acta physica slovaca vol. 46 No. 3, 231 – 240 June 1996

PAUL TRAP MULTI-QUANTUM INTERACTIONS[1]

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Received 7 June 1996, accepted 7 June 1996

We study the dynamics of a single ion moving in an explicitly time dependent Paul trap and interacting with a classical laser field.

1. Introduction

The recent success [1] in creating non-classical states of the motion of a single ion in a Paul trap [2] has triggered the question of measuring [3] the density operator of the vibrational degree of freedom. Paul trap endoscopy [4,5] is the answer to this question of state measurement when one takes into account the explicit time dependence [6] of the trap. This technique relies on the rotating wave approximation (RWA) simplifying a rather complicated time dependent interaction Hamiltonian to a multi-quantum nonlinear Jaynes-Cummings model. In the present paper we study the underlying model in more detail and, in particular, focus on the question of the validity of the RWA.

The article is organized as follows: In Sec. 2 we outline the model [7] which couples the internal degree of freedom to the center-of-mass motion by a classical laser field. We derive an exact expression for the interaction Hamiltonian in the interaction picture, which brings out the explicit time dependence of the problem and which is the starting point for the rotating wave approximation. We devote Sec. 3 to a discussion of the Rabi frequencies and investigate various limiting cases. In Sec. 4 we perform the rotating wave approximation and study its validity in Sec. 5. We conclude by summarizing our main results in Sec. 6.

[1] Presented at the **4th central-european workshop on quantum optics**, Budmerice, Slovakia, May 31 - June 3, 1996

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0323-0465/96

2. The model Hamiltonian

In this article we study the system of a single two-level ion moving in one dimension in a Paul trap and interacting with a classical laser field. The complete Hamiltonian

$$\hat{H}(t) = \hat{H}_{\rm cm}(t) + \hat{H}_{\rm a} + \hat{H}_{\rm int}(t) \tag{1}$$

consists of three parts which describe the center-of-mass motion, the internal structure of the ion, and the interaction.

The one-dimensional center-of-mass motion of the ion with mass m follows from the Hamiltonian

$$\hat{H}_{\rm cm}(t) = \tfrac{1}{2m}\,\hat{p}^2 + \tfrac{1}{2}\,m\omega^2(t)\,\hat{x}^2\,, \tag{2}$$

where

$$\omega^2(t) = \tfrac{1}{4}\,\omega_{\rm rf}^2\,[a + 2\,q\,\cos(\omega_{\rm rf}t)] \tag{3}$$

denotes the time-dependent steepness of the harmonic oscillator potential. The dimensionless parameters a and q are proportional [2] to the applied DC and AC voltages, respectively, and $\omega_{\rm rf} \equiv 2\pi/T$ is the (radio) frequency of the AC voltage.

We describe the internal structure of the two-level ion with transition frequency $\omega_{\rm a}$ by the Hamiltonian

$$\hat{H}_{\rm a} = \tfrac{1}{2}\hbar\omega_{\rm a}\hat{\sigma}_z \tag{4}$$

with $\hat{\sigma}_z$ being the Pauli spin matrix.

The interaction of the classical laser field with the two levels of the ion reads in the rotating wave approximation [7]

$$\hat{H}_{\rm int}(t) = \hbar\,g\Big(\hat{\sigma}^+\,\exp\left[-i\,(\omega_L t - k\hat{x})\right] + h.c.\Big)\,. \tag{5}$$

Here, g and k denote the interaction strength and the wave vector of the light field with frequency ω_L, and the Pauli matrix $\hat{\sigma}^+$ is the raising operator for the internal levels of the ion.

We now concentrate on the interaction Hamiltonian Eq. (5), which we transform into the interaction picture

$$\hat{\tilde{H}}_{\rm int}(t) = \hat{U}_{\rm cm}^\dagger(t)\,\hat{U}_{\rm a}^\dagger(t)\,\hat{H}_{\rm int}(t)\,\hat{U}_{\rm a}(t)\,\hat{U}_{\rm cm}(t)\,. \tag{6}$$

via the unitary evolution operators

$$\hat{U}_{\rm a}(t) \equiv \exp\left(-\tfrac{i}{2}\,\omega_{\rm a}\,\hat{\sigma}_z\,t\right) \tag{7}$$

and

$$\hat{U}_{\rm cm}(t) \equiv \hat{\mathcal{T}}\,\exp\left[-\tfrac{i}{\hbar}\int_0^t dt'\,\hat{H}_{\rm cm}(t')\right]\,, \tag{8}$$

where $\hat{\mathcal{T}}$ is the time ordering operator. We recall the relations

$$\hat{U}_{\rm a}^\dagger\,\hat{\sigma}^\dagger\,\hat{U}_{\rm a} = \hat{\sigma}^\dagger\,e^{i\omega_{\rm a}t} \tag{9}$$

and [8,9]

$$\hat{U}_{\rm cm}^\dagger(t)\,\hat{x}\,\hat{U}_{\rm cm}(t) = \sqrt{\frac{\hbar}{2m\omega_r}}\,\left(\epsilon^*(t)\,\hat{b} + \epsilon(t)\,b^\dagger\right)\,, \tag{10}$$

where $\hat{b}$ and $\hat{b}^\dagger$ are the annihilation and creation operators of a time-independent reference harmonic oscillator with frequency ω_r and the complex-valued function $\epsilon(t)$ satisfies the classical Mathieu differential equation

$$\ddot{\epsilon}(t) + \omega^2(t)\,\epsilon(t) = 0 \tag{11}$$

with the initial conditions $\epsilon(0) = 1$ and $\dot{\epsilon}(0) = i\omega_r$. As Glauber has shown [9], the choice of the frequency ω_r determines the natural basis of the center-of-mass motion of the ion.

With the help of Eqs. (9) and (10) we arrive at

$$\hat{\tilde{H}}_{\rm int}(t) = \hbar\,g\left(\sigma^\dagger e^{-i\Delta t}\hat{D}[\alpha(t)] + h.c.\right)\,, \tag{12}$$

where the displacement operator $\hat{D}(\alpha) \equiv \exp(\alpha\,\hat{b}^\dagger - \alpha^*\,\hat{b})$ involves the time-dependent displacement $\alpha(t) \equiv i\,\eta\,\epsilon(t)$ with the Lamb-Dicke parameter $\eta \equiv k\,[\hbar/(2m\omega_r)]^{1/2}$, and $\Delta \equiv \omega_L - \omega_{\rm a}$ is the detuning between the laser frequency and the two-level transition frequency.

Note, that the interaction Hamiltonian Eq. (12) involves all possible multi-phonon transitions. To bring this out most clearly we express the displacement operator $\hat{D}(\alpha)$ in the basis of the energy eigenstates $|n\rangle$ of the reference oscillator. When we recall the relations [10]

$$\langle n|\hat{D}(\alpha)|m\rangle = \left[\frac{n!}{m!}\right]^{1/2}\,\exp\left(-\tfrac{1}{2}|\alpha|^2\right)\,(-\alpha^*)^{m-n}\,L_n^{m-n}\left(|\alpha|^2\right) \tag{13}$$

for $m \geq n$ and

$$\langle n|\hat{D}(\alpha)|m\rangle = \left[\frac{m!}{n!}\right]^{1/2}\,\exp\left(-\tfrac{1}{2}|\alpha|^2\right)\,\alpha^{n-m}\,L_m^{n-m}\left(|\alpha|^2\right) \tag{14}$$

for $m \leq n$, we can write the interaction Hamiltonian in the form [4]

$$\hat{\tilde{H}}_{\rm int}(t) = \sum_{n=0}^{\infty}\sum_{s=-n}^{\infty}\hbar\,\Omega^{(n,n+s)}(t)\,\hat{\sigma}^+\,|n\rangle\langle n+s| + h.c.\,. \tag{15}$$

Here, we have introduced the substitution $m = n+s$ and the time-dependent generalized Rabi frequencies

$$\Omega^{(n,n+s)}(t) = g\,\left[\frac{n!}{(n+s)!}\right]^{1/2}\,\exp(-i\,\Delta\,t)\,[i\,\eta\,\epsilon^*(t)]^s\,\exp\left(-\tfrac{\eta^2}{2}|\epsilon(t)|^2\right)\,L_n^s(\eta^2\,|\epsilon(t)|^2) \tag{16}$$

for $s \geq 0$ and

$$\Omega^{(n,n+s)}(t) = g \left[\frac{(n+s)!}{n!}\right]^{1/2} \exp(-i\,\Delta\,t)\,[i\,\eta\,\epsilon(t)]^{-s}\,\exp\left(-\tfrac{\eta^2}{2}|\epsilon(t)|^2\right)\,L_{n+s}^{-s}(\eta^2\,|\epsilon(t)|^2) \tag{17}$$

for $-n \leq s \leq 0$. From Eqs. (16) and (17) we recognize that the detuning Δ and the complex-valued function $\epsilon(t)$ determines the time dependence of the Rabi frequencies.

