acta physica slovaca vol. 46 No. 3, 337 – 342 June 1996

MICROMASER DRIVEN BY ENTANGLED ATOM PAIRS[1]

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Received 31 May 1996, accepted 7 June 1996

A micromaser driven by a beam of entangled pairs of atoms is studied. Two schemes are considered where the atom pairs are prepared in coincidence states, $|a', a''\rangle + e^{i\phi}|b', b''\rangle$, or in anticoincidence states, $|a', b''\rangle + e^{i\phi}|b', a''\rangle$. In the absence of losses, we solve the former scheme analytically for the steady state of the micromaser field. At the presence of losses, the master equation for the latter scheme is derived and solved for the steady-state photon statistics. We argue why analytical solutions are not found in the alternative schemes.

1. Introduction

The so-called micromaser or one-atom maser [1] has become a landmark in quantum optics for its success in studying the quantum nature of the interaction of matter and light. In this system, a sparse beam of two-level atoms traverses a high-Q microwave cavity in such a way that there is at most one atom in the cavity at a time interacting resonantly with a single mode of the electromagnetic field. Due to the high-Q value it is a reasonable theoretical assumption to separate the single-atom + single-mode interaction given by the Jaynes-Cummings model [2] from the damping of the field due to cavity losses. In this case the problem becomes simple enough to be theoretically tractable [3]. As experimental techniques improved drastically due to a combined application of Rydberg-state spectroscopy and superconducting microwave cavities, experimentalists managed to realize extremely high-Q (3×10^{10}) micromaser systems in the laboratory [4]. In this way, experimental checks of the predictions of an exactly solvable theoretical model have become available – not a very frequent constellation in physics. It turned out that this rather simple system is rich of interesting physics and can be used to demonstrate various quantum phenomena. Prominent examples are the collapse/revival of Rabi oscillations [5], or generation of nonclassical states of fields [1,3–7] including quantum superpositions of separated micromaser fields [8]. Such entanglement between

[1]Presented at the **4th central-european workshop on quantum optics**, Budmerice, Slovakia, May 31 - June 3, 1996

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nonlocal fields has also been used in proposals for tests of fundamentals of quantum mechanics in the so-called "quantum eraser" setup [9].

In the present paper we also address the problem of entanglement. We want to know what the implications of prepared entanglement in the pumping atomic beam are on the behavior of the micromaser field. In particular, two schemes are considered where every consecutive pair of atoms is prepared in coincidence states, $|a',a''\rangle + e^{i\phi}|b',b''\rangle$ or in anticoincidence states, $|a',b''\rangle + e^{i\phi}|b',a''\rangle$. It is assumed throughout the paper that there is at most one atom in the cavity at a time. However, since the atoms are in correlated states the atom-field interactions are not independent from atom to atom. This is where the departure from ordinary micromaser operation originates from.

The paper is organized as follows. In the next section we present an analytical solution for the pure steady state of the field in the coincidence atomic state scheme in the absence of losses. We argue why solution cannot be found in the other scheme. In Sect. 3 the master equation for the anticoincidence scheme is derived and solved for the steady-state photon statistics in the presence of losses. It is argued why analytical solution is not found in the alternative scheme. We note, however, that results of numerical simulations for this scheme were presented at the workshop. Finally, Sect. 4 is devoted to conclusions.

2. Steady-state behavior in the absence of losses

Let us consider a microwave field in a cavity of no losses pumped by a beam of two-level atoms where every consecutive pair is in the coincidence entangled state. The initial state of an atom pair + field system reads as

$$|\Psi\rangle \otimes \frac{1}{\sqrt{2}}\left(|a',a''\rangle + e^{i\phi}|b',b''\rangle\right). \tag{1}$$

