acta physica slovaca vol. 46 No. 3, 387 – 392 June 1996

SINGLE-PHOTON WAVEPACKETS AT ABSORBING BARRIERS [1]

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Received 31 May 1996, accepted 7 June 1996

Using a recently developed formalism of quantization of radiation in the presence of absorbing dielectric bodies, the problem of photon tunneling through multilayer dielectric barriers is studied. It is shown that losses in the layers may considerably change the photon tunneling times observed in two-photon interference experiments. The results further reveal that for sufficiently large numbers of layers interference fringes are observed that cannot be related to a single traversal time.

1. Introduction

Stimulated by recent experiments [1] - [3], the problem of photon tunneling through multilayer dielectric barriers has been of increasing interest. In order to determine the time that is spent by a photon inside such a barrier, the effects of dispersion and absorption should be considered very carefully. Calculations that have been performed so far are based on real refractive indices of the layers [1] - [4], so that a number of questions, such as the influence of absorption on the measured traversal times [5], have been open. Clearly, in frequency intervals where the bulk materials are nearly transparent the action of multilayer barriers can be described in terms of unitary transformations that relate the operators of the outgoing fields to those of the incoming fields (see, e.g., Refs. [6] and [7]). The concept of unitary transformations and the underlying quantization scheme (see, e.g., Refs. [8] - [11]) of course fail when the effects of absorption cannot be disregarded.

Various approaches to the problem of quantization of radiation in the presence of absorbing dielectric bodies have been developed [12] - [23]. In the present paper we use a Green function expansion of the operator of the (transverse) vector potential [23], which applies to radiation in both homogeneous and inhomogeneous dielectric matter. Applying the method to the calculation of input–output relations of radiation at absorbing multilayer dielectric barriers, we can systematically study the effects of

[1]Presented at the **4th central-european workshop on quantum optics**, Budmerice, Slovakia, May 31 - June 3, 1996

dispersion and absorption on the propagation of single-photon pulses through such barriers.

The paper is organized as follows. In Sec. the quantization scheme developed in [23] is applied to radiation falling on multilayer barriers. Input–output relations are derived which are used in Sec. to calculate barrier traversal times measurable in two-photon interference experiments. Finally, a summary of the results is given in Sec. .

2. Input–output relations

As has been shown in [23], the operator of the vector potential of linearly polarized light propagating in x direction may be represented as

$$\hat{A}(x) = \int_0^\infty \mathrm{d}\omega \int \mathrm{d}x'\, G(x,x',\omega)\, \frac{\omega}{c^2} \sqrt{\frac{\hbar}{\pi\epsilon_0 \mathcal{A}}\, \epsilon_{\mathrm{i}}(x',\omega)}\, \hat{f}(x',\omega), \tag{1}$$

where $G(x,x',\omega)$ is the classical Green function that satisfies the equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2}\epsilon(x,\omega)\right] G(x,x',\omega) = \delta(x-x') \tag{2}$$

and vanishes in the limit when $x \to \pm\infty$. Here, $\hat{f}(x,\omega)$ is a basic bosonic field, $\mathcal{A}$ denotes a normalization area perpendicular to x, and $\epsilon_{\mathrm{i}}(x,\omega)$ is the imaginary part of the complex permittivity $\epsilon(x,\omega)$. Let us now consider a dielectric barrier consisting of $(N-2)$ layers $(N \geq 3)$,

$$\epsilon(x,\omega) = \sum_{j=1}^{N} \lambda_j(x)\, \epsilon_j(\omega), \quad \text{with} \quad \lambda_j(x) = \begin{cases} 1 & \text{if } x_{j-1} < x < x_j, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $\epsilon_j(\omega)$ is the permittivity of the jth layer $[x_0 \to -\infty,\ x_N \to \infty,\ \sqrt{\epsilon_j(\omega)} = n_j(\omega) = \beta_j(\omega) + i\,\gamma_j(\omega)]$. The form of $G(x,x',\omega)$ implies that [23]

$$\hat{A}(x) = \sum_{j=1}^{N} \lambda(x) \int_0^\infty \mathrm{d}\omega \sqrt{\frac{\hbar\beta_j(\omega)}{4\pi c\omega\epsilon_0\epsilon_j(\omega)\mathcal{A}}} \times \left[e^{i\beta_j(\omega)\omega x/c}\hat{a}_{j+}(x,\omega) + e^{-i\beta_j(\omega)\omega x/c}\hat{a}_{j-}(x,\omega)\right] + \mathrm{H.c.}, \tag{4}$$

where the quasi-mode operators $\hat{a}_{j+}(x,\omega)$ and $\hat{a}_{j-}(x,\omega)$ are associated with the (damped) waves propagating to the right and left, respectively (for the relations to the basic field operators $\hat{f}(x,\omega)$, see Ref. [25]). In particular when $\mathrm{Im}[\epsilon_j(\omega)] \to 0$, then the operators $\hat{a}_{j\pm}(x,\omega) \to \hat{a}_{j\pm}(\omega)$ are ordinary (x-independent) bosonic free-field operators [23].

