acta physica slovaca vol. 46 No. 3, 367 – 372 June 1996

QUANTUM PRISM FOR ATOMIC BEAMS: DEFLECTION BY A SINGLE CAVITY MODE IN FEW PHOTON LIMIT[1]

P. Domokos[2], P. Adam, J. Janszky
Research Laboratory for Crystal Physics, Hungarian Academy of Science
PO Box 132, H–1502 Budapest, Hungary

A. Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25
A–6020 Innsbruck, Austria

Received 31 May 1996, accepted 7 June 1996

It is shown that the deflection of an atom de Broglie wave at a non–resonant weak cavity field mode can yield an entangled state in which discernable atomic beams are entangled to photon number and internal atomic states.

1. Introduction

Deflection of an atomic de Broglie wave at electromagnetic field has received a considerable attention in the recent literature. For example, matter-wave interferometry can make use of diffraction off an intense standing light wave as a beam splitting mechanism [1]. On the other hand, if the field intensity is weak enough, interesting quantum phenomena occur in the course of the atomic wave diffraction process. Quantized [2] and field statistics sensitive [3] momentum transfer, quantum demolition [4] and nondemolition [5] measurement of the photon number distribution, quantum lens [6], etc. has recently been studied. In this paper we propose a scheme for a de Broglie wave beam splitter that is governed by microscopic quantum degrees of freedom of the atom–field system.

Let us consider a two-level atom crossing a high-Q cavity. The atomic transition $| e\rangle \leftrightarrow | g\rangle$ is strongly coupled to a slightly detuned Gaussian mode of the cavity. The dipole coupling strength is described by the inhomogeneous Rabi frequency $\Omega f(\mathbf{r})$ where $f(\mathbf{r})$ is the dimensionless rms vacuum field amplitude. Provided the inequality $\delta >> \sqrt{n+1}\Omega$ holds for the detuning, no photon exchange occurs between the field and the atom, so the energy of the field as well as the internal atomic state is a constant of

[1] Presented at the **4th central-european workshop on quantum optics**, Budmerice, Slovakia, May 31 - June 3, 1996

[2] E-mail address: domokos@sparc.core.hu

motion during the atom's traversal across the cavity. The influence of this type of non-resonant interaction is to yield a phase shift of the total state vector. If the atom moves slowly enough, the "dressed atom" system follows adiabatically a position-dependent level which can be regarded as a potential for the atomic center-of-mass motion. Within the adiabatic approximation, the unitary time evolution operator can be written as

$$\hat{U}(z) = | g\rangle\langle g | \otimes e^{-i\epsilon(z)\hat{a}^\dagger\hat{a}} + | e\rangle\langle e | \otimes e^{i\epsilon(z)(\hat{a}^\dagger\hat{a}+1)} , \qquad (1)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the creation and annihilation operators of the field mode, k is its wavevector, z gives the position where the atom crosses the cavity, ϵ characterizes the experimental parameters

$$\epsilon(z) = \frac{\Omega^2 l_{cav}}{\delta v_{at}} \cos^2 kz . \qquad (2)$$

Here v_{at} is the atomic velocity supposed to be constant. The z dependence of the mode function was explicitely used in this expression. l_{cav} is the effective cavity length of which dependence on z can be considered negligible in the case of Gaussian modes.

Using an arbitrary phase-space picture, one can see the effect of the interaction on the field state. For example let us describe the field mode by the Husimi Q function. If the atom is prepared in the state $| g\rangle$, then the following relation connects the output to the input field state:

$$Q_{|out\rangle}(\alpha) = Q_{|in\rangle}(\alpha e^{-i\epsilon(z)}) , \qquad (3)$$

which expresses a rotation in phase space with an angle $\epsilon(z)$. Note, that the angle of rotation depends on the position where the atom crosses the cavity. The atom prepared in the state $| e\rangle$ would rotate the field state in the opposite direction.

2. Diffraction of atomic waves off a quantum field

The unitary operator in Eq.(1) describes the dispersive interaction between the field mode and an atom which was supposed to cross the cavity along a straight line at a given position z. What happens if a transversely extended atom de Broglie wave impinges on the cavity? In the Raman–Nath regime, the z dependent evolution operator (1) can be applied on the wavefunction. Accordingly, the initial atomic wave experiences an inhomogeneous phase shift yielding a transformation of the initial wavefront. There is a certain regime of the mode function between a nodal and an adjacent antinodal of the standing wave, where the cosine–square function $\cos^2 kz$ can be approximated by the linear function $1/2 + kz$. In this regime the interaction with the field gives rise to a linear phase shift in function of z, which can be interpreted as if the cavity mode acted as a *prism* for the atomic wave. However, the characteristic of the prism, i. e. the angle of refraction, depends on the internal quantum states of the system. One can see from Eq.(1) that two constants of motion, namely the field energy, $\hat{a}^\dagger\hat{a}$, and the atomic population, $| e\rangle\langle e |$ and $| g\rangle\langle g |$, determine the angle of deflection. For an energy eigenstate of the field, i. e. Fock state $| n\rangle$ it is

$$\alpha(n, g) = n\epsilon\frac{k}{k_{at}} \qquad (4)$$

for an atom initially in the state $| g\rangle$, and

$$\alpha(n, e) = -(n+1)\epsilon\frac{k}{k_{at}} \qquad (5)$$

for an atom initially in the state $| e\rangle$. k_{at} is the atomic wavevector, and $\epsilon = \Omega^2 l_{cav}/\delta v_{at}$.