We now concentrate on a particular solution $\epsilon^{(F)}(t)$ of the differential equation (11). With the specific choice [9]

$$\omega_r^{(F)} \equiv \mu + \omega_{\rm rf} \sum_{n=-\infty}^{\infty} n\,c_n \tag{18}$$

of the reference frequency ω_r, the solution of Eq. (11) takes on the Floquet form [11]

$$\epsilon^{(F)}(t) = \exp(i\mu t)\,\phi(t)\,. \tag{19}$$

Here, μ is the so-called characteristic exponent, and the function

$$\phi(t) = \phi(t+T) \equiv \sum_{n=-\infty}^{\infty} c_n\,\exp\,(in\,\omega_{\rm rf} t) \tag{20}$$

is periodic in t with period $T = 2\pi/\omega_{\rm rf}$. We find the expansion coefficients c_n and the characteristic exponent μ by substituting the Floquet solution Eq. (19) into Eq. (11). For the specific case Eq. (3) of the time dependent frequency $\omega^2(t)$ of the Paul trap, this yields a three-term recurrence relation for c_n which fixes μ via the Hill determinant [11]. The coefficients c_n follow from the resulting linear set of equations. In the stable region of the Mathieu equation [12] the coefficients c_n and the characteristic exponent μ are purely real [9]. The frequency μ then gives the secular frequency of the motion of the ion.

We now substitute the Floquet solution Eq. (19) into the Rabi frequencies Eqs. (16) and (17). When we expand the T-periodic part of $\Omega^{(n,n+s)}(t)$ in Fourier series, we arrive at

$$\Omega^{(n,n+s)}(t) = \sum_{l=-\infty}^{\infty} \omega_l^{(n,n+s)}\,\exp[i(l\,\omega_{\rm rf} - s\,\mu - \Delta)\,t]\,, \tag{21}$$

where the expansion coefficients $\omega_l^{(n,n+s)}$ for $s \geq 0$ read

$$\omega_l^{(n,n+s)} \equiv g \left[\frac{n!}{(n+s)!}\right]^{1/2} (i\,\eta)^s \int_{-T/2}^{T/2} \frac{dt}{T}\,[\phi^*(t)]^s\,e^{-\frac{\eta^2}{2}|\phi(t)|^2}\,L_n^s(\eta^2\,|\phi(t)|^2)\,e^{-il\omega_{\rm rf}t} \tag{22}$$

and similarly

$$\omega_l^{(n,n+s)} \equiv g \left[\frac{(n+s)!}{n!}\right]^{1/2} (i\,\eta)^{-s} \int_{-T/2}^{T/2} \frac{dt}{T}\,[\phi(t)]^{-s}\,e^{-\frac{\eta^2}{2}|\phi(t)|^2}\,L_{n+s}^{-s}(\eta^2\,|\phi(t)|^2)\,e^{-il\omega_{\rm rf}t} \tag{23}$$

for $-n \leq s \leq 0$. After substituting Eq. (21) into Eq. (15), the interaction Hamiltonian in the interaction picture takes on the form

$$\hat{\tilde{H}}_{\rm int}(t) = \sum_{n=0}^{\infty} \sum_{s=-n}^{\infty} \sum_{l=-\infty}^{\infty} \hbar\,\omega_l^{(n,n+s)}\,\exp[i(l\,\omega_{\rm rf} - s\,\mu - \Delta)\,t]\,\hat{\sigma}^+\,|n\rangle\langle n+s| + h.c.\,. \tag{24}$$

We emphasize that this representation is exact. Note, that the time dependence of the interaction Hamiltonian in the interaction picture is governed by harmonics of the specific combination $l\,\omega_{\rm rf} - s\,\mu - \Delta$, that is by all harmonics of the frequencies $\omega_{\rm rf}$ and μ and the detuning Δ.

