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EXPLICIT EXPRESSION FOR THE DEUTERON OVERLAP INTEGRAL IN THE $^3D_1$ STATE

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Explicit expression for the deuteron overlap integral occurring in the reactions with deuterons has been obtained. Harmonic oscillator wave functions have been used for nucleons whereas $(^3S_1 +^3 D_1)$ - state wave function obtained as a solution of the Schrödinger equation with tensor forces was employed for the deuteron. A good agreement with experimental data is obtained.

1. Introduction

The majority of the reduced widths for various channels of light nuclei has been calculated and compared with experimental data [1-2]. In the paper [2], its theoretