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ENTANGLED FIELD STATES IN MULTIPLE MICROMASER CAVITIES AND FUNDAMENTAL TESTS OF QUANTUM MECHANICS[1]

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Highly entangled states of more than one quantum system are necessary for conceptual tests of quantum mechanics and other alternative theories. These tests include local hidden variables theories, pre- and postselection, QND measurements and quantum computing, and tests of quantum mechanics itself against, e.g., the so-called causal communication constraint. We show how to produce nonlocal states of the electromagnetic field which are located in several cavities and are highly entangled. We first discuss a two-cavity state, one of the four Bell basis states for this system, in which both cavities are in a one-photon state or both are in the vacuum state. We then discuss a straightforward generalization to a similar n-cavity state. These states can be produced by sending apropriately prepared atoms through the cavities. As applications we present a test of quantum mechanics against the causal communication constraint using the two-cavity state and a test of the pre- and postselection quantum mechanics using the three-cavity state.

1. Introduction

The state of two coupled quantum systems is said to be entangled if it cannot be expressed in any basis as a product of states of the individual systems. This implies that the two systems are correlated. In fact, if the degree of entanglement is large enough, Bell's inequality can be violated [1]. Consequently, entangled states feature prominently in investigations of the foundations of quantum mechanics. For example, a recent proposal by Greenberger, Horne and Zeilinger for a very strong test of local hidden variables theories involves the use of a highly entangled state of three systems

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[2]. Tests of quantum mechanics itself also require that highly entangled states be used [3,4].

Experimental realizations of these tests require that methods of producing entangled states be found. Previous work has used cavity QED techniques and has concentrated primarily on producing entangled states of atoms. Cirac and Zoller showed how to produce a maximally entangled state of two two-level atoms [5]. They could also produce a GHZ state of three atoms if the cavity through which the atoms pass is prepared in a superposition of a three photon state and the vacuum. A method of producing entangled pairs of atoms using two micromaser cavities was discussed by Bogar and Bergou [6]. Sleator and Weinfurter, as a byproduct of their work on teleportation and quantum logic gates, found how to create an entangled state of one cavity and an arbitrary number of two-level atoms [7]. Recently, Gerry has developed a scheme to generate GHZ states of four two-level atoms [8]. A method of generating particular entangled states of two cavities occured as an intermediate step in the teleportation procedure proposed by Davidovich, Zagury, Brune, Raimond, and Haroche [9].

Here we shall show how entangled states of photons in spatially separated cavities can be produced using the techniques of cavity quantum electrodynamics. First, we shall discuss the necessary theoretical background. This includes atomic Rabi oscillations induced by the interaction of an atom with an applied classical field and both resonant and nonresonant interactions of an atom with a field in a cavity. A combination of these can be used to produce a state of two cavities which is a superposition of a state in which each cavity has one photon and one in which both are in the vacuum state. A straightforward generalization allows us to manufacture a GHZ state for three cavities and, in principle, a maximally entangled state of N cavities can also be produced. Finally we sahall discuss various tests of quantum mechanics.

2. Theoretical Background

We shall consider two-level atoms with upper state $|a\rangle$, lower state $|b\rangle$, and energy separation E_0. In the interaction picture the resonant interaction of this atom with a single-mode field is described by the Hamiltonian

$$H = ig(\sigma^{(+)}a - \sigma^{(-)}a^\dagger), \tag{1}$$

where g is the atom-field coupling constant, $a^\dagger$ and a are the mode creation and annihilation operators, and $\sigma^{(+)}$ and $\sigma^{(-)}$ are the atomic raising and lowering operators. If the field is in a highly excited coherent state the field operators can both be replaced by the real c-number field amplitude ξ_0.

Let us look at the classical field case first. If the atom and the field interact for a time t the evolution of the atomic states is given by

$$\begin{aligned} |a\rangle &\rightarrow \cos\theta_1(t)|a\rangle - \sin\theta_1(t)|b\rangle \\ |b\rangle &\rightarrow \sin\theta_1(t)|a\rangle + \cos\theta_1(t)|b\rangle, \end{aligned} \tag{2}$$

where $\theta_1(t) = \xi_0 gt$. Later we shall be interested in specific values of $\theta_1(t)$, but we shall leave it unspecified for now.

When the field is quantized we have to give both the state of the atom and the number of photons to completely specify the state of the system. That is, our states are of the form $|a,n\rangle$ or $|b,n\rangle$, where n is the photon number. The field is assumed to exist inside a cavity which the atom traverses. We are particularly interested in the cases $n = 0$ and $n = 1$. For $n = 0$ we have

$$\begin{aligned} |a,0\rangle &\rightarrow \cos\theta_2(t)|a,0\rangle - \sin\theta_2(t)|b,1\rangle \\ |b,0\rangle &\rightarrow |b,0\rangle, \end{aligned} \tag{3}$$

and for $n = 1$ we are only interested in what happens to an atom injected in its ground state

$$|b,1\rangle \rightarrow \sin\theta_2(t)|a,0\rangle + \cos\theta_2|b,1\rangle. \tag{4}$$

Here we have that $\theta_2(t) = gt$.

Finally, we want to see what happens when a two-level atom which is far off resonance with a quantized field mode interacts with it. We consider, in particular, the situation treated by Brune, Haroche, Raimond, Davidovich, and Zagury of a 3-level Rydberg atom in a cavity [10]. The lowest levels are our states $|a\rangle$ and $|b\rangle$, and there is a third level $|i\rangle$ above level $|a\rangle$. The cavity mode is slightly detuned from the $a \rightarrow i$ transition with the detuning denoted by δ. If the vacuum Rabi frequency of the $a \rightarrow i$ transition, Ω, and the cavity photon number, n, satisfy the relation $\Omega^2 n/\delta^2 << 1$, then in the $a-b$ subspace the effective Hamiltonian of the atom is, in the interaction picture,

$$H_{eff} = \frac{\Omega^2}{\delta} a^\dagger a \sigma^{(+)}\sigma^{(-)}, \tag{5}$$

where $\sigma^{(+)}$ and $\sigma^{(-)}$ are, as before, the atomic raising and lowering operators for the a and b levels. Note that this Hamiltonian does not change either the photon number or the atomic excitation.

We are interested in the time evolution generated by this Hamiltonian when the photon number is either zero or one. If it is zero the states of the atom are unaffected. If it is one, we have

$$\begin{aligned} |a\rangle &\rightarrow e^{i\theta_3(t)}|a\rangle \\ |b\rangle &\rightarrow |b\rangle, \end{aligned} \tag{6}$$

where $\theta_3(t) = (\Omega^2/\delta)t$. That is, the $|b\rangle$ state is unchanged and the $|a\rangle$ state is multiplied by a phase factor.

We now have the basic interactions we need to create entangled cavity states. It is now only necessary to arrange them in the proper sequence and to choose the proper values of θ_1, θ_2, and θ_3.

3. Two-Cavity Entangled State

Our object is to produce a maximally entangled state of two cavities. We begin with two cavities in their vacuum states. An atom in the state $|b\rangle$ is sent through a region where it interacts with a classical field (Ramsey zone) with $\theta_1 = \pi/4$, putting the atom in the state

$$|+\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle). \tag{7}$$

This atom is then sent through the first cavity, with which it is resonant, and where the interaction time has been adjusted so that $\theta_2 = 3\pi/2$. This gives us the atom-cavity state

$$\frac{1}{\sqrt{2}}(|1\rangle_1 + |0\rangle_1)|0\rangle_2|b\rangle. \tag{8}$$

The states $|0\rangle_1$ and $|1\rangle_1$ are, respectively, the zero and one photon states of cavity one and $|0\rangle_2$ and $|1\rangle_2$ are those for cavity two. This first step is only necessary to establish the appropriate initial condition, Eq. (8), for the two cavity system. The first atom now disappears from the scene, and a second atom in its ground state is sent into the system. The atom is first sent through the Ramsey zone which prepares it in the state $|+\rangle$. It then passes through both cavities and a second Ramsey zone between them. The atom interacts off-resonantly with the first cavity and the interaction time has been chosen so that $\theta_3 = \pi$. After this interaction the state of the full atom cavity system is

$$\frac{1}{\sqrt{2}}(|1\rangle_1|-\rangle + |0\rangle_1|+\rangle)|0\rangle_2, \tag{9}$$

where

$$|-\rangle = \frac{1}{\sqrt{2}}(|b\rangle - |a\rangle). \tag{10}$$

The atom now passes through a Ramsey zone with $\theta_1 = 7\pi/4$ which has the effect

$$\begin{aligned} |+\rangle &\rightarrow |b\rangle \\ |-\rangle &\rightarrow -|a\rangle, \end{aligned} \tag{11}$$

so that the total atom-cavity state is now

$$\frac{1}{\sqrt{2}}(-|1\rangle_1|a\rangle + |0\rangle_1|b\rangle)|0\rangle_2. \tag{12}$$

Finally the atom passes through the second cavity where it interacts resonantly with $\theta_2 = \pi/2$. The resulting state of the system is

$$\frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_2 + |0\rangle_1|0\rangle_2)|b\rangle. \tag{13}$$

The atom can now be discarded and we are left with the final highly entangled two-cavity field state

$$|\Phi_2\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_2 + |0\rangle_1|0\rangle_2). \tag{14}$$

This is just one of the four Bell basis states.