Internal degrees of freedom of the system govern the propagation of the matter wave. Therefore the linear regime of a non–resonant, single–mode quantum field in a high–Q cavity can be considered to be a *"quantum prism"* for the atom de Broglie wave.

3. Resolution

In what follows we prove that the quantum prism has one–photon resolution. In other words, atomic waves deflected by different Fock states are distinguishable, even if there is only one photon difference in the cavity.

For the sake of simplicity we suppose that the atomic wave is prepared to approximate a plane wave in front of the cavity, and in addition, we fix the atomic state to be $| g\rangle$. The transverse extension of the atomic wave is confined to the linear regime by placing an aperture of width A in front of the cavity. Without loosing any important physical effect we can limit the treatment to two dimensions, i. e., the system in the y direction is assumed to be uniform. According to the evolution operator in Eq.(1), the injected plane wave $\psi_{in}(x, z) = \mathcal{N} \exp ik_{at}x$ is tilted into the plane wave $\psi_{out}(x, z) = \mathcal{N} \exp(ik_{at}x + in\epsilon[1/2 + kz])$ when exactly n photon is present in the cavity. The outcoming plane wave would have a well–defined momentum in the z direction, but the finite size of the aperture introduces a broadening in the momentum distribution. To each photon number we can assign a peak having a half–width λ_{at}/A. Resolution requires that the broadening has to be dominated by the distinction of the peaks in the momentum distribution associated with adjacent photon numbers. From Eq.(4) the following condition can be obtained:

$$\epsilon k A \geq 2\pi \qquad (6)$$

This condition expresses the possibility of distinguishing atomic beams corresponding to different photon numbers. In the position distribution of the atomic wave this resolution becomes detectable in the far–field limit. Using the unitary operator describing the free–evolution of matter waves, one can derive the wavefunction of the system associated with the internal state $| n, g\rangle$ far from the cavity

$$\begin{aligned}\Psi_{far}(x,z) &= \mathcal{N} e^{ik_{at}(x+z^2/2x)} \frac{k_{at}}{2\pi i x} \times e^{in\epsilon/2} \int_{-A/2}^{A/2} e^{i(n\epsilon k - k_{at}z/x)\zeta} d\zeta \\ &\propto \frac{\sin\eta}{\eta} , \quad \text{where} \quad \eta = \frac{k_{at}A}{2}\left[\alpha(n,g) - \frac{z}{x}\right] \qquad (7)\end{aligned}$$

The corresponding position distribution is then a $\sin^2\eta/\eta^2$ type function. This function has one peak and oscillatory wings. The overlap between distributions corresponding to different photon numbers is minimized when the peak of one distribution coincides with a zero point of the other. This imposes a supplementary periodic condition on the size of the aperture

$$\epsilon k A = j \times 2\pi \,, \quad j = 1, 2, \ldots \tag{8}$$

The significance of this periodic condition can be elucidated by using a different approach.

Far enough from the cavity, the detection in the position $z_{det} = \alpha(m,g)x_{det}$ has the field part jump into the corresponding number state $| m\rangle$. Furthermore, a QND readout of the photon number takes place this way since the energy stored in the cavity is a constant of motion by virtue of the non-resonant feature of the interaction. The collapse of the field state can be traced by inserting the relation $z_{det} = \alpha(m,g)x_{det}$ into Eq. (7). Substituting $\varphi = \epsilon k \zeta$ and $\varphi_{min} = -\epsilon k A/2 = -\varphi_{max}$, if $\varphi_{max} - \varphi_{min} = 2\pi$ in accordance with Eq. (8), one can see that the output field state appears as the action of the projection operator

$$| m\rangle\langle m | = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i(m-\hat{a}^{\dagger}\hat{a})\varphi} d\varphi \tag{9}$$

on the arbitrary initial one. Applying this operator on a cavity field initially in the coherent state $| \beta\rangle$, the well-known representation of the $| m\rangle$ number state [7] can be recognized

$$| m\rangle = \text{const} \times \int_{-\pi}^{\pi} e^{-im\varphi} \, | \beta e^{i\varphi}\rangle d\varphi \,, \tag{10}$$

The expansion of the Fock state $| m\rangle$ in terms of coherent states having the same mean energy but different phases in phase space reflects that the back action of the energy measurement appears in increasing the uncertainty of the phase observable. As it follows from Eq.(3), each coherent state in the integral corresponds to one trajectory, or in other words, to one rotation angle in the phase space induced by the non-resonant interaction between the atom and the field. The condition for the boundaries, that is $\varphi_{max} - \varphi_{max}$ be an integer multiple of 2π, is required for the complete erasure of the phase information stored in the initial field. It ensures the possibility of reading out a photon number in a single measurement. This condition naturally coincides with (8) that was deduced by using different considerations.

4. Discussion

The quantum feature of the proposed atom optical prism becomes evident, if one applies it being in a superposition state. In a general case the atom is prepared in $a_g \, | g\rangle + a_e \, | e\rangle$, while the cavity field mode is set to an arbitrary state $\sum C_n \, | n\rangle$. Fulfillment of the conditions (6) and (8) ensures that each photon number and internal atomic state specifies a distinct direction of propagation. Hence, the impinging wave separates to partial waves that are *labelled* by the microscopic quantum numbers, i. e. the highly entangled state is produced:

$$| \, field + atom\rangle = \sum_n C_n a_g \, | n, g, \vec{\alpha}(n,g)\rangle + C_n a_e \, | n, e, \vec{\alpha}(n,e)\rangle \,, \tag{11}$$

where $\vec{\alpha}$ symbolizes a discernable atomic partial wave propagating into the given direction α. This entangled state is similar to that occuring in a Stern-Gerlach experiment, where the spin of the electron determines its path. The atomic states $| e\rangle$ and $| g\rangle$ can be regarded as "pseudo–spin" states that determine whether the beam is deviated up or down. However, the quantum prism creates a more composite situation inasmuch as the state of the cavity field mode also maps onto the diffraction pattern. Note that we assumed a non-resonant interaction between the atom and the field that requires $\Omega\sqrt{n+1} << \delta$. This condition limits the maximum photon number at which transitions between internal states are avoided and hence gives the maximum number of trajectories.

The entangled state of Eq.(11) is very intriguing in view of applications. It can be viewed as a device producing separate atomic partial waves from a single input one, where these beams are numbered by the quantum states $| n\rangle$. Controlling the photon statistics, i. e., the complex coefficients C_n in the cavity, one can manipulate the atomic beams. In the most elementary case, when the cavity mode is initially in vacuum state and the atom is prepared in the superposition $1/\sqrt{2}(| g\rangle + | e\rangle)$, the beam entangled to $| g\rangle$ crosses the cavity without altering its direction while in the state $| e\rangle$ the wave is subject to a deflection by an angle $\epsilon \frac{k}{k_{at}}$. The vacuum limit of the optical Stern–Gerlach experiment [8] can be realized this way.

In turn, information of the field state can be extracted from the measurement of the atom's distribution. As an evident possibility, QND readout of the photon number can be performed. But before detecting the atom, which results in a collapse of the highly entangled state (11), one could use an additional atom interferometer [1] to recombine different trajectories. For example, by the use of an intense, classical standing–wave field, the neighboring beams can be led to the same position at the observation plane. On detecting the atom in such a position, the field reduces to a superposition of the two corresponding number states. The probability of detecting the atom in this position is proportional to the squared modulus of the *sum* of these number state coefficients. Hence the quantum interference in the Hilbert space appears in the probability distribution of the atom at the detection plane. The phase of the complex coefficients can also be extracted from the mapping of the atom's position distribution in repeated measurements. Thus, beside the measurement of the absolute values of the Fock coefficients, the quantum prism makes possible to get complete information of the quantum state of the field mode. This type of experiment is referred to as quantum state reconstruction.

Acknowledgement This research has been supported by the National Scientific Research Foundation of Hungary (OTKA) under Contracts Nos. F017380 and T017386.

References

[1] E.M. Rasel, M.K. Oberthaler, H. Batelaan, J.Schmiedmayer, A. Zeilinger: *Phys. Rev. Lett.* **75** (1995) 2633; D.M. Giltner, R.W. McGovan, Siu Au Lee: *Phys. Rev. Lett.* **75** (1995) 2638;

[2] S. Stenholm: *Rev. Mod. Phys.* **58** (1986) 699;

[3] P. Meystre, E.Schumacher, S. Stenholm: *Opt. Comm.* **73** (1989) 443; V.M. Akulin, Fam Le Kien, W. P. Schleich: *Phys. Rev. A* **44** (1991) R1462;

[4] A.M. Herkommer, V.M. Akulin, W. P. Schleich: *Phys. Rev. Lett.* **69** (1992) 3298; M. Freyberger, A.M. Herkommer: *Phys. Rev. Lett.* **72** (1994) 1952;

[5] M.J. Holland, D.F. Walls, P. Zoller: *Phys. Rev. Lett.* **67** (1991) 1716; S. Haroche, M. Brune, J.M. Raimond: *Appl. Phys. B* **54** (1992) 355; P. Domokos, J. Janszky, P. Adam: *Phys. Rev. A* **50** (1994) 3340;

[6] Sh. Averbukh, V.M. Akulin, W. P. Schleich: *Phys. Rev. Lett.* **72** (1994) 437;

[7] P. Domokos, P. Adam, J. Janszky: *Phys. Rev. A* **50** (1994) 4293;

[8] T. Sleator, T. Pfau, V. Balykin, J. Mlynek: *Appl. Phys. B* **54** (1992) 375; F. Treussart, J. Hare, L. Collot, V. Lefèvre, D. Weiss, V. Sandoghdar, J.M. Raimond, S. Haroche: *Opt. Lett.* **19** (1994) 1651;