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QUANTUM NOISE IN AMPLIFICATION AND ATTENUATION[1]

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The reservoir can be modelled by an infinite array of beam splitters and the superposition of input fields in a lossless beam splitter can be described by the convolution of quasiprobability functions. We use the convolution relation and the beam splitter model to derive the Fokker-Planck equation for a system coupled with a phase-sensitive reservoir. Solving the Fokker-Planck equation, we test the coincidence of the loss of well-known nonclassical properties with the appearance of the positive well-defined Glauber-Sudarshan P function.

1. Introduction

The evolution of the quantum noise in an amplifier or an attenuator has been developed extensively ([1,2] and references therein). The amplifier/attenuator can be modelled by an infinite array of amplifier/attenuator sets each of which gives an infinitesimal gain/loss. We follow the usual treatment where the matter is traced out and the statistics is reduced to an optical constant. Under this assumption the distribution of the absorption and stimulated emission centred in a real medium is well modelled by the uniform regular distribution of the amplifier/attenuator sets. The amplifier/attenuator

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superposes noise onto the signal while the signal is amplified or attenuated, which is analogous to the action of a beam splitter. In the beam splitter, the signal is attenuated and superposed with the noise field. The superposition between the attenuated signal and the noise is described by the convolution relation for their quasiprobability functions. The amplifier can also be modelled by a parametric amplifier and a similar convolution relation was found. Using the convolution relations the Fokker-Planck equations are found for the evolution of the signal field in the amplifier/attenuator.

It has been well-known that the quasiprobabilities, especially Glauber-Sudarshan P function (P function), can be used as the measure of the nonclassicality of the given field [3]. In fact the P function can be defined for any state including a Fock state [4]. However we are interested in the P function as a probability density-like function. Throughout this paper the P function is said to exist only when the P function is well-behaved. Among quantum-mechanical pure states the coherent state is the only state with a definite form of the P function [5]. The Husimi Q function (Q function) is always positive definite while the Wigner function always exists but can be negative as well. The fields with classical correspondence should be represented by positive well-behaved quasiprobabilities in phase space.

In this paper we shall discuss the existence and negativity of the quasiprobabilities of the field state during attenuation and amplification processes connected with a finite temperature bath. We shall compare the moments when the quasiprobability functions become positive definite with the moments when some nonclassical features such as quadrature squeezing or sub-Poissonian photon number distribution disappear.

The photon number fluctuations can be measured by the photon number variance. More conveniently, the normal-ordered photon number variance shows the photon number variance less the mean photon number so that the normal-ordered photon number variance is useful to show the relative size of the noise compared with the amplified signal. Even though the photon number variance monotonously grows even with the phase-sensitive amplification, the signal to noise ratio can be enhanced. It was earlier suggested that if we use the phase-sensitive amplifier the photon number fluctuations can be reduced so that a super-Poissonian input can turn into sub-Poissonian during the amplification [6]. Despite the work by Barnett and Gilson who proved that it is impossible to generate the sub-Poissonian output from the super-Poissonian input by the amplification [7], it is true that the problem has not been clearly unravelled yet. In this paper solving the Fokker-Planck equation for the phase-sensitive linear amplifier we study the photon statistics of the amplified field.

In a real optical amplifier, distributed loss can concurrently exist with the distributed gain. Also, one of the main subjects in long-haul fibreoptic communication is how to amplify the optical signal to compensate the transmission loss, and such amplification can be realised by using an Er-doped fibre, which is a lossy distributed amplifier.

We derive a very general Fokker-Planck equation for the distributed amplification in Section 2 when the added noise is phase-sensitive. As a special case we consider the evolution of an initial Fock state coupled with the thermal bath to test the coincidence between the lost of sub-Pöissonicity and appearance of the positive well-behaved P function in Section 3. The quantum noise in distributed amplifiers is analysed in Section 4. As a special case the possibility of amplifying the super-Poissonan field into the sub-Poissonian field is discussed.

2. Fokker-Planck equation

Consider that two optical fields are fed to the two input ports of a beam splitter and are superposed. The beam splitter is assumed to be lossless, and its reflection and transmittance coefficients are respectively denoted by R and T. For convenience we refer to the two fields as the signal and the noise. The signal and noise fields can be represented in phase space by their Wigner functions, $W_s(\alpha)$ and $W_{n2}(\alpha)$, respectively, and the transmitted signal through the lossless beam splitter is described by the convolution law [1,2]:

$$W_{at}(\alpha) = \frac{1}{T}\int d^2\phi\, W_{n2}(\phi) W_s\left(\frac{\alpha - \sqrt{R}\phi}{\sqrt{T}}\right), \tag{1}$$

where $W_{at}(\alpha)$ is the Wigner function for the transmitted signal. This shows that the signal is attenuated as $T \leq 1$ and superposed with the noise. As T goes to zero, the signal loses its initial property and is substituted by the noise.

The noise is introduced usually by the thermal reservoir, which is phase-insensitive. However, the phase-sensitive noise can also be considered based on establishment of squeezed light[8]. The Wigner function $W_{sq}(\beta)$ for the squeezed thermal field is Gaussian:

$$W_{sq}(\beta) = \frac{1/\pi}{\sqrt{(\frac{1}{2}+N)^2 - M^2}} \exp\left(-\frac{\beta_1^2}{\frac{1}{2}+N-M} - \frac{\beta_2^2}{\frac{1}{2}+N+M}\right), \tag{2}$$

where N is the phase-insensitive parameter representing the mean photon number of the squeezed thermal field and M is the phase-sensitive parameter, whose value is restricted by

$$M^2 \leq N(N+1); \qquad M \text{ real.} \tag{3}$$

Linear amplification is modelled by a parametric amplifier. In the amplification process, energy is poured into the system by pumping. Differently from the ordinary beam splitter case, to cope with the pumping we remove the energy conservation constraint, $R + T = 1$, in the amplification beam splitter model. The Wigner function $W_{am}(\alpha)$ for the amplified signal is then described by the convolution relation [1]

$$W_{am}(\alpha) = \frac{1}{g}\int d^2\lambda\, W_{n1}(\lambda) W_s\left(\frac{\alpha - \sqrt{g-1}\,\lambda}{\sqrt{g}}\right), \tag{4}$$

where $W_{n1}(\lambda)$ is the Wigner function for the noise, and g is the gain factor of the amplifier. When $g = 1$, the signal is kept constant without amplification.

Let us assume an amplifier-attenuator set (am-at set). In the first half of the am-at set, the signal field is amplified and is superposed with the noise field during the amplification. In the second half of the am-at set, the amplified signal is attenuated, and the other type noise associated with the dissipation is added. In this paper we derive a formalism for a general case where the amplification noise factors N_1 and M_1 are different from the attenuation noise factors N_2 and M_2.

From Eqs. (1) and (4), we find that the Wigner function $W(\alpha)$ of the signal field after passing through the am-at set is represented by the convolution relation

$$W(\alpha) = \frac{1}{gT}\int d^2\xi\, W_{no}(\xi) W_s\left(\frac{\alpha - \sqrt{1-gT}\,\xi}{\sqrt{gT}}\right), \tag{5}$$

where the Wigner function W_{no} for the combined noise is defined as

$$W_{no}(\xi) = \frac{|1-gT|}{\pi\sqrt{AB}} \exp\left[-|1-gT|\left(\frac{\xi_1^2}{A} + \frac{\xi_2^2}{B}\right)\right], \tag{6}$$

with the noise parameters

$$\begin{aligned} A &\equiv T(g-1)(1/2+N_1-M_1) + (1-T)(1/2+N_2-M_2) \\ B &\equiv T(g-1)(1/2+N_1+M_1) + (1-T)(1/2+N_2+M_2)\,. \end{aligned} \tag{7}$$