At this point a brief discussion of the experimental feasibility of our scheme is in order. We shall do this within the context of the micromaser experiments performed by the Garching group [11-13]. The coupling costant was found to be $g = 40$ kHz. In these experiments the adjustable parameters are the cavity-field detuning, δ, the atom-field interaction, τ, which is determined by the velocity of the atoms, and the angle of Rabi rotation which is controlled by the applied Ramsey fields. The Rabi angle should be tunable between 0 and π, and for a given atomic velocity (interaction time) it can be varied by changing the intensity of the Ramsey field. For the interaction time in the resonant case we require $g\tau_{res} = \pi/2$, or $\tau_{res} \approx 39\mu s$. This corresponds to an atomic velocity of about 700 m/s, well within the experimental range. The size of the detuning and the interaction time in the off-resonant case can be obtained from two somewhat conflicting requirements. First, for the interaction to be off-resonant we need $\delta > g$ (possibly $\delta >> g$) which implies that $g' = g^2/\delta << g$. On the other hand, one needs $g'\tau_{off-res} \sim \pi$ to conditionally interchange the $|+\rangle$ and $|-\rangle$ states. Hence we need $\tau_{off-res}/\tau_{res} \sim \delta/g$. A good practical compromise could be the choice $\delta = 3g$ and $\tau_{off-res} = 3\tau_{res} = 120\mu s$. Again, this interaction time, i. e. the atomic velocity, is within the experimental range. The amount of detuning necessary for the onset of the dispersive interaction can be produced by a variety of experimental techniques including simple mechanical squeezing of the cavity and Stark shifting with an applied electric field. Thus we can conclude that the method we are proposing for generating entangled states of two spatially separated cavities is experimentally feasible.

4. Three-Cavity Correlated State

Next we wish to prepare a three cavity state similar to that in Eq. (14), i. e. a state which is a superposition of a state in which each cavity has one photon and one in which each cavity is in its vacuum state. This is the type of state which is needed for an experimental realization of the GHZ test of local hidden variables theories.

We start with two cavities in the state $|\Phi_2\rangle$ and add a third cavity in its vacuum state. We now send an atom initially in its ground state through all three cavities. The atom first traverses a Ramsey zone, with $\theta_1 = \pi/4$, so that it enters the first cavity in the state $|+\rangle$. It interacts off-resonantly with cavities 1 and 2, in each case with $\theta_3 = \pi/2$. At this point the system is in the state

$$\frac{1}{\sqrt{2}}(|-\rangle|1\rangle_1|1\rangle_2 + |+\rangle|0\rangle_1|0\rangle_2)|0\rangle_3. \tag{15}$$

Before entering the third cavity the atom is sent through a Ramsey zone with $\theta_1 = 7\pi/4$ transforming the system state to

$$\frac{1}{\sqrt{2}}(-|a\rangle|1\rangle_1|1\rangle_2 + |b\rangle|0\rangle_1|0\rangle_2)|0\rangle_3. \tag{16}$$

The atom now interacts resonantly with cavity 3 with $\theta_2 = \pi/2$. The final state of the system is

$$\frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_2|1\rangle_3 + |0\rangle_1|0\rangle_2|0\rangle_3)|b\rangle, \tag{17}$$

i. e. a product of the desired field state and the atom in its ground state.

It is clear that this process can be continued to produce entangled states of even more cavities. In general, if an entangled state of N cavities

$$|\Phi_{N+1}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1 \ldots |0\rangle_N + |0\rangle_1 \ldots |0\rangle_N), \tag{18}$$

has been produced it can be used to produce the $N+1$ cavity state

$$|\Phi_{N+1}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1 \ldots |0\rangle_{N+1} + |0\rangle_1 \ldots |0\rangle_{N+1}). \tag{19}$$

This is done by sending an atom in the state $|+\rangle$ through the first N cavities, which are in the state $|\Phi_N\rangle$, where it interacts off resonantly with $\theta_3 = \pi/N$. Before entering cavity $N+1$, which is in the vacuum state, the atom is sent through a Ramsey zone with $\theta_1 = 7\pi/4$. It then passes through cavity $N+1$ and the result is the state $|\Phi_{N+1}\rangle|b\rangle$.