Here, the primes and double-primes respectively denote the first and second atom of the pair entering the cavity, and the field is given by

$$|\Psi\rangle \equiv \sum_n \Psi_n|n\rangle. \tag{2}$$

After the two atoms have interacted with the field the final state reads as

$$\begin{aligned}\frac{1}{\sqrt{2}}\sum_n \Psi_n \big(&C'_{n+1}C''_{n+1}|a',a'',n\rangle - iC'_{n+1}S''_{n+1}|a',b'',n+1\rangle \\ &-iS'_{n+1}C''_{n+2}|b',a'',n+1\rangle - S'_{n+1}S''_{n+2}|b',b'',n+2\rangle \\ &+e^{i\phi}C'_nC''_n|b',b'',n\rangle - ie^{i\phi}C'_nS''_n|b',a'',n-1\rangle \\ &-ie^{i\phi}S'_nC''_{n-1}|a',b'',n-1\rangle - e^{i\phi}S'_nS''_{n-1}|a',a'',n-2\rangle\big),\end{aligned} \tag{3}$$

where $C'_n \equiv cos(g'\tau'\sqrt{n})$ and $S'_n \equiv sin(g'\tau'\sqrt{n})$ correspond to the first atom with atom-field coupling constant g' and interaction time τ', and similarly C''_n and S''_n stand for the second atom of the pair. It is apparent from the above equation that the Fock space of the field is separated by the so-called trapping states into dynamically disconnected blocks. The lower boundary of a block is given by the "downward" trapping state, $|N_d\rangle$, defined by $S_{N_d} = 0$, and the upper boundary by the "upward" trapping state, $|N_u\rangle$, defined by $S_{N_u+1} = 0$. This means, that a field state initially bounded within one block completely will remain in the same block for any time, i.e., $\Psi_n \neq 0$ for $N_d \leq n \leq N_u$, but zero otherwise. This is the necessary condition for the existence of a normalizable steady state of the system in the absence of dissipation.

$|\Psi\rangle$ is a steady state of the field if, after the interaction, the atom pair + field system factorizes again as

$$e^{i\theta}|\Psi\rangle \otimes \left(c_{a',a''}|a',a''\rangle + c_{b',b''}|b',b''\rangle + c_{a',b''}|a',b''\rangle + c_{b',a''}|b',a''\rangle\right), \tag{4}$$

where $c_{i,j}$ and θ are independent of n and $\sum_{i,j}|c_{i',j''}|^2 = 1$. Comparing Eqs.(3) and (4) we arrive at the following system

$$e^{i\theta}c_{a',a''}\Psi_n = \frac{1}{\sqrt{2}}\left(\Psi_nC'_{n+1}C''_{n+1} - e^{i\phi}\Psi_{n+2}S'_{n+2}S''_{n+1}\right), \tag{5}$$

$$e^{i\theta}c_{b',b''}\Psi_n = \frac{1}{\sqrt{2}}\left(-\Psi_{n-2}S'_{n-1}S''_n + e^{i\phi}\Psi_nC'_nC''_n\right), \tag{6}$$

$$e^{i\theta}c_{a',b''}\Psi_n = \frac{-i}{\sqrt{2}}\left(\Psi_{n-1}C'_nS''_n + e^{i\phi}\Psi_{n+1}S'_{n+1}C''_n\right), \tag{7}$$

$$e^{i\theta}c_{b',a''}\Psi_n = \frac{-i}{\sqrt{2}}\left(\Psi_{n-1}S'_nC''_{n+1} + e^{i\phi}\Psi_{n+1}C'_{n+1}S''_{n+1}\right). \tag{8}$$

Substituting $n = N_d$ and $n = N_u$ into Eqs. (7) and (8), respectively, we find the solutions for the coincidence atomic amplitudes at steady-state given by

$$c_{a',a''} = \frac{e^{-i\theta}}{\sqrt{2}} \quad \text{and} \quad c_{b',b''} = \frac{e^{-i\theta}}{\sqrt{2}}e^{i\phi}. \tag{9}$$

As a consequence of this, the anticoincidence amplitudes are zero,

$$c_{a',b''} = c_{b',a''} = 0. \tag{10}$$

It can be seen that, apart from a physically irrelevant overall phase factor of $e^{-i\theta}$, the atom pair is bound to exit the cavity in the same coincidence entangled state as the initial one at steady state (compare Eqs.(1) and (4)).

