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QUANTUM CONTROL OF CHAOS INSIDE A CAVITY[1]

M.Fortunato[2], W. P. Schleich[3]
Abteilung für Quantenphysik, Universität Ulm, Albert-Einstein-Allee 11
D-89069 Ulm, Germany

G. Kurizki
Department of Chemical Physics, The Weizmann Institute of Science
Rehovot 76100, Israel

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By sending many two-level atoms through a cavity resonant with the atomic transition, and letting the interaction times between the atoms and the cavity be randomly distributed, we end up with a predetermined Fock state of the electromagnetic field inside the cavity if we perform after the interaction with the cavity a conditional measurement of the internal state of each atom in a coherent superposition of its ground and excited states. Differently from previous schemes, this procedure turns out to be very stable under fluctuations in the interaction times.

1. Introduction

In the last decade, a great deal of attention has been dedicated to the problem of quantum state preparation [1-5], since the availability of non-classical states can allow the investigation of fundamental problems in Quantum Mechanics [1]. Among them, Fock states [2] are particularly intriguing because they do not present intensity fluctuations. Two major approaches to achieve this goal have been proposed: the first one is based on unitary evolution [3], that is on finding the right Hamiltonian which evolves the initial state to the desired final one. The second approach is based on the conditional measurement (CM) scheme [4] in which the desired state is achieved after a measurement is performed on one of two interacting systems. The CM approach has the disadvantage that unsuccessful runs (experiments in which the measurement does not give the right result) must be discarded, and therefore it has a success probability which is always less than unity. On the other hand, it has the clear advantage of a simple Hamiltonian evolution, as, for example, the Jaynes-Cummings model.

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[2] E-mail address: for@physik.uni-ulm.de
[3] Also at Max-Planck Institut für Quantenoptik, D-85748 Garching bei München, Germany

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In this paper, we present a new scheme which—differently from previous ones [5]—allows the preparation of Fock states inside a cavity in the presence of even large fluctuations in the interaction times between the two-level atoms and the cavity field. It is based on the CM approach and on the quantum interference between the two possible final states of the atom. The presence of both these effects performs a strong suppression of the fluctuations in the atomic velocities, and makes the convergence of the photon-number distribution towards that of a number state possible.

Our proposal connects as well to the recently introduced field of chaos control [6]. In fact, it has been shown theoretically and experimentally that it is possible to use the extreme sensitivity of chaotic systems to stabilise regular periodic orbits in the chaotic dynamics. The classical version of our model, implemented via non-selective measurements (NSMs), is indeed chaotic even for a small spread in the interaction times [5]. Our CM scheme could then be interpreted as a "quantum way" of controlling chaos. In this view, our method is a new scheme which can effectively restore fixed points in the quantum dynamics of a classically chaotic system.

2. The model

We consider a model in which many two-level atoms are sent through a cavity whose frequency is resonant with the atomic transition. The atoms cross the cavity sequentially (one at a time) so that at most one atom is present inside the cavity. In the general case, the atoms are initially prepared in a coherent superposition of their ground and excited states [7] with the help of two classical fields E_1 (resonant) and E_2 (non-resonant). After the preparation, the state of the kth atom is $|\phi_k^{(i)}\rangle = \alpha_k^{(i)}|e\rangle + \beta_k^{(i)}|g\rangle$, where $|g\rangle$ and $|e\rangle$ are the ground and the excited state of the atom, respectively. On the other hand, the cavity field is initially prepared by a classical oscillator E_3 in a coherent state

$$|\psi_0\rangle = \sum_{n=0}^{\infty} d_n^{(0)}|n\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle\,. \tag{1}$$