5. Tests of Quantum Mechanics

We would like to describe briefly how a state such as $|\Phi_2\rangle$ can be used to test quantum mechanics. Most such tests, such as those proposed by Bell and by Greenberger, Horne, and Zeilinger, compare the predictions of quantum mechanics to those of local hidden-variables theories [2]. Here we propose something a bit different. Quantum mechanics constructs probabilities, from an underlying Hilbert space structure, hence are restrictions on the kinds of correlations quantum mechanics can produce. The first of these was discovered by Tsirelson in 1980 [3]. The existence of these constraints provides a method for developing very stringent tests of quantum mechanics. These tests do not depend on dynamics; they depend only on the way in which probabilities are calculated in quantum mechanics. The test itself consists of performing measurements to see if any correlations which are not allowed by quantum mechanics occur.

To be specific suppose a source produces two- particle states, and that one of the particles is measured at detector 1 and the other at detector 2. At detector 1 we can measure the observables X_1 or Z_1, and at detector 2 we can measure X_2 or Z_2. We can think of each detector as being equipped with a switch. When it is in one position the detector measures the X variable and when it is in the other it measures the Z variable. Let us suppose that each of these observables can take only the values 1 or -1. On very general grounds, quantum mechanics predicts that if [4]

$$\langle X_1 X_2\rangle = \langle Z_1 Z_2\rangle = 1, \tag{20}$$

then

$$\langle X_1 Z_2\rangle = \langle X_2 Z_1\rangle. \tag{21}$$

This result is independent of dynamics and depends only on the way quantum mechanics constructs probabilities.

The results from a large number of runs of such an experiment can be summarized by writing down 16 probabilities: $p(X_1 = 1, X_2 = 1)$, $p(X_1 = 1, X_2 = -1)$, etc.

These probabilities should satisfy what we call the causal communication constraint. This simply means that if we look at only one detector, the results we obtain do not depend on the switch setting on the other detector. It is possible to write down sets of probabilities which describe the results of our Gedanken experiment, are consistent with causal communication, but yet cannot be produced by quantum mechanics [4]. It is, in fact, possible to find such sets of probabilities which satisfy Eq. (20) but not Eq. (21). The quantum mechanical constraint expressed by these equations allows us to test quantum mechanics by finding a state for which Eq. (20) is true and then verifying experimentally that Eq. (21) is satisfied.

For two cavities define

$$\begin{aligned} Z_j &= |1\rangle_{j\,j}\langle 1| - |0\rangle_{j\,j}\langle 0| \\ X_j &= |1\rangle_{j\,j}\langle 0| + |0\rangle_{j\,j}\langle 1|, \end{aligned} \tag{22}$$

for $j = 1, 2$. These operators have only 1 and -1 as eigenvalues. We also note that

$$\langle\Phi_2|X_1X_2|\Phi_2\rangle = \langle\Phi_2|Z_1Z_2|\Phi_2\rangle = 1. \tag{23}$$

It is not difficult to measure the variables X_j and Z_j. Let us look at the case $j = 1$ to be definite. In order to measure Z_1 we simply send an atom, initially in its ground state, through the cavity where it interacts resonantly with the field and where $\theta_2 = \pi/2$. The inversion of the exiting atom is equivalent to the Z_1 of the initial field state. The measurement of X_1 is similar except that after exiting the cavity the atom now traverses a Ramsey zone with $\theta_1 = \pi/4$. The inversion of the atom after passing through both the cavity and the Ramsey zone is equivalent to the X_1 of the initial field state. In both cases the atomic inversion can be measured.

It is, therefore, feasible to prepare the cavities in the state $|\Phi_2\rangle$ and to measure X_j and Z_j for $j = 1, 2$. Consequently, cavity QED provides us with a method of testing quantum mechanics.

6. Conclusion

Highly entangled states are useful in testing local hidden variables theories and also in testing quantum mechanics itself. They also feature prominently in certain schemes to transmit quantum information such as teleportation [14] and quantum cryptography [15]. One, therefore, wants to have a method of producing them. In the lecture given at the meeting further tests of pre- and postselective quantum mechanics [16] as well as the connection of these schemes with quantum computing and quantum nondemolition measurements [17] have also been briefly discussed. For space constraints these and other consideration will be discussed elsewhere.

As we have shown cavity QED gives us the necessary tools to carry out these tests. It is possible to produce maximally entangled states of two, three, and, in principle, N cavities. It should be possible to use these states to process and transmit quantum information.

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