We now derive the Fokker-Planck equation for the infinite array of am-at sets using (5). The total duration of time when the field is coupled with the am-at sets is denoted by t, the total number of the am-at sets by $\aleph$, and the interval between the adjacent am-at sets by $\Delta\tau$ with $\aleph = t/\Delta\tau$. The am-at sets are first taken to be discrete components. Under the assumption that the amplification and attenuation in each am-at set is very small, i.e.,

$$0 < 1-T = R \ll 1 \quad \text{and} \quad 0 < g-1 = \epsilon \ll 1 \tag{8}$$

Eq. (5) can be written as

$$W(\alpha) \approx (1+R-\epsilon)\int d^2\xi\, W_{no}(\xi) W_s\left(\frac{\alpha - \sqrt{1-gT}\,\xi}{\sqrt{gT}}\right). \tag{9}$$

We define $W(\tau;\alpha)$ as the Wigner function of the signal field incident on the am-at set at time τ, $W(\tau+\Delta\tau;\alpha)$ as the Wigner function for the signal leaving from the am-at set. From Eq. (9), we obtain the relation

$$W(\tau+\Delta\tau;\alpha) = (1+R-\epsilon)\int d^2\xi\, W_{no}(\xi) W_s\left(\tau;\frac{\alpha - \sqrt{1-gT}\,\xi}{\sqrt{gT}}\right), \tag{10}$$

where the argument $(\alpha - \sqrt{1-gT}\,\xi)/\sqrt{gT}$ is expanded as

$$\frac{\alpha - \sqrt{1-gT}\,\xi}{\sqrt{gT}} \approx \alpha + \frac{R-\epsilon}{2}\alpha - \sqrt{R-\epsilon}\,\xi \equiv \alpha + \Delta\alpha\,, \tag{11}$$

where up to the first order terms of R and ϵ are kept. The usual Taylor expansion for a real function having a complex argument is used to expand the Wigner function of the propagating signal field,

$$\begin{aligned} W(\tau;\alpha+\Delta\alpha) &= W(\tau;\alpha) + \frac{R-\epsilon}{2}\left[\frac{\partial W}{\partial\alpha_1}\alpha_1 + \frac{\partial W}{\partial\alpha_2}\alpha_2\right] \\ &+ \frac{R-\epsilon}{2}\left[\frac{\partial^2 W}{\partial\alpha_1^2}\xi_1^2 + \frac{\partial^2 W}{\partial\alpha_2^2}\xi_2^2 + \frac{\partial^2 W}{\partial\alpha_1\partial\alpha_2}\xi_1\xi_2\right], \end{aligned} \tag{12}$$

where the real and imaginary parts of α and ξ are, respectively, denoted by α_1, α_2 and ξ_1, ξ_2.

To simulate the distributed amplifier, we consider the am-at sets forming a continuous array by taking the limits $\aleph \to \infty$ and $\Delta\tau \to 0$, while keeping $\aleph\Delta\tau$ constant. In these limits, we define the attenuation coefficient, $\kappa = \lim_{\Delta\tau\to 0} R/\Delta\tau$, and the amplification coefficient, $\gamma = \lim_{\Delta\tau\to 0}\epsilon/\Delta\tau$. In the above limit, with Eqs. (2), (10) and (12), we obtain the Fokker-Planck equation for the field which propagates in the infinite array of the am-at sets:

$$\frac{\partial}{\partial\tau}W(\tau;\alpha) = \left(\frac{\delta}{2}\frac{\partial}{\partial\alpha_1}\alpha_1 + \frac{\delta}{2}\frac{\partial}{\partial\alpha_2}\alpha_2 + \frac{a}{4}\frac{\partial^2}{\partial\alpha_1^2} + \frac{b}{4}\frac{\partial^2}{\partial\alpha_2^2}\right) W(\tau;\alpha)\,, \tag{13}$$

where $\delta = \kappa - \gamma$ and the diffusion coefficients are

$$\begin{aligned} a &= \gamma(1/2+N_1-M_1) + \kappa(1/2+N_2-M_2) \quad \text{and} \\ b &= \gamma(1/2+N_1+M_1) + \kappa(1/2+N_2+M_2)\,. \end{aligned} \tag{14}$$

It is obvious that the Fokker-Planck equation (13) has the same form when each am-at set is replaced by the attenuation-amplification set where infinitesimal amplification follows infinitesimal attenuation.

