8}$$

and

$$V_2 = 2\left(\frac{|r_1|}{|t_1|}\frac{|r_2|}{|t_2|}\frac{|t_3|}{|t_4|} + \frac{|t_1|}{|r_1|}\frac{|t_2|}{|r_2|}\frac{|t_4|}{|t_3|}\right)^{-1}. \tag{9}$$

It is readily seen that V_1 and V_2 attain their maxima

$$V_1 = 1 \quad \text{if} \quad \frac{|r_1|}{|t_1|}\frac{|t_2|}{|r_2|}\frac{|t_3|}{|t_4|} = 1, \tag{10}$$

and

$$V_2 = 1 \quad \text{if} \quad \frac{|r_1|}{|t_1|}\frac{|r_2|}{|t_2|}\frac{|t_3|}{|t_4|} = 1. \tag{11}$$

The expressions (8) and (9) may be understood as functions of only three independent variables, for reflectance and transmittance are coupled by Eq. (2) and just the ratio $\frac{|t_3|}{|t_4|}$ is of significance, as will be seen later.

We first consider a case when $\frac{|t_3|}{|t_4|} = 1$, thereby reducing the number of independent variables to two, say r_1 and r_2. It is now possible to graphically illustrate the degradation of V_1 and V_2 when BS1 and BS2 are not precise 50:50 beam splitters (see Fig. 3). For a better idea, power reflectances $|r_1|^2$ and $|r_2|^2$ are plotted along the horizontal axes. Visibilities V_1 and V_2 simultaneously reach unity for $|r_1|^2 = |r_2|^2 = 0.5$, as expected. If $|r_1|^2 = |r_2|^2 \neq 0.5$, V_1 remains unity while V_2 decreases to

$$V_2 = 2\left(\frac{|r_1|^2}{|t_1|^2} + \frac{|t_1|^2}{|r_1|^2}\right)^{-1}. \tag{12}$$

In such a case, it is evident to be more advantageous to perform measurements, if possible, on D_1 only. If $|r_1|^2 = |t_2|^2$, V_1 and V_2 exchange one another. A top view of V_1 is shown in Fig. 4(a).

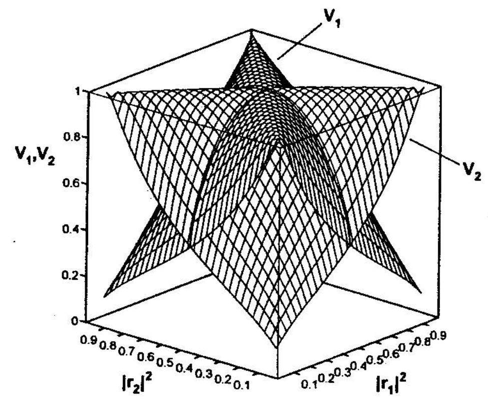

Fig. 3. A plot of V_1 and V_2 as functions of $|r_1|^2$ and $|r_2|^2$ if $\frac{|t_3|}{|t_4|} = 1$.

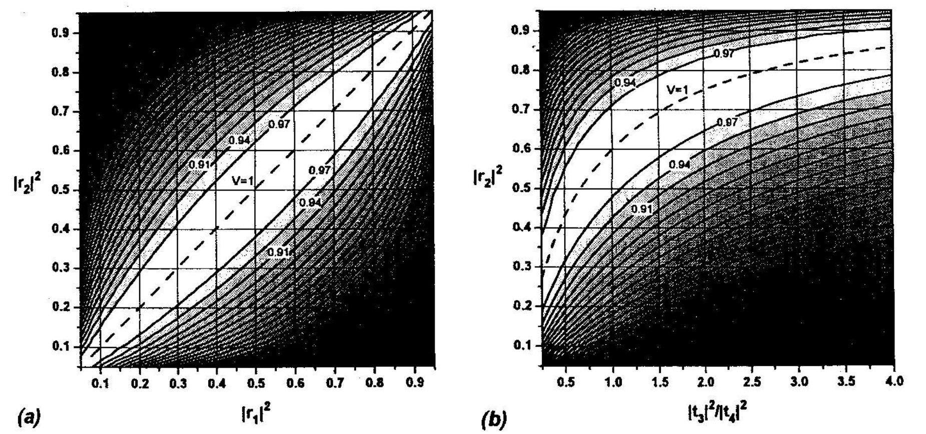

Fig. 4. A top view of V_1 *(a)* as a function of $|r_1|^2$ and $|r_2|^2$ if $|t_3|/|t_4| = 1$, *(b)* as a function of $|t_3|^2/|t_4|^2$ and $|r_2|^2$ if $|r_1|^2 = 0.6$. The lighter grey the higher visibility. The dashed lines exhibit $V_1 = 1$. Each next line indicates a visibility decrease by 3%.

Eq. (8) implies that the visibility at D_1 can be set up to unity even if $|r_1|^2 \neq |r_2|^2$. Such a situation can be achieved by inducing an appropriate amount of losses in one of the arms of the interferometer, i.e., by setting the ratio $\frac{|t_3|}{|t_4|}$ such that the condition (10) is satisfied. However, if done so, V_2 decreases to

$$V_2 = 2\left(\frac{|r_2|^2}{|t_2|^2} + \frac{|t_2|^2}{|r_2|^2}\right)^{-1}. \tag{13}$$

In the same way, we can improve the visibility at D_2, whereupon V_1 drops to the value given by the right-hand side of Eq. (13). Note that the expression (13) is a function of one independent variable characterizing BS2. It indicates that beam splitter BS2 is more significant than BS1. If BS2 were an ideal 50:50 beam splitter, it would be possible to reach unity visibilities at both detectors at a time. The conditions that must hold in order that $V_1 = V_2 = 1$ have the form

$$|r_2|^2 = 0.5 \qquad \text{and} \qquad \frac{|r_1|}{|t_1|}\frac{|t_3|}{|t_4|} = 1. \tag{14}$$

However, if $|r_2|^2 \neq 0.5$, a unity visibility can only be reached at one of detectors D_1 or D_2. Hence the beam splitter whose splitting ratio is closer to 50:50 should be used as BS2, i.e., as a combiner. If only one of the equalities in (14) is true, visibilities V_1 and V_2 are equal but lower than 1.

Fig. 4*(b)* shows the dependence of V_1 on $\frac{|t_3|^2}{|t_4|^2}$ and $|r_2|^2$ while the imperfections of beam splitter BS1 are (for a better illustration) assumed to be such that its real splitting ratio is 60:40 rather than 50:50.

As for the ratio $\frac{|t_3|^2}{|t_4|^2}$, in practice it should not be thought of as approaching unity. It may substantially differ from 1, especially when there are some devices inserted in the interferometer, such as phase shifters, U-brackets, polarization controllers, etc. For example, an air gap of a 2dB insertion loss, placed in the lower arm, can thus cause visibilities V_1 and V_2 to decrease by 2.5%, and if BS1 and BS2 were in addition imperfect, say 52:48 and 51:49, then V_1 would drop to 97% and V_2 even to 96%.

The analysis presented here has shown that the same amount of losses in both arms of a Mach-Zehnder interferometer has no effect on fringe visibility, whereas unequal losses and beam-splitter imperfections diminish it, as has been expected. By inserting an appropriate amount of losses in one of the arms, the effects of the above factors may completely be eliminated if only one detector is necessary for the measurement, or at least minimized when two detectors must be used. In the latter case, the beam splitter whose splitting ratio is closer to 50:50 should be used as the combiner.

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