FORMATION TIME AND RESCATTERING OF HADRONS IN HEAVY ION COLLISIONS

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The aim of this paper is to start a study of the effects of the formation time of hadrons [9] on the dynamics of heavy ion collisions. For this purpose we have developed our version of the rescattering model of Humanic [1]. In the paper we sketch briefly the internal structure of our program and we also present first results obtained.

1. Introduction

Heavy ion collision experiments (HIC) have been attracting increasing attention among the high energy particle physics community during the last decade because of the beauty of physics involved, the new phenomenological approaches required and finally the possibility of studying dense hot nuclear matter in laboratories on Earth.

One of the methods of the extraction of information about physical processes participating in the process of heavy ion collision is interferometry analysis [2]. At the present time the interferometry in HIC has finished its "first order" stage and more sensitive methods of correlation analysis are being used now. The NA35 experimantal group in CERN has reported its results [3] about the different sizes of the radii of pion sources for different regions of transversal momentum. The dependence of the parameters extracted by interferometry on the momentum of selected pairs was predicted a decade ago [4] and the new approaches in the explanation of this behaviour have arisen [1].

In this paper we present a rescattering model developed according to the description of the model described in [1]. Our model reproduces the main results of [1] - the p_t dependence of radii R_t measured by NA35 and additionally it uses a new concept - formation time - and therefore it addresses a slightly different physics of the rescattering process. Moreover new questions and phenomena can be and already are being investigated using the model. The paper is organized as follows: In section 2. we describe the basic structure, properties and elements of physics used in our rescattering model. In section 3. we introduce a new concept of the scattering scenario which is different from that used in [1]. The results and conclusions can be found in sections 4. and 5.

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2. Computer Simulation of Rescattering

At the CERN SPS about 700 pions are detected in one S-Pb 200 GeV/nucl. central collision. These pions during their flight from the places of creation scatter each other and form in this way an interacting expanding pion gas. A small number of particles in the investigation makes it possible to simulate on the computer the whole process in the real $\vec{x}$ space in time. In this simulation the pions are treated as pointlike objects. Momentum and position of each pion is known during the simulation of the rescattering process. This exact determination of $\vec{p}$ and $\vec{x}$ is not in agreement with the principal law of quantum mechanics - the uncertainty principle. The description using wave packets for the pions should be used to keep the uncertainty principle fulfilled. However the representation of pions as pointlike objects in the case of the rescattering phenomenon is accepted in HIC community e.g. [1] and also we would be doubtful to claim that the wave packet simulation would lead to results substantially different to the results obtained by the pointlike approach.

Our program is based on the description of the computer simulation in the excellent paper of T.J. Humanic [1] where perhaps a better and more understandable explanation of the concepts used can be found. Nevetherless we shall sketch briefly the internal structure of our program in the next subsections.

2.0 General Structure of Program

The program consists of the three main parts shown on Fig.1. In this subsection we present general interface information about the program which does not require any knowlege of the internal structure of its parts.

The Initial Generation (IG) unit produces the input information for the Time Evolution (TE) unit (see Fig.1) in the form of records for each pion (particle) and saves it into the file *IG.dat*. The file *IG.dat* is a text ASCI file therefore its content can be read, understood and printed out without any supplementary formatting procedures. Each line of this file contains information about one particle in the form:

$$q, t, x, y, z, m, p_x, p_y, p_z \tag{1}$$

where q - the charge of particle is integer number, t is real number in fm/c, x, y, z are real numbers in fm=10^{-15}m, m is mass of particle in MeV/c^2 and p_x, p_y, p_z represent the momentum components in MeV/c units. This set of variables determines fully the kind of particle, its position and momentum and thus allows us to include other particles (kaons, nucleons or resonances) into the simulation.

Unit TE reads the information from file *IG.dat*, simulates the rescattering process and writes the results into the output files *TEx.dat* and *TEi.dat*. File *TEx.dat* contains information of positions and momenta of particles after the rescattering process (typically after 20 fm/c). *TEi.dat* contains the information about the time, positions of last interaction in the rescattering process and momentum after this interaction for every particle. The structure of these two files is the same as the structure of file *IG.dat* (1).

The Data Analysis (DA) unit reads the information from the output files of TE part, adds the physical consequences of B-E interference phenomenon for like-sign bosons, produces the graphs, histograms and performs the fit procedures.