The interaction between the atoms and the cavity is described [8] by the resonant Jaynes-Cummings model, namely, the total Hamiltonian of the system (atom and field) is given by

$$\hat{H} = \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\omega\left(\hat{a}^\dagger\hat{a}\right) + \hbar g\left(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right) = \hat{H}_0 + \hat{H}_{\rm int}\,, \tag{2}$$

where ω is the resonance frequency of the atoms and of the cavity, $\hat{a}$ and $\hat{a}^\dagger$ are the usual annihilation and creation operators for the field mode, $\hat{\sigma}_i$ are the Pauli operators and $\hat{H}_{\rm int} = \hbar g\left(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right)$. In Eq. (2) g denotes the coupling constant between the atoms and the field mode. The atoms are detected, after they have passed through the cavity, in the coherent superposition $|\phi_k^{(f)}\rangle = \alpha_k^{(f)}|e\rangle + \beta_k^{(f)}|g\rangle$, again thanks to two classical fields with which the atoms interact after they exit the cavity: E_4 (non-resonant) and E_5 (resonant), like in the preparation region but in reverse order [7].

The problem is such that it can be treated iteratively, finding the recurrence relation between the coefficients of the Fock basis expansion of the field state inside the cavity after the interaction and the conditional measurement of the kth atom and the corresponding coefficients before [after the $(k-1)$th atom]. Then, by repeatedly applying such a recurrence relation, we can compute the coefficients in the number basis of the final field state (after a sequence of N atoms), starting from the initial coherent state, Eq. (1). For convenience we will work in the interaction picture [where $H_{\rm int}$ is regarded as the interaction part of the Hamiltonian (2)], and we will assume that the resonant fields E_1, E_3, and E_5 are phase-locked. In what follows we neglect spontaneous emission (since the transit time of the atoms is much smaller than the typical decay time) and any dissipation inside the cavity, assuming that the time required for the whole sequence of atoms is much smaller than the cavity lifetime.

Computing the evolved atom-field entangled state through the unitary evolution given by the Hamiltonian (2), and then projecting it onto the final atomic state, the following recurrence relation between the state of the field $|\psi_k\rangle = \sum_n d_n^{(k)}|n\rangle$ after the kth atom and the corresponding state after the $(k-1)$th atom can be found

$$\begin{aligned} d_n^{(k)} = P_k^{-1/2}\Big\{\Big[\alpha_k^{(i)}\alpha_k^{(f)*}C_n^{(k)} &+ \beta_k^{(i)}\beta_k^{(f)*}C_{n-1}^{(k)}\Big]d_n^{(k-1)} - i\alpha_k^{(i)}\beta_k^{(f)*}S_{n-1}^{(k)}d_{n-1}^{(k-1)} \\ &- i\beta_k^{(i)}\alpha_k^{(f)*}S_n^{(k)}d_{n+1}^{(k-1)}\Big\}\,, \end{aligned} \tag{3}$$

where $C_n^{(k)} = \cos(g\tau_k\sqrt{n+1})$, $S_n^{(k)} = \sin(g\tau_k\sqrt{n+1})$, and P_k is the success probability of the CM, which is given by the norm of the projection onto the final atomic state. In Eq. (3) it is understood that $d_{-1}^{(k-1)} = 0$.

3. Field state dynamics in the presence of random fluctuations

In this section we study the behaviour of the final field state (after many atoms have passed through the cavity) when we allow a spread in the atomic velocities, that is in the interaction times of the atoms with the cavity. The JC model has already been proposed [5] for the production of Fock states of the electromagnetic field inside a cavity, in connection with NSMs. That model, however, is very sensitive to even a small spread in atomic velocities [5], which eventually makes the system escape any fixed points in the evolution of the photon number distribution. As a consequence, such a scheme—notwithstanding its great pioneering value—is of no practical use in the production of Fock states, since any velocity selector for atomic beams allows a spread in the atomic velocities. In that approach, the convergence to a Fock state is due to the existence of the well known "trapping states" in the Jaynes-Cummings evolution [8]. However, in the case of $|e\rangle \to |e\rangle$ (elastic CMs) or $|e\rangle \to |g\rangle$ (inelastic CMs) schemes, if the interaction times τ_k fluctuate randomly with k, there is a critical value of the spread $\Delta\tau$ above which the number distribution will broaden rather than converge. We can estimate this critical value $\Delta\tau_c$ as the difference between the trapping and the anti-trapping interaction times for a given n [8], namely,

$$\Delta\tau_c \cong \frac{\pi}{g\sqrt{n+1}} - \frac{\pi}{2g\sqrt{n+1}} = \frac{\pi}{2g\sqrt{n+1}}\,, \tag{4}$$