The output operators $\hat{a}_{1-}(x,\omega)$ and $\hat{a}_{N+}(x,\omega)$ can be calculated step by step starting from a single-slab plate $(N=3)$. After some lengthy calculation [25] we find that

$$\begin{pmatrix} \hat{a}_{1-}(x_1,\omega) \\ \hat{a}_{N+}(x_{N-1},\omega) \end{pmatrix} = \tilde{\mathbf{T}}(\omega) \begin{pmatrix} \hat{a}_{1+}(x_1,\omega) \\ \hat{a}_{N-}(x_{N-1},\omega) \end{pmatrix} + \tilde{\mathbf{A}}(\omega) \begin{pmatrix} \hat{g}_{+}(\omega) \\ \hat{g}_{-}(\omega) \end{pmatrix}, \tag{5}$$

where the input operators $\hat{a}_{1+}(x,\omega)$ and $\hat{a}_{N-}(x,\omega)$ and the bosonic noise operators $\hat{g}_\pm$ are commuting quantities. The characteristic transformation matrix $\tilde{\mathbf{T}}(\omega)$ describes the effects of transmission and reflection of the input fields [24], whereas the losses inside the barrier give rise to an absorption matrix $\tilde{\mathbf{A}}(\omega)$. Explicit expressions for the matrices $\tilde{\mathbf{T}}(\omega)$ and $\tilde{\mathbf{A}}(\omega)$ and the noise operators $\hat{g}_\pm(\omega)$ [as linear functionals of the field $\hat{f}(x,\omega)$ inside the barrier] are given in Ref. [25].

3. Photon tunneling

To study the influence of dispersion and absorption on photon tunneling through multilayer dielectric barriers, let us consider a two-photon experiment of the type described in Ref. [1] (Fig. 1). Pairs of down-conversion photons are directed by mirrors to impinge on the surface of a 50%:50% beam splitter and the output coincidences are measured. One photon (I) of each pair travels through air, while the conjugate photon (II) passes a barrier. The coincidences attain a minimum when the two photons' wavepackets overlap perfectly at the beam splitter. This can be achieved by translating a prism in one arm of the interferometer in order to compensate for the delay owing to the barrier.

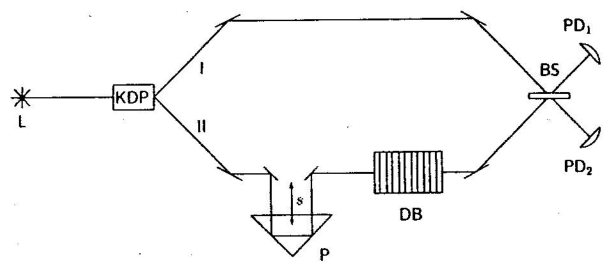

Fig. 1. Scheme of the two-photon interference experiment [1, 2] for the determination of photon traversal times through multilayer dielectric barriers (L, laser; P, prism; DB, dielectric barrier; BS, beam splitter; PD_1, PD_2, photodetectors).

Let us assume that the barrier is in the ground state and the two correlated photons are prepared in a state

$$|\Psi\rangle = \int_0^\infty \mathrm{d}\Omega\, \alpha(\Omega) \int_0^\Omega \mathrm{d}\omega\, f(\omega) f(\Omega-\omega)\, \hat{a}_{\mathrm{I}}^\dagger(\omega)\, \hat{a}_{\mathrm{II}}^\dagger(\Omega-\omega)\, |0\rangle, \tag{6}$$

where $\alpha(\Omega)$ and $f(\omega)$ are the bandwidth functions of the laser and down-conversion photons, respectively, $f(\omega)$ being centered at $\Omega/2$. From photodetection theory it is well known (see, e.g., [28]) that the overall coincidences R can be obtained as the time-integrated normally ordered intensity correlation function,