When γ is smaller than κ the signal is attenuated and when κ is smaller than γ the signal is amplified. If the gain factor g is equal to unity, i.e., $\gamma = 0$, then the infinite array of the am-at sets becomes an ordinary attenuator. Similarly, if the transmission coefficient T is equal to unity, i.e., $\kappa = 0$, then the system becomes an ordinary amplifier. From Eq. (13), we can obtain the Fokker-Planck equations for the pure amplifier ($\kappa = 0$) and for the pure attenuator ($\gamma = 0$).

3. Amplification and attenuation of Fock states for M=0

We take a Fock state $|m\rangle$, with m photon numbers, to be attenuated in a phase-insensitive heat bath ($N \neq 0$, $M = 0$). The Wigner function for the Fock state is

$$W_m(\alpha) = \frac{2}{\pi}(-1)^m L_m(4|\alpha|^2), \tag{15}$$

where $L_m(x)$ is a Laguerre polynomial. When $\gamma = 0$, we solve the Fokker-Planck equation (13) for the attenuation of the initial Fock state.

$$\begin{aligned} W_m(\alpha) &= \frac{2[1+2N(1-e^{-\kappa t}) - 2e^{-\kappa t}]^m}{\pi[1+2N(1-e^{-\kappa t})]^{m+1}} \exp\left(-\frac{2|\alpha|^2}{1+2N(1-e^{-\kappa t})}\right) \\ &\times L_m\left[-\frac{4e^{-\kappa t}|\alpha|^2}{[1+2N(1-e^{-\kappa t})][1+2N(1-e^{-\kappa t}) - 2e^{-\kappa t}]}\right], \end{aligned} \tag{16}$$

where $t > 0$.

There are quasiprobabilities other than the Wigner function, for example, the Q and the P functions. Other quasiprobabilities can be obtained from the Wigner function through the following convolution relation [5]:

$$R(\alpha, s) = \int d^2\beta W(\beta)\left[\frac{-2}{\pi s}\exp\left(\frac{2|\alpha-\beta|^2}{s}\right)\right], \tag{17}$$

where $R(\alpha, s)$ is the Q function for $s = -1$ and the P function for $s = 1$. When $s = 0$, $R(\alpha, s)$ is the Wigner function.

The P function does not exist for the initial Fock state but using the relation (17) and Eq.(16) we can see that the P function exists as soon as the system is influenced by the heat bath even though it shows negativity at some α's. By the properties of the Laguerre polynomials we find that the P function $P_{m,t}(\alpha)$ is positive at any α when the decay time t is larger than t_1, where t_1 is defined by the relation

$$e^{-\kappa t_1} = \frac{N}{1+N}. \tag{18}$$

We find an interesting result that the characteristic decay time t_1 does not depend on the photon number m of the initial Fock state but only on the average thermal photon number N of the heat bath. If the Fock state decays into the vacuum the P function is never positive definite.

The moments of the field are derived with the use of the Wigner function:

$$\langle\{(a^\dagger)^m a^n\}_s\rangle = \int d^2\alpha(\alpha^*)^m\alpha^n W(\alpha), \tag{19}$$

where $\{(a^\dagger)^m a^n\}_s$ represents the symmetrical-ordering of bosonic operators. Substituting the quasiprobability $W_m(\alpha, t)$ in Eq.(16) into Eq.(19), the symmetrical-ordered moments are calculated.

Let us now consider when the well-known nonclassical properties are lost during attenuation. The initial Fock state is diagonalised and the heat bath is also diagonalised. When an intial diagonalised field gradually approaches to a final diagonalised state the off-diagonal terms do not appear. Thus neither quadrature squeezing nor amplitude-squared squeezing appears during the attenuation process. The Fock state is highly sub-Poissonian with the photon-number variance $(\Delta n)^2 = 0$ by definition. With the use of Eq.(19) we find the normal-ordered photon-number variance for the evolution of the initial Fock state in the heat bath:

$$:(\Delta n)^2: = (N^2 - m - 2mN)e^{-2\kappa t} + 2N(m-N)e^{-\kappa t} + N^2. \tag{20}$$