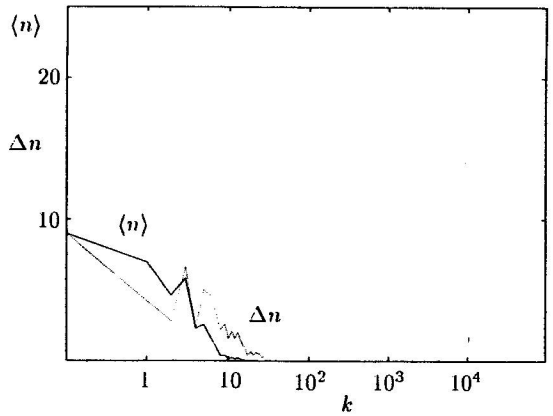

Fig. 1. Evolution of $\langle n\rangle$ and Δn versus k (number of atoms) in the scheme of elastic CMs in the case of a large spread ($\Delta\tau = \Delta\tau_c$) in the interaction times. The initial coherent state is $|\alpha\rangle = |3\rangle$.

where the sub-ensemble of $|e\rangle \to |e\rangle$ CMs has been considered. This phenomenon is shown in Fig. 1, where we plot for $\Delta\tau = \Delta\tau_c$ the mean value $\langle n\rangle$ and the rms spread $\Delta n = (\langle n^2\rangle - \langle n\rangle^2)^{1/2}$ as a function of the number of atoms injected into the cavity in their excited state and detected afterwards in the same excited state. Even though for $\Delta\tau \ll \Delta\tau_c$ a convergence towards a Fock state is still possible, for $\Delta\tau \approx \Delta\tau_c$ such a convergence is completely destroyed: the system escapes every fixed point because the trapping condition is different for each atom. In spite of this, we will show that it is possible to restore the convergence to fixed points even for large fluctuations ($\Delta\tau > \Delta\tau_c$), if we allow the presence of quantum interference between the sub-ensembles $|e\rangle \to |e\rangle$ and $|e\rangle \to |g\rangle$.

We explain this effect with a simple argument. Let us suppose that the initial state of each atom is the excited one so that the general transformation (3) simplifies to

$$d_n^{(k)} = P_k^{-1/2}\left[\alpha_k^{(f)*} C_n^{(k)} d_n^{(k-1)} - i\beta_k^{(f)*} S_{n-1}^{(k)} d_{n-1}^{(k-1)}\right], \tag{5}$$

where the coefficients of the final atomic superposition are expressed [9] in terms of the Rabi frequency $\Omega_k^{(f)}$ and of the interaction time $T_k^{(f)}$ with the resonant classical field E_5 according to $\alpha_k^{(f)} = \cos\left(\Omega_k^{(f)} T_k^{(f)}/2\right)$ and $\beta_k^{(f)} = \sin\left(\Omega_k^{(f)} T_k^{(f)}/2\right) e^{i\varphi_f}$. If we now choose $\varphi_f \simeq -\pi/2$, and neglect the difference between n and $n-1$, that is $d_n^{(k)} \simeq d_{n-1}^{(k)}$ and $S_{n-1}^{(k)} \simeq S_n^{(k)}$, Eq. (5) approximately reads

$$d_n^{(k)} \simeq P_k^{-1/2} \cos\left(\Omega_k^{(f)} T_k^{(f)}/2 - g\tau_k\sqrt{n+1}\right) d_n^{(k-1)}. \tag{6}$$

Since the atoms (with thermal velocity) cross first the cavity and then the classical field, $T_k^{(f)}$ and τ_k in Eq. (6) are correlated, even if they are random. This yields a strong

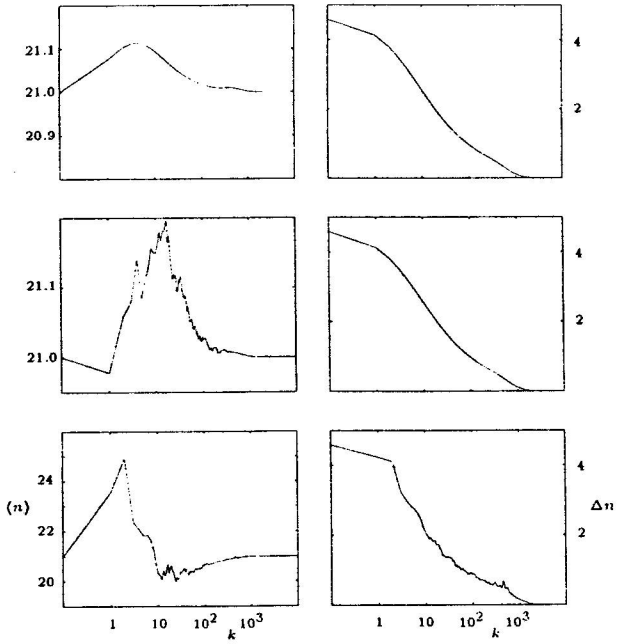

Fig. 2. Convergence towards a Fock state—even for a large spread in the interaction times—in the scheme $|e\rangle \to |e\rangle + |g\rangle$. Top: behaviour of $\langle n\rangle$ (left) and Δn (right) versus k (number of atoms) in the case of fixed interaction times; middle: the same for small fluctuations in the interaction times ($\Delta\tau = \Delta\tau_c/10$); bottom: the same for large spread in the interaction times ($\Delta\tau = 2\Delta\tau_c$). In all cases the initial coherent state is $|\alpha\rangle = |\sqrt{21}\rangle$.

suppression of the fluctuations in the argument of cosine in (6). This is shown in Fig. 2 where we plot (for the scheme $|e\rangle \to |e\rangle + |g\rangle$) $\langle n\rangle$ and Δn for fixed interaction times, and for small and large spreads in the interaction times. Notwithstanding the large fluctuations, the convergence towards the desired Fock state is still very good. This is confirmed by the final photon-number distribution $P(n)$, which corresponds to that of a number state: all the $P(n)$ vanish except one, $P(21) = 1$.

4. Discussion and Conclusions

In this paper we have presented a novel scheme which is able to produce preselected Fock states inside a cavity in which a coherent state was initially prepared. The final number state is achieved by sending many two-level atoms in their excited state through the cavity and by performing a conditional measurement of their internal degree of freedom in a superposition of the ground and excited states after they leave the cavity. The proposed scheme—differently from previous ones [5]— is quite effective and immune even to large fluctuations in the interaction times between the atoms and

the cavity field. This is achieved essentially thanks to two basic ingredients: (a) the conditional measurement of the final state of the atom, and (b) the quantum interference between the two possible atomic states ($|e\rangle$ and $|g\rangle$) after the interaction with the cavity. Since the classical NSMs counterpart of our model is chaotic (in the regime of random interaction times), and has a quantum dynamics similar to the classical one, such a striking behaviour suggests an analogy with recently proposed methods [6] of controlling classical chaos. These methods, mainly based on classical feedback, use the extreme sensitivity of chaotic systems to small perturbations in order to stabilise regular periodic orbits in the chaotic dynamics. In this perspective, our method can be considered as a novel (fully quantum) way of stabilising—even for large fluctuations—fixed points in the quantum dynamics of a system which is classically chaotic.

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