3. The Rabi frequencies

In this section we focus on the Rabi frequencies $\Omega^{(n,n+s)}(t)$, Eq. (21), and their time-independent expansion coefficients $\omega_l^{(n,n+s)}$ given by Eqs. (22) and (23). We first note, that the quantities $\omega_l^{(n,n+s)}$ are purely real for s even and purely imaginary for s odd. This results from the fact that the integrals in Eqs. (22) and (23) are real, since Eq. (20) yields the symmetry relation $\phi(-t) \equiv \phi^*(t)$ for c_n real.

In Fig. 1 we display the modulus $|\omega_l^{(n,n+s)}|$ as a function of l and n for the six different transitions $s = 1,2,3,4,5,6$. Here we use the trap parameters $a = 0$, $q = 0.4$, and the Lamb-Dicke value $\eta = 1$. From this figure we recognize an oscillatory behavior of $|\omega_l^{(n,n+s)}|$ for l fixed, which is due to the nonlinear coupling between the internal levels and the vibrational degree of freedom. Note, that for the chosen trap parameters the explicit time dependence of the trap potential is important.

However, in the limit $a, q \to 0$ the vibrational degree of freedom of the ion is well described [7] by a time-independent harmonic oscillator with frequency μ. In this case, Eq. (20) simplifies to $\phi(t) \simeq 1$ and hence we find from Eq. (19) the expression $\epsilon^{(F)}(t) \simeq e^{i\mu t}$. As a consequence, the expansion coefficients $\omega_l^{(n,n+s)}$ all vanish except for $l \equiv 0$ and hence the Rabi frequencies $\Omega^{(n,n+s)}(t)$ simplify to

$$\Omega^{(n,n+s)}(t) \simeq g \left[\frac{n!}{(n+s)!}\right]^{1/2} (i\eta)^s\,e^{-\frac{\eta^2}{2}}\,L_n^s(\eta^2)\,e^{-i(s\,\mu+\Delta)\,t} \tag{25}$$

for $s \geq 0$ and

$$\Omega^{(n,n+s)}(t) \simeq g \left[\frac{(n+s)!}{n!}\right]^{1/2} (i\eta)^{-s}\,e^{-\frac{\eta^2}{2}}\,L_{n+s}^{-s}(\eta^2)\,e^{-i(s\,\mu+\Delta)\,t} \tag{26}$$

for $-n \leq s \leq 0$. Analogous expressions in the case of a standing light field as the coupling mechanism were first derived in Ref. [13].

Finally we concentrate on the Lamb-Dicke limit for which $\eta \ll 1$. When we expand the coefficients Eqs. (22) and (23) up to first order in η, only the terms with $s = 0, \pm 1$ survive. Here we focus on the case $s = \pm 1$ since the term with $s = 0$ does not change the

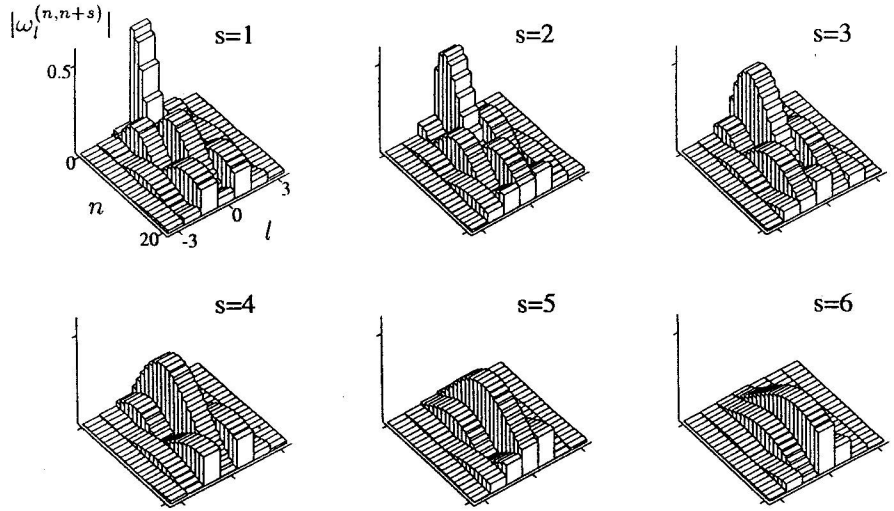

Fig. 1. The modulus $|\omega_l^{(n,n+s)}|$ for the different values $s = 1, 2, 3, 4, 5, 6$ as a function of n and l. Here we use the trap parameters $a = 0$, $q = 0.4$, and the Lamb-Dicke value $\eta = 1$.