The corresponding steady state of the field is determined by the following recursion formula,

$$S_{n+1}\left(S_{n+2}\Psi_{n+2} + e^{-i\phi}S_{n+1}\Psi_n\right) = 0. \tag{11}$$

Apparently, since every second photon numbers are coupled only this recursion provides two possible amplitude sequences. However, examining the formula carefully the first factor S_{n+1} proves to be important to avoid contradiction. We find that in the case when the trapping states, $|N_d\rangle$ and $|N_u\rangle$, bounding the field in the Fock space are of the same parity the only normalizable solution is given by

$$\Psi_{N_d+2n} = \Psi_{N_d}\left(-e^{-i\phi}\right)^n \prod_{k=1}^{n}\left(\frac{S_{N_d+2k-1}}{S_{N_d+2k}}\right). \tag{12}$$

The amplitudes range as $\Psi_{N_d}, \Psi_{N_d+2}, \ldots, \Psi_{N_u-2}, \Psi_{N_u}$, and zero otherwise. In the case when N_d and N_u are of opposite parity we obtain

$$\Psi_{N_d+2n+1} = \Psi_{N_d+1}\left(-e^{-i\phi}\right)^n \prod_{k=1}^{n}\left(\frac{S_{N_d+2k}}{S_{N_d+2k+1}}\right) \tag{13}$$

ranging as $\Psi_{N_d+1}, \Psi_{N_d+3}, \ldots, \Psi_{N_u-2}, \Psi_{N_u}$, and zero otherwise. This means that the pure steady states of the field are parity states including either even, or odd photon numbers only. They are bounded by trapping states in a disconnected block in the Fock space (overlap of several blocks is not considered here). The parity of the states is determined by the relative parity of the bounding trapping states. The case of Eq. (12) with even-even trapping states may be the most important for its including vacuum, $N_d = 0$, that is always an even parity trapping state.

Clearly, the existence of pure steady state in the present scheme is a consequence of quantum interference. Any possible state, $|a', a''\rangle$, $|b', b''\rangle$, $|a', b''\rangle$, and $|b', a''\rangle$, in the final state vector can be reached via two alternative paths. In the case of $|a', a''\rangle$, for example, the two paths are $|a', a''\rangle \to |a', a''\rangle \to |a', a''\rangle$ and $|b', b''\rangle \to |a', b''\rangle \to |a', a''\rangle$. The two paths change the photon number of the field in different ways that opens up the possibility to find a solution for field amplitudes separating the field state from the atomic state. We want to note, that this mechanism is similar to the one in the case of tangent/cotangent states of the micromaser field [6]. Actually, the difference between the two systems resides in the existence of atomic entanglement only. Therefore, a comparison of the two is a good way to see the effect of entanglement.

Without going into details we note here that after carrying out similar calculations for the anticoincidence scheme we found no solutions for the derived equations, i. e., the system has no steady state. There are interfering paths in this scheme too. However, they do not change the photon number of the field in different ways as it happened in the coincidence scheme but both alternatives correspond to the same photon number. For example, evolving into the final atomic state $|a', a''\rangle$ results in a decrease of the photon number by one any path you take. Similarly, $|a', b''\rangle$ and $|b', a''\rangle$, are reached with unchanged photon number, while $|b', b''\rangle$ corresponds to an increase of one for any paths and any initial atomic state. It follows, that there cannot be such field amplitudes that would separate the field degrees of freedom from those of the atomic.

3. Steady-state behavior in the presence of losses

Let us look at the scheme where the cavity field is pumped by atom pairs in anticoincidence entangled states, and the pairs enter the cavity according to a Poissonian statistics. We assume that the field can decay between the arrival of the atom pairs but not during the interaction of a pair with the field. In this way we can make maximize the effect of atomic entanglement on the field.