$$R = \xi^2 \int \mathrm{d}t_1 \int \mathrm{d}t_2 \left\langle \hat{E}^{(-)}(t_1)\hat{E}^{(-)}(t_2)\hat{E}^{(+)}(t_1)\hat{E}^{(+)}(t_2)\right\rangle, \tag{7}$$

where $\hat{E}^{(\pm)}(t_1)$ and $\hat{E}^{(\pm)}(t_2)$ are the fields at the detectors in the two output channels of the beam splitter (ξ, detection efficiency). Applying the input–output relations (5) and using Eq. (6), after some lengthy but straightforward calculation we find that

$$R = 2\pi^2 \mathcal{N}^4 \int_0^\infty d\Omega\, \alpha^2(\Omega)\, F(\Omega), \tag{8}$$

$$F(\Omega) = \int_0^\Omega d\omega \left|f^2(\omega) f^2(\Omega-\omega)\right| \omega(\Omega-\omega) T_{12}^*(\Omega-\omega)\left[T_{12}(\Omega-\omega) - e^{-2i\Omega s} e^{4i\omega s} T_{12}(\omega)\right], \tag{9}$$

where s is the translation length of the prism (cf. Fig. 1), and $\mathcal{N} = \sqrt{\xi\hbar/(4\pi c\epsilon_0 \mathcal{A})}$.

The translation length $s = s_0$ corresponding to the minimum of $R(s)$ is usually used to distinguish between superluminal and subluminal behaviour (positive and negative values of s_0, respectively) of the photon passing through the barrier. In the numerical calculations we have considered $H(LH)^k$ structured plates (H, titanium dioxide; L, fused silica) of $\lambda/4$-layers. The calculation of the function $T(\omega)$ follows the lines given in Ref. [26]. A dependence on frequency of the (complex) refractive indices of the layers has not been taken into account. The line shape function of the exciting laser, $\alpha(\Omega)$, has been assumed to be sufficiently small, so that $F(\Omega) \approx F(\omega_0)$ in Eq. (8), where ω_0 is the centre frequency ($\omega_0 = 5.37 \times 10^{15}\mathrm{s}^{-1}$). Introducing the single-photon pulse shape function $f(t) = (2\pi)^{-1/2} \int d\omega\, \exp[-i\omega t] f(\omega)$, we have performed calculations for both Gaussian pulses $f(t) \propto \exp[i\omega_0 t/2 - (t/t_0)^2]$ and time-limited non-Gaussian pulses $f(t) \propto \exp\{i\omega_0 t/2 - [1 - [t/(2t_0)]^2]^{-1}\}$ if $|t| < 2t_0$ and $f(t) = 0$ elsewhere, where $t_0 = 20$fs.

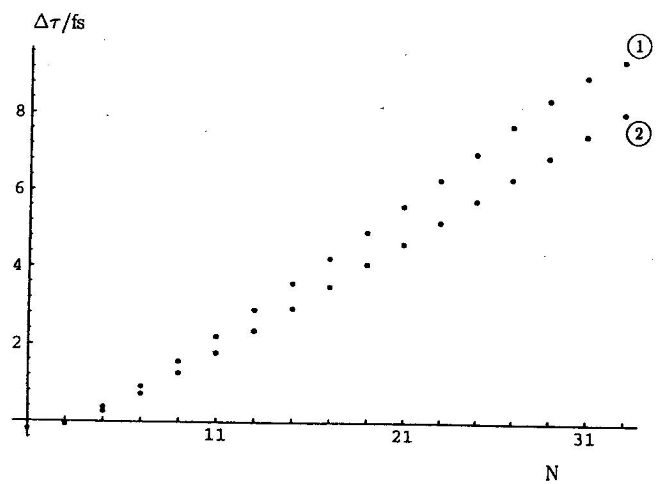

Fig. 2. The temporal "lead" $\Delta\tau = 2s_0/c$ that corresponds to the position s_0 of the minimum of $R(s)$ is shown as a function of the number of layers, $N = 2k+1$, for a $H(LH)^k$ structured plate of $\lambda/4$-layers of the type described in Ref. [3]; curve (1): lossless barrier ($n_{TiO_2} = 2.22$, $n_{SiO_2} = 1.41$), curve (2): absorbing barrier ($n_{TiO_2} = 2.22$, $n_{SiO_2} = 1.41 + 0.0372i$ [27]).