The normal-ordered photon-number variance is zero when the field is Poissonian and less (more) than zero when the field is sub-Poissonian (super-Poissonian). The normal-ordered photon-number variance exceeds the Poisson (quantum) limit when the decay time is larger than t_2, where t_2 is defined by the relation,

$$e^{-\kappa t_2} = \frac{N}{N+K}, \tag{21}$$

where the parameter $K = \sqrt{m^2+m} - m$ and, as m is positive definite, $0 \le K < 1/2$. We find that the time when for the decaying Fock state the P function becomes positive does not coincide with the time when the sub-Poissonian nature of the initial state is lost. Whereas the characteristic time t_1 depends only on the mean photon number of the heat bath N, t_2 depends on the initial photon number m as well. When the P function is positive definite the field is always super-Poissonian. However the inverse statement is not always true. For example, in the time interval between t_1 and t_2 the field is super-Poissonian without the presence of a positive P function.

The oscillations in the photon number distribution is another nonclassical feature of the field [9]. The photon number distribution P(n) is defined as the probability of having n photons in the given field. This can be obtained as a scalar product of the Wigner function $W(\alpha)$ with the n-photon Fock state Wigner function of (15),

$$P(n) = \pi\int d^2\alpha W(\alpha)W_n(\alpha). \tag{22}$$

Substituting Eq.(16) into Eq.(22), the photon number distribution can easily be plotted for the initial Fock state decaying into the heat bath, from which we can see no oscillations [10]. It is straightforward to show that the higher-order moments in photon number distribution, for example $(\Delta n)^4$, also indicate that the field is nearly Poissonian at the decay time t_2.

The amplification of a field is inevitably accompanied by added noise. Taking $\kappa = 0$ we solve the Fokker-Planck equation (13) for the amplified Fock state. The solution is analogous to Eq. (16) as we replace N by $-N-1$ and $\exp(-\gamma t)$ by $G = \exp(\gamma t)$. Thus the P function becomes positive at any α when $G > G_1$ where G_1 is defined as

$$G_1 = \frac{N+1}{N}. \tag{23}$$

By the same analogy we also find that the sub-Poissonicity is lost when $G > G_2$ where

$$G_2 = \frac{N+1}{N+1-K}. \tag{24}$$

The characteristic amplification factor G_1 is larger than G_2 so that the amplified field becomes super-Poissonian before the $P(\alpha)$ function becomes positive definite. Similarly to the dissipation case the field is represented by diagonal terms throughout the amplification so neither quadrature squeezing nor amplitude-squared squeezing occurs.

4. Quantum noise in distributed amplifiers

We first solve the Fokker-Planck equation (13) for an arbitrary signal in the distributed amplifier and analyse the added noise [11]. The Wigner function for an arbitrary input state of the single-mode field can be represented as a weighted integral of complex Gaussian functions

$$W_s(\alpha) = \frac{2}{\pi}\int d^2\mu d^2\nu P(\mu,\nu)\exp\left[-2(\alpha_1 - E)^2 - 2(\alpha_2 - F)^2\right], \tag{25}$$

where the weight function $P(\mu,\nu)$ is the positive P function which is a quasiprobability function defined in four-dimensional phase space. The complex variables E and F have been defined as

$$E = (\mu+\nu)/2 \quad \text{and} \quad F = -i(\mu-\nu)/2 \,. \tag{26}$$

The initial Gaussian function of the quantum system remains Gaussian in a linear Fokker-Planck equation such as Eq.(13) with time-dependent parameters [11]. The Wigner function $W_{\rm out}(\alpha,\tau)$ for the output signal can then be written in the form

$$W_{\rm out}(\alpha,\tau) = \frac{1/\pi}{\sqrt{C^2(\tau)-D^2(\tau)}}\int d^2\mu d^2\nu P(\mu,\nu) \times \exp\left\{-\frac{[\alpha_1-E(\tau)]^2}{C(\tau)+D(\tau)} - \frac{[\alpha_2-F(\tau)]^2}{C(\tau)-D(\tau)}\right\}, \tag{27}$$

where the displacement of the signal in phase space is represented by

$$E(\tau) = E\exp(-\delta\tau/2) \quad \text{and} \quad F(\tau) = F\exp(-\delta\tau/2)\,. \tag{28}$$