vibrational state. With the help of the property [12] $L_n^1(0) = n + 1$ of the generalized Laguerre polynomial we find from Eqs. (16) and (17) the Rabi frequencies

$$\Omega^{(n,n+1)}(t) \simeq g\, i\eta\, [\epsilon^{(F)}(t)]^* \sqrt{n+1}\, e^{-i\Delta t} \tag{27}$$

and

$$\Omega^{(n,n-1)}(t) \simeq g\, i\eta\, \epsilon^{(F)}(t) \sqrt{n}\, e^{-i\Delta t}\,. \tag{28}$$

Here we recover the well-known square root dependence of the Rabi frequency on the quantum number n of the ordinary one-quantum Jaynes-Cummings model.

4. The rotating wave approximation

We now return to Eq. (24), which shows, that the interaction Hamiltonian in the interaction picture involves all different multi-phonon transitions between energy eigenstates $|n\rangle$ of the reference oscillator with frequency $\omega_r^{(F)}$. Note, that so far we have not yet specified the detuning Δ. By an appropriate choice of Δ we can now select a specific interaction, which allows only such transitions which involve a certain number of phonons [8,13]. This happens, when one of the terms in the sums over l and s in Eq. (24) depends only slowly on time whereas all the others are rapidly oscillating. Indeed, when we choose the detuning Δ such that $s_0\,\mu + \Delta \equiv l_0\,\omega_{\rm rf}$ provided that $(l - l_0)\,\omega_{\rm rf} - (s - s_0)\,\mu \neq 0$ for all $l \neq l_0$ and $s \neq s_0$, we arrive with the help of the rotating wave approximation at the time averaged Hamiltonian

$$\hat{\tilde{H}}_{\rm int}^{\rm RWA} = \sum_{n=0}^{\infty} \hbar\, \omega_{l_0}^{(n,n+s_0)}\, \hat{\sigma}^+\, |n\rangle\langle n + s_0| + h.c.\,, \tag{29}$$

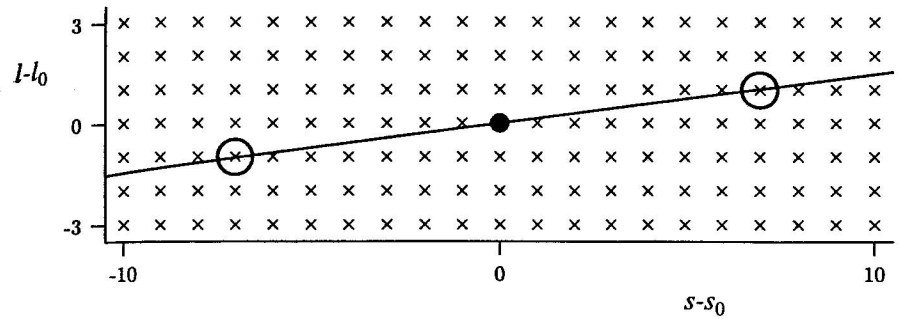

Fig. 2. The rotating wave approximation and possible resonances. The distance between the straight line $y = \mu/\omega_{\rm rf} \cdot x$ and the possible resonances at $(l - l_0, s - s_0)$ shown by crosses determines the quality of the RWA. The filled circle at the origin marks the single term $(l \equiv l_0, s \equiv s_0)$ which survives the RWA. Note, that the terms at $(\pm 1, \pm 7)$ are almost resonant but can be neglected provided they violate the inequality Eq. (30). For the trap parameters $a = 0$, $q = 0.4$, and $\eta = 1$ used here we find the value $\mu/\omega_{\rm rf} \simeq 0.14$.

which represents the s_0-phonon nonlinear Jaynes-Cummings Hamiltonian [13]. In order to achieve a large coupling we choose Δ such that only a term with large coefficients $\omega_{l_0}^{(n,n+s_0)}$ survives the time averaging. Note, that these generalized Rabi frequencies coincide with the Rabi frequencies of the ordinary Jaynes-Cummings model only in the Lamb-Dicke limit.