First, we find the master equation of the system for the density operator of the field, ρ. It reads as

$$\dot{\rho} = r^{(2)}\left(M^{(2)} - 1\right)\rho + \mathcal{L}\rho, \tag{14}$$

where $r^{(2)}$ is the average injection rate for the atom pairs, $\mathcal{L}\rho$ is the usual term accounting for the cavity losses and thermal radiation, and

$$M^{(2)}\rho = Tr_{at.}\left(U''U'\rho_{at.} \otimes \rho U'^{\dagger}U''^{\dagger}\right). \tag{15}$$

Here, U' and U'' denote the Jaynes-Cummings time-evolution operators for the single-atom + single-mode field interaction for the first and second atom, respectively, $\rho_{at.}$ is the two-atom density matrix, and $Tr_{at.}$ indicates that the trace is taken over the atomic degrees of freedom.

In the number representation the master equation exhibits a simple form of diagonal coupling and takes on the detailed balance equation given by

$$\dot{\rho}_{n,n} = \mathcal{J}_{n,n} - \mathcal{J}_{n-1,n-1}, \tag{16}$$

where

$$\begin{aligned}\mathcal{J}_{n,n} \;=\; & \frac{r_{(2)}}{2}\Big[\rho_{n+1,n+1}\left|S'_{n+1}C''_{n+1}e^{i\phi/2} + C'_{n+2}S''_{n+1}e^{-i\phi/2}\right|^2 \\ & -\rho_{n,n}\left|S'_{n+1}C''_{n+1}e^{i\phi/2} + C'_{n}S''_{n+1}e^{-i\phi/2}\right|^2\Big] \\ & +\gamma_c\,(n+1)\left[\rho_{n+1,n+1}\,(\bar{n}_b + 1) - \rho_{n,n}\bar{n}_b\right]\end{aligned} \tag{17}$$

At steady state $\mathcal{J}_{n,n} = 0$. Solving this equation for $\rho_{n,n}$ we arrive at the steady-state photon statistics of the field reading as

$$\rho_{n,n} = \rho_{0,0}\prod_{k=1}^{n}\left[\frac{\frac{r^{(2)}}{2}\left|S'_kC''_ke^{i\phi/2} + C'_{k-1}S''_ke^{-i\phi/2}\right|^2 + \gamma_c\bar{n}_bk}{\frac{r^{(2)}}{2}\left|S'_kC''_ke^{i\phi/2} + C'_{k+1}S''_ke^{-i\phi/2}\right|^2 + \gamma_c\,(\bar{n}_b+1)\,k}\right], \tag{18}$$

where γ_c is the cavity decay rate and $\bar{n}_b$ is the average number of thermal photons present in the cavity. Due to limitations of space we cannot illustrate this formula with specific examples here (some of them were discussed at the workshop). As a summary we can say that these photon statistics exhibit a similar "phase-like" structure as that of the ordinary micromaser. However, the photon numbers are in general characteristically smaller, new kinds of trapping states show up, and the system is very sensitive to the relative phase in the atomic entanglement.

Without going into details (detailed results were presented at the workshop) we want to mention here that analytical results for the coincident scheme were not found. The reason is that the coupling is not simple diagonal in this case but includes the first and second off-diagonals too. This makes the analytical treatment rather difficult.

However, we carried out numerical simulations of this system. It was found that the density matrix of the field includes large coherence terms at steady-state. Similarly to the diagonal terms where the probabilities of even photon numbers are enhanced against the odd ones resulting in oscillatory photon statistics, nonzero coherence terms are significant in the even-numbered off-diagonals. These results are reminiscent of the characteristics of the parity states found analytically in the previous section and show the effect of cavity losses and finite thermal radiation on the system.

4. Conclusions

We conclude that according to our calculations entanglement in the pumping atomic beam modifies the micromaser field drastically. The experimental verification of these predictions, however, appears to be a formidable challenge due to the difficulties in preparing the atoms in the proper entangled states. We nevertheless beleive that this calculation is a good illustration of and may encourage further research on the effects of entanglement on quantum optical systems.

Acknowledgements We thank J. Bergou for useful discussions, P. Zoller for hospitality at the University of Innsbruck, and the Max-Planck-Gesellschaft (Quantenoptik) for support.

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