The time-evolution of the noise parameters in Eq.(27) are

$$C(\tau)\pm D(\tau) = \frac{1}{2}e^{-\delta\tau} + \left[\frac{\gamma}{\delta}\left(\frac{1}{2}+N_1\mp M_1\right)+\frac{\kappa}{\delta}\left(\frac{1}{2}+N_2\mp M_2\right)\right](1-e^{-\delta\tau})\,. \tag{29}$$

A. Quantum noise in amplifiers

We can also consider the phase-sensitive amplifier which can be implemented as a stream of three-level atoms in a ladder configuration with equispaced levels injected into the cavity where the initial state of the field has been prepared [6, 12]. We denote the population in the uppermost state by ρ_{aa}, the population in the lowest state by ρ_{cc} and the coherences between them by ρ_{ac} and ρ_{ca}. The atomic coherences ρ_{ac} and ρ_{ca} bring about the phase-sensitive effect in the two-photon linear amplifier. The parameters N and M can then be represented by the atomic variables

$$N = \frac{\rho_{cc}}{\rho_{aa}-\rho_{cc}}, \qquad M = \frac{|\rho_{ac}|}{\rho_{aa}-\rho_{cc}}\,. \tag{30}$$

With use of Eq.(19) and Eq.(27) we find the normally-ordered photon number variance

$$:(\Delta n)^2: = G^2 :(\Delta n)^2_s: + \zeta, \tag{31}$$

where $\langle\ \rangle_s$ stands for the expectation value of the signal field and the additive noise is

$$\zeta = \frac{(G-1)^2}{(\rho_{aa}-\rho_{cc})^2}(\rho_{aa}^2+|\rho_{ac}|^2) + \frac{G(G-1)}{\rho_{aa}-\rho_{cc}}[2\rho_{aa}\langle a^\dagger a\rangle_s - |\rho_{ac}|(\langle a^2\rangle_s + \langle (a^\dagger)^2\rangle_s]. \tag{32}$$

If the additive noise is negative, the amplified field has less photon number fluctuation than the input field. It is clearly seen that if the atomic coherence, ρ_{ac}, is zero the additive noise is always positive. However as the atomic coherence is nonzero we can have the negative noise to enhance the signal to noise ratio.

If each atom injected into the cavity is in atomic coherences we have the relation $\rho_{aa}\rho_{cc} = |\rho_{ac}|^2$ and the additive noise (32) can be written as

$$\zeta = G(G-1)\frac{c|\rho_{ac}|}{\rho_{aa}-\rho_{cc}} + 2G(G-1)\frac{\rho_{aa}-\rho_{cc}}{\rho_{aa}-|\rho_{ac}|}\langle a^\dagger a\rangle_s + (G-1)^2\frac{\rho_{aa}}{(\rho_{aa}-\rho_{cc})^2}, \tag{33}$$

where

$$c = 2\langle a^\dagger a\rangle_s - (\langle a^2\rangle_s + \langle (a^\dagger)^2\rangle_s), \tag{34}$$

which has to be negative to have the additive noise negative. The bosonic operators a and $a^\dagger$ have a simple restriction, $2\langle a^\dagger a\rangle - (\langle a^2\rangle + \langle (a^\dagger)^2\rangle) \geq -1$. It is thus required that

$$-1 \leq c < 0 \tag{35}$$

for the noise reduction in the amplified signal. The noise reduction in the photon number fluctuations seems to be possible if the input field satisfies Eq.(35). However we should not fail to notice that the condition (35) is related to the initial photon number fluctuations. Because the expectation value of an operator times its hermitian conjugate is again positive,

$$c \geq -\frac{:(\Delta n)^2_s:}{\langle a^\dagger a\rangle}. \tag{36}$$

It is easily seen from Eqs.(35) and (36) that the input field should be super-Poissonian to have a possibility to reduce the photon number fluctuations by the amplification. If the input field is Poissonian there is no intersection between the two conditions (35) and (36) so that we can say that the Poissonian field does not become sub-Poissonian during the amplification.