We illustrate this with the help of the previous example of the trap parameters $a = 0$, $q = 0.4$, and the Lamb-Dicke value $\eta = 1$. In this case [5] the ratio between the secular frequency μ and the trap frequency $\omega_{\rm rf}$ reads $\mu/\omega_{\rm rf} \simeq 0.14$. In Fig. 2 we show the straight line $y = \mu/\omega_{\rm rf} \cdot x$ together with all possible resonances at integer numbers of $y = l - l_0$ and $x = s - s_0$. We recognize, that this line approaches very closely the resonances at $l - l_0 = \pm 1$ and $s - s_0 = \pm 7$. Note, that the coupling strength g determines, whether these resonances indeed yield important contributions, since it sets the time scale $\tau \sim 1/g$ on which the wave function of the system changes significantly. Hence all terms in the interaction Hamiltonian which oscillate faster than g average out on the time scale τ. Therefore, we expect that only terms which satisfy the condition

$$|(l - l_0)\,\omega_{\rm rf} - (s - s_0)\,\mu| < g \tag{30}$$

give some contribution besides the term $l = l_0$ and $s = s_0$. In addition, the ratios of the corresponding Rabi frequencies $\omega_l^{(n,n+s)}/\omega_{l_0}^{(n,n+s_0)}$ determine how large these contributions are.

5. The validity of the rotating wave approximation

In the preceeding section we have taken a time average of the explicitly time dependent Hamiltonian Eq. (24) and have so arrived at the s_0-phonon nonlinear Jaynes-Cummings Hamiltonian Eq. (29). Here the detuning Δ played a key role in selecting the number

s_0 of phonons involved in the transition. At this point it is important to recall that the time average of Eq. (24) is already the second rotating wave approximation. Indeed, the interaction Hamiltonian Eq. (5), which served as our starting point, was already in the RWA since we have neglected terms oscillating with twice the optical frequency. Now we realize that this approximation was indeed justified since in order to select the s_0-phonon transition the detuning Δ is of the order of the secular frequency μ or the radio frequency $\omega_{\rm rf}$. Both are in the MHz range and therefore far away from the optical regime. We emphasize, that a choice of Δ in the optical range would allow us to investigate the influence of the anti-resonant terms. However, this is outside of the scope of the present paper.

We now turn to the second RWA and test its quality for the case of the one-phonon transition $s_0 = 1$ and the trap parameters $a = 0$, $q = 0.4$, and $\eta = 1$. For this purpose we calculate numerically the probability $P_e(t)$ of finding the ion at time t in the excited state using the exact time-dependent interaction Hamiltonian Eq. (24). We compare the so-calculated probability to the probability

$$P_{\rm e}^{\rm RWA}(t) = \sum_{n=0}^{\infty} \langle n|\langle e| \exp\left(-\frac{i}{\hbar}\hat{\tilde{H}}_{\rm int}^{\rm RWA} t\right) |\Psi(0)\rangle\langle\Psi(0)| \exp\left(\frac{i}{\hbar}\hat{\tilde{H}}_{\rm int}^{\rm RWA} t\right) |e\rangle|n\rangle \qquad (31)$$

obtained in the rotating wave approximation. As the initial total state vector $|\Psi(0)\rangle = |\psi_{\rm cm}(0)\rangle \otimes |\psi_{\rm a}(0)\rangle$ we use the direct product of the vibrational coherent state $|\psi_{\rm cm}(0)\rangle = |\alpha\rangle$ with $\alpha = 1.5$ and the superposition state $|\psi_{\rm a}(0)\rangle = \frac{1}{2}(|g\rangle + |e\rangle)$ of the ground state $|g\rangle$ and the excited state $|e\rangle$ of the two-level ion.

In Fig. 3 we show by solid lines the exact curve $P_e(t)$ and by dashed lines the approximation Eq. (31). We emphasize the excellent agreement between the two curves.

6. Summary

In this article we study multi-quantum interactions of a single two-level ion in the Paul trap. In contrast to related work we take into account the explicit time dependence of the harmonic trap potential. This time dependence carries over to the generalized Rabi frequencies which we investigate in detail. In two limiting cases, namely when the time dependence of the trap potential is not important, and in the Lamb-Dicke limit, we recover well-known results.

Moreover we show, that despite of the complexity of the system originating from the time dependence of the binding force we can simplify the complicated interaction Hamiltonian with the help of a rotating wave approximation. The resulting interactions are of the Jaynes-Cummings type with modified Rabi frequencies. In particular, we discuss the conditions under which the rotating wave approximation is valid and demonstrate its quality by comparing it to the exact numerical solution.

We conclude by noting that this model opens a new way of interaction engineering: By appropriately choosing the detuning Δ we can select an interaction which involves only a pre-described number of phonons. Moreover the trap parameters a and q together with the Lamb-Dicke parameter η create a large set of Rabi frequencies $\omega_l^{(n,n+s)}$ which we can use to engineer a large set of different interaction Hamiltonians.

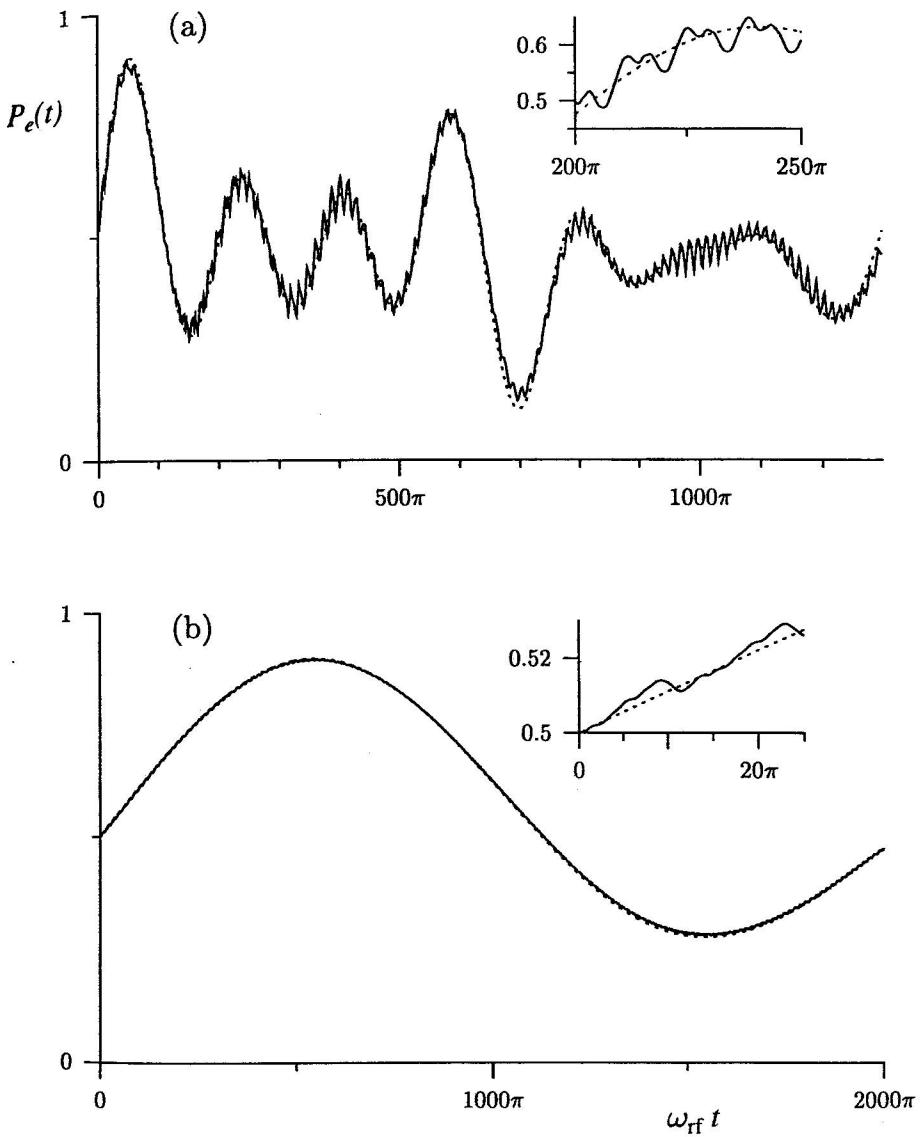

Fig. 3. The quality of the rotating wave approximation for the one-phonon transition $s_0 = 1$ and the trap parameters $a = 0$, $q = 0.4$, and $\eta = 1$. The solid lines give the exact probability $P_e(t)$ of finding the ion at time t in the excited state, whereas the dashed curves represent the results of the rotating wave approximation. The magnifications in the insets illustrate that the exact curves oscillate around the approximations. In (a) we use the the coupling strength $g = 0.01 \cdot \omega_{\rm rf}$, whereas in (b) we use $g = 0.001 \cdot \omega_{\rm rf}$.

Acknowledgement We (P.J.B. and C.L.) gratefully acknowledge the support of the DFG.

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