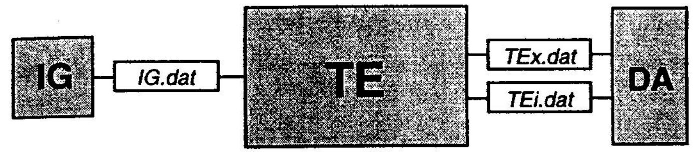

Fig. 1 **The structure of the program**

This interface structure of the program allows us to change, upgrade and run[2] the three main parts IG,TE and DA independently. For example it is planned to replace the IG unit by the Cascading Generator program [5] in the near future. At present the units IG and TE are written in object pascal language and they can be run on both PC Turbo Pascal and workstation HP Pascal platforms. The IG is usually run on PC, the TE unit on HP Apollo workstation. The DA unit is written in FORTRAN, it uses the CERN Library procedures and is run on the HP Apollo workstation. The most time consuming part of program is the TE unit. It is planned to translate it to C or rewrite it in FORTRAN in order to use machines of higher computational power.

In the following subsections we describe briefly some details of the internal structure and physics contained in IG, TE and DA units.

2.1 Initial Generation Unit

The program simulates a central S-Pb heavy ion collision at the energy 200 GeV/nucleon. In such collisions typically 700 pions are created. The Initial Generation Unit generates the momenta, positions and time of creation for these 700 pions. As the simplest space distribution for pions the random distribution in the sphere of radius $R = 3.8$ fm was chosen. This choice does not reproduce precisely the real physical situation however it is acceptable at the first stage also because of the planned replacement of the IG unit by the Cascading Generator [5] in the future, also because of the low sensitivity of the results of simulation to the initial distribution $\varrho(\vec{x})$ verified in the model [1]. The time of creation for the pions is generated randomly in the interval 0.0-2.0 fm/c. Initial momenta of pions are distributed in the same way as in [1]. We just repeat here that the p_t is generated according to the experimentally measured distribution:

$$\phi(p_t) = \alpha . p_t . e^{-\sqrt{p_t^2+m^2}/T} \tag{2}$$

and the azimuthal orientation is chosen randomly. The longitudinal momenta of pions are generated according to $\psi(p_l)$ distribution in agreement with the experimentally measured rapidity distribution:

$$\varphi(y) = \beta . e^{-y^2/2\sigma^2} \tag{3}$$

where α, β are normalization constants and the values of parameters T, σ in (2) and (3) are: $T = 176$MeV and $\sigma = 1.5$ - the same as in [1]. Thus the initial distribution of

[2] This opens the possibility for parallel processing technique

pions $\Phi(\vec{x},\vec{p},t)$ is generated as

$$\Phi(\vec{x},\vec{p},t) = \chi(t).\varrho(\vec{x}).\phi(\vec{p}_t).\psi(p_l) \tag{4}$$

where $\chi(t)$ represents our random distribution of the points of creation in time. It is clear that the initial distribution for pions (4) is momentum-space uncorrelated.

2.2 Time Evolution Unit

During the process of simulation the particles are evolved in space as pointlike objects in small time steps $\Delta t = 0.1$fm/c according to the equation of motion

$$\vec{x}_i(t+\Delta t) = \vec{x}_i(t) + \vec{v}_i(t,\vec{p}(t)) \cdot \Delta t \tag{5}$$

in the Global Frame of Collision (GFC). The components of velocity $\vec{v}$ are determined from the momentum of the particle, taking into account relativistic kinematics in the following way:

$$\vec{v} = \frac{\vec{p}}{m} \cdot \sqrt{\frac{m^2c^2}{\vec{p}^2 + m^2c^2}} \tag{6}$$

where m is the rest mass of the particle. During each time step for all pairs of particles the distance $d_{i,j} = |\vec{x}_i - \vec{x}_j|$ in space is computed in GFC. The collision of pions occurs when the condition

$$d_{i,j} < d_c(s) \tag{7}$$

is valid. Here d_c (typically smaller than 2fm) depends on $s = \sqrt{(p_i+p_j)^2}$ and is determined from the circular geometrical interpretation of the cross-section $\sigma(s) = \pi d_c^2(s)$, where $\sigma(s)$ is the elastic $\pi\pi$ cross-section taken from [6]. At the present stage, the $\sigma(s)$ is the isospin averaged cross-section and therefore its value is the same for any pair combination of π^+, π^-, π^0.

The structure of the TE unit is ready to use different $\sigma(s)$ for different isospin combinations for pion pairs and also for different particles $(K, N, ...)$. For the physical phenomena studied at present the isospin averaged approach is acceptable. We would like to mention here that condition (7) brings a Lorentz non-invariance to our simulation. However this consequence of non-local condition was already encountered in other computer simulation models of HIC and its influence on the results is estimated not to exceed the statistical errors [7].

The position and time of every collision are stored in memory for further generation of the last interaction information to be saved in *TEi.dat* file. The internal structure of TE unit allows each particle to be excluded from the rescattering process tested by (7) for an arbitrary time interval at any time (e.g. after the collision or the creation of particle). The particles can also be annihilated or created during the time evolution though this possibility is not used in TE unit at present.

After 20 fm/c typically 98% of collisions happen and they are saved into the *TEx.dat* file, the last interaction information is saved to *TEi.dat* file.

2.3 Data Analysis Unit

DA unit reads the information from files *TEx.dat* and *TEi.dat*. The files for many events are analyzed by DA unit at the same time. The desired cuts in rapidity and transversal momentum are applied and the histograms are filled via HBOOK CERN Library procedures. The graphs of results obtained from analysis of *TEx.dat* files for 24 events are shown on Fig.4 and discussed in section 4. The last interaction information about the scattered particles contained in *TEi.dat* files is the input for B-E correlation analysis. We shall describe it briefly here.

The correlation function is experimentally measured by counting the pairs of particles in each event $N(\vec{p}_1,\vec{p}_2)$ and normalizing it by the non-correlated distributions $N(\vec{p}_1).N(\vec{p}_2)$ of pions in momentum space:

$$C(\Delta\vec{p}) = \frac{N(\vec{p}_1,\vec{p}_2)}{N(\vec{p}_1).N(\vec{p}_2)} \tag{8}$$

In a real experiment the non-correlated distributions are obtained by the event mixing prescription [3] and the correlated pairs are measured in the separate events. In the case of our simulation the B-E interference phenomenon is added artificially after the TE process during the data analysis in DA unit. Therefore the non-correlated distributions $N(\vec{p}_1), N(\vec{p}_2)$ are contained directly in *TEi.dat* files and so the event mixing technique does not need to be applied. Distribution $N(\vec{p}_1,\vec{p}_2)$ is obtained in the same way as in [1] according to the formula

$$N(\vec{p}_1,\vec{p}_2) = \sum_{i,j} |\Psi_{i,j}(\Delta\vec{p},\Delta\vec{x})|^2 \tag{9}$$

as a result of discrete integration of the analytical expression for $N(\vec{p}_1,\vec{p}_2)$

$$N(\vec{p}_1,\vec{p}_2) = \int d\vec{x}_1 dt_1 d\vec{x}_2 dt_2 \left[g(\vec{x}_1,\vec{p}_1,t).g(\vec{x}_2,\vec{p}_2,t)|\Psi|^2\right] \tag{10}$$

where $|\Psi_{i,j}(\Delta\vec{p},\Delta\vec{x})|^2 = 1 + \cos[(\vec{x}_i - \vec{x}_j)\cdot(\vec{p}_i - \vec{p}_j)]$ for the bosons emitted at the same time and $g(\vec{x},\vec{p},t)$ is the last-interaction distribution of bosons. The summation in (9) includes all pairs of like-sign bosons in one event.

Two-dimensional correlation fuctions in the transversal relative momentum $(\Delta p_x, \Delta p_y)$ space obtained from the analysis of 24 simulated events are shown on Fig. 2. The extraction of transversal radius parameters was performed by the fit of one-dimensional correlation function $C(\Delta p_t)$ (see Fig.6) where $\Delta p_t = \sqrt{\Delta p_x^2 + \Delta p_y^2}$ because of low statistics visible as the fluctuations of the two-dimensional correlation functions on Fig.2.

Procedure HFITGA from HBOOK Library fits the data to the function

$$F(\Delta p, p_o, \sigma) = e^{-(\Delta p - p_o)^2/2\sigma^2} \tag{11}$$

($p_o = 0$ in our case) and therefore from the comparison to the analytical correlation function of Gaussian source [8]

$$C_G(\Delta p) - 1 = e^{-\Delta p^2 R^2/2} \tag{12}$$

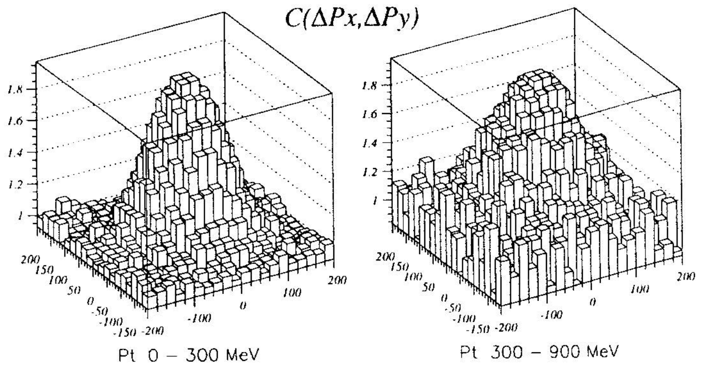

Fig. 2 Correlation functions in $\Delta\vec{p}_t$ variable. The width of the Gaussian peak is bigger in the case of the $p_t \in (300, 900)$ MeV region what indicates the smaller R_t parameter.

the source parameter R is obtained from the result of fit as $R = 1/\sigma$. Higher statistics will allow the two and three dimensional analysis of the correlation functions.

3. Formation Time Scenario

In the rescattering simulation [1] the following scenario of collision is used: The pions are allowed to interact immediatelly after they are created but after the collision the pions involved in the collision are forbidden to interact for the time period τ_d (see Fig.3, S_a).

Our model includes this collision scenario but we have tested a different concept of the collision (see Fig.3, S_b) including the formation time parameter [9]. In this new scenario of collision the pions are forbidden to scatter immediately after their creation. The ability to interract is postponed for a time period T_f in Global Frame of Collision which is obtained by the Lorentz dilatation of τ_f - formation time parameter [9]:

$$T_f = \frac{\tau_f}{\sqrt{1 - \vec{v}^2/c^2}} \qquad (13)$$

The value of τ_f influences substantially the number of collisions per pion. The analysis of simulations performed so far does not indicate any substantial differences in the results obtained for different scenarios of collision - S_a or S_b on Fig. 3.

However one could expect slight differences because of a momentum sensitivity of scenario S_b due to the momentum dependent Lorentz dilatation factor in equation (13). Further discussion of this topic can be found in the following section.

4. Results

Initial source distributions for 24 events were generated according to equation (4) and used as the input for the TE unit which had been run for diffemt scenarios of collisions

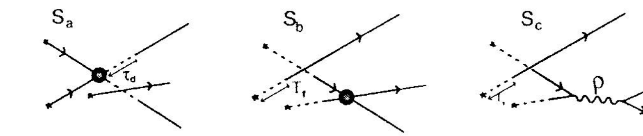

Fig. 3 The scenarios of collision. S_a is the scenario used in model [1], S_b is our scenario with the formation time parameter. S_c - possible description of resonances.

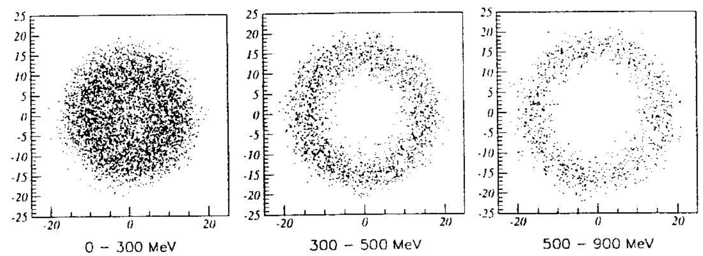

Fig. 4 The $\rho(\vec{x}_t)$ distributions of pions after 20fm/c obtained from $TEx.dat$ files. Axes values are in fm.

S_a, S_b and different values of τ_f. The pions with a rapidity in the range $\langle -1, +1 \rangle$ were selected for analysis in the DA unit. Before the discussion of the correlation analysis we want to emphasize the result seen directly from Figure 4.

Different shapes of $\varrho(\vec{x})$ distributions for different transversal momentum regions indicate a momentum-space ($\vec{x}$-$\vec{p}$) correlation of the distribution of pions after 20 fm/c. Therefore we do not find it surprising that the distribution of the points of last interaction is also $\vec{x}$-$\vec{p}$ correlated (Fig.5). This is an important result which indicates that the assumption of $\vec{x}$-$\vec{p}$ correlation of distribution functions in analytical calculations is fully justified for the interacting (scattered) particles whatever mechanism of production is assumed. We regard this production mechanism independent result to be quite important.

One dimensional correlation function shown on Fig.6 was constructed from the generated events according to formula (11). Table 1. shows the results of the fitting procedure applied to such correlation functions for different p_t regions, different scenarios S_a, S_b and different values of parameter τ_f. The first column of Table 1. indicates that our simulation reproduces the main result of simulation described in [1].

The numerical results for R_t are in agreement with the results presented in [1]. In the column x we give radii parameters R_t obtained from the initial pion distributions, that is from $IG.dat$ - the output files of IG unit. The fact that the radius parameter does not depend on the p_t momentum region if the rescattering is not present is a consequence of our non-correlated initial distribution (4). The variation of parameters is in the range of statistical errors assessed to be 5% for R_t parameters.

Three other columns show the results for our new scenario of collision S_b. From the row Coll/π of Table 1. we see that average number of collisions per pion is sensitive to the value of τ_f. Our new scenario of collision also reproduces the p_t dependence of R_t

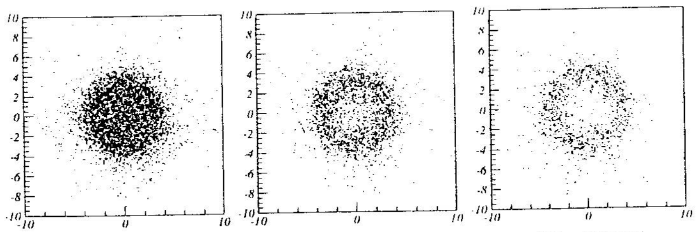

Fig. 5 The last interaction distributions in transversal plane. Axes values are in fm.

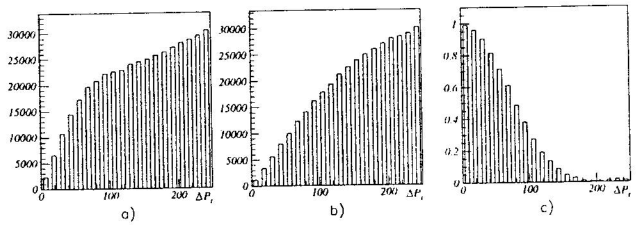

Fig. 6 Construction of the one-dimensional correlation function c) from correlated a) and uncorrelated b) pair distributions (8) in Δp_t variable. Histogram c) = [a) / b)] - 1.

parameters measured by [3].

The slope of p_t dependence of R_t is however slightly smaller in the case of scenario S_b than in the case S_a. This seems to be valid for the whole region of $\tau_f \in \langle 0.2, 0.5 \rangle$. We regard this weaker dependence of R_t on p_t region in this case to be a consequence of momentum sensitivity of the S_b mechanism of collision already mentioned in Section 3. The column for τ_f = 1.0fm indicates that the number of collisions per π should be higher than 1.0 in order to reproduce the experimentally measured p_t dependence of R_t. The higher value of R_t for the p_t > 600MeV region than for the 0 < p_t < 600MeV region in this column is quite strange especially because of $R_t \approx 2.5$ without the influence of rescattering. This behaviour is a consequence of the special directional momentum property of $\varrho(\vec{p}, \vec{x})$ last interaction distribution which is studied nowadays in collaboration with the Regensburg group using the results of the model.

5. Summary

We have briefly described our version of the rescattering computer simulation model. The model reproduces the p_t dependence of R_t radius parameters identified by CERN SPS NA35 [3] group in agreement with the rescattering model [1]. Based on the results of our work we want to emphasize the following conclusions:

The uncorrelated initial source distribution (4) leads to the momentum-space correlated distribution of points of last interactions of interacting particles. This result justifies the momentum-space correlated analytical distributions considered in theoretical calculations.

$\tau_i - \tau_d$	0.0 - 0.5	0.2 - 0.0	0.5 - 0.0	1.0 - 0.0	x
P_t [MeV]	R [fm]	R [fm]	R [fm]	R [fm]	R [fm]
0-300	3.24	3.12	3.08	2.94	2.54
300-600	2.43	2.38	2.44	2.72	2.52
600 - Inf.	2.09	2.14	2.21	3.03	2.57
Coll./π	4.5	11.2	3.8	0.6	0.0

Tab. 1 The results of the fit procedures. The errors for the values of R_t parameters are 5%.

We have shown that for the different scenario of collision process - the scenario using the formation time τ_f the p_t dependence of R_t is easily reproduced. This gives the possibility of including a more natural description of the collision process (see S_c on Fig.3) and allows us to include resonances in our simulation.

We put the upper limit on the formation time parameter τ_f < 1.0fm because of p_t dependence of R_t presented in Table 1.

The architecture of our computer representation of the rescattering model is prepared for further enhancements. The program is already being used in the study and prediction of interesting phenomena in HIC interferometry experiments.

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