The values of $\Delta\tau = 2s_0/c$ that are shown in Fig. 2 are valid for both Gaussian and time-limited pulse shape functions. From $\Delta\tau$ a characteristic traversal time $\tau_t = l/c - \Delta\tau$ can be derived (l, thickness of the barrier). The Fig. 2 reveals that the "lead" of the second photon, $\Delta\tau$, increases with the number of layers of the barrier, $N = 2k+1$, and tends to a linear function of N. Disregarding the losses, its slope is simply given by the inverse velocity of light in vacuum, which indicates that τ_t is independent of N. The effect of losses is seen to decrease this slope and to increase τ_t.

The interval of N in which $\Delta\tau$ (linearly) increases with N is limited by an upper boundary, which depends on the pulse shape function of the photon at the entrance plane. For the system under consideration the increase of $\Delta\tau$ with N ends when $N \approx 35$ (lossless barrier) or $N \approx 41$ (absorbing barrier) for the time-limited pulse, whereas for the Gaussian pulse the boundary value of N is substantially increased. The increase of $\Delta\tau$ with N is of course not in contradiction to causality, but can be explained by a shift of the pulse maximum towards earlier times owing to pulse reshaping in the barrier.

The spectral line shape function of the outgoing photon, $\overline{f}(\omega) \propto f(\omega) T_{12}(\omega)$, which sensitively depends on the two competing quantities $f(\omega)$ and $T_{12}(\omega)$, can essentially differ from that of the incoming photon. For sufficiently large N the incoming and outgoing photons' wavepackets lose all resemblance to each other. In this case the measured coincidences are expected to be a more or less complicated function of the translation length, the structure of which does not allow one to define uniquely a traversal time. From Fig. 3 we see that with increasing N interference fringes are observed. They correspond to various possibilities of overlapping of the undisturbed and the multi-peaked outgoing photons' wavepackets at the beam splitter.

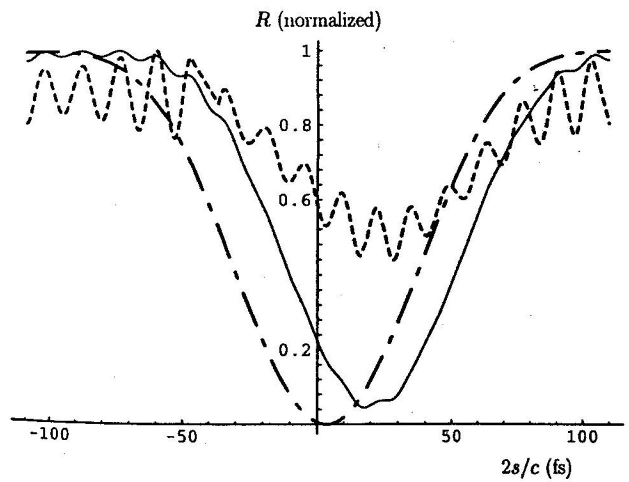

Fig. 3. The (normalized) coincidences $R(s)$ are shown in dependence on the translation length s for a time-limited pulse of the incoming photon ($2t_0 = 40$ fs) and various numbers of the layers of an absorbing barrier: $N = 11$ (dotted-dashed line), $N = 41$ (full line), $N = 49$ (dashed line). The data of the absorbing barrier are the same as in Fig. 2.

Compared to a time-limited pulse, the wings of the spectral line shape function $f(\omega)$ of a Gaussian pulse decrease substantially faster. Hence, the transformed line shape function $f(\omega) T_{12}(\omega)$ of a Gaussian pulse reflects the frequency response of the transmittance of the barrier, $T_{12}(\omega)$, less sensitively than that of a time-limited pulse. This explains the above mentioned difference in the boundary values of N.

4. Summary and Conclusions

On the basis of a Green function approach to the quantization of radiation in inhomogeneous, dispersive and absorptive linear dielectrics we have derived quantum optical input-output relations for optical fields at multilayer dielectric plates, which are generalizations of unitary transformations. Applying the theory to photon tunneling through absorbing barriers, we have shown that relatively small imaginary parts of the refractive indices of the layers can already give rise to observable effects in two-photon interference experiments as performed recently.

The results reveal that only up to an upper boundary value of layers the measured coincidences can be used for extracting from them a characteristic time that may be regarded as traversal time. The boundary value sensitively depends on both the spectral line shape function of the photon at the barrier and the dependence on frequency of the transmittance of the barrier, which can be substantially different for absorbing and non-absorbing barriers. For sufficiently large numbers of layers the photon's wavepacket can be distorted in the barrier to such an extend that the observed coincidences show a number of interference fringes which correspond to different time constants.

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