B. Quantum noise in distributed amplifier for $\gamma = \kappa$

We are interested in the case when the amplification just compensates the distributed attenuation. For a special case of $\kappa = \gamma$ the mean photon number is calculated using the relation (19) and the time evolution of the Wigner function (27):

$$\langle \hat n\rangle = \langle \hat n\rangle_s + (N_1+N_2+1)\gamma\tau\,. \tag{37}$$

This shows that even when $\gamma = \kappa$ the mean energy grows due to the noise added into the signal. In this case the mean field is not changed.

The fluctuation in the photon number is considered using the normal-ordered variance of the photon number:

$$\langle:(\Delta\hat n)^2:\rangle = \langle:(\Delta\hat n)^2:\rangle_s + \gamma\tau[2(N_1+N_2+1)\langle\hat n\rangle_s + (M_1+M_2)(\langle\hat a^2\rangle_s + \langle(\hat a^\dagger)^2\rangle_s)] + \gamma^2\tau^2[(N_1+N_2+1)^2+(M_1+M_2)^2]. \tag{38}$$

The normal-ordered variance (38) is a quadratic equation with regard to $\gamma\tau$ and the coefficient for the quadratic and constant terms are positive for initial super-Poissonian fields. To reduce the noise in the photon number fluctuation the coefficient of the first-order term should be negative so that the minimum value of the quadratic equation exists when $\gamma\tau > 0$.

When $N \gg 1$, the maximally squeezed state has $M \simeq -(N+1/2)$ and the coefficient of the first-order term in $\gamma\tau$ of Eq.(38) is

$$\begin{aligned} 2(N_1+N_2+1)\langle\hat{n}\rangle_s &+ (M_1+M_2)(\langle\hat{a}^2\rangle_s + \langle(\hat{a}^\dagger)^2\rangle_s) \\ &\geq (N_1+N_2+1)[2\langle\hat{n}\rangle_s - (\langle\hat{a}^2\rangle_s + \langle(\hat{a}^\dagger)^2\rangle_s)] \end{aligned} \quad (39)$$

for $\langle\hat{a}^2\rangle_s$ and $\langle(\hat{a}^\dagger)^2\rangle_s$ positive. With a simple restriction for bosonic operators [12]

$$2\langle\hat{n}\rangle - (\langle\hat{a}^2\rangle + \langle(\hat{a}^\dagger)^2\rangle) \geq -\frac{:(\Delta n)^2:}{\langle\hat{n}\rangle} \quad (40)$$

we find that when the signal is Poissonian the photon number fluctuation does not reduce under the quantum limit.

We now consider the quadrature noise added by the distributed amplification. The quadrature amplitude operator is defined as

$$\hat{\chi}_\theta = \frac{1}{2}(\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}). \quad (41)$$

The quadrature variance for the signal field in the distributed amplifier is

$$\langle(\Delta\hat{\chi}_{\theta=0})^2\rangle_{out} = \langle(\Delta\hat{\chi}_{\theta=0})^2\rangle_s + \frac{1}{2}(1+N_1-M_1+N_2-M_2)\gamma\tau \quad (42)$$

which is *linearly* dependent on time. When the noise field is maximally squeezed with a proper choice of the phase, i.e., $M_{1,2} \approx N_{1,2} + \frac{1}{2}$ for $N \gg 1$, the time-dependent term vanishes in Eq.(42). In other words, no noise is added into $\theta = 0$ quadrature. It does not mean that no noise is added into any of the quadratures. After a little calculation, we find that the maximum noise has been added, for example, into $\theta = \pi/2$ quadrature. If $M_{1,2} \approx -N_{1,2} - \frac{1}{2}$ for $N \gg 1$, the maximum noise is added into the $\theta = 0$ quadrature while no noise is added into the $\theta = \pi/2$ quadrature.

5. Discussion

We have considered quantum noise in amplification and attenuation. It has been shown that the Fock state loses its sub-Poissonicity before the P function becomes to be positive in the whole phase space during its amplification and attenuation in heat baths. The super-Poisson field never becomes sub-Poissonian during the phase-sensitive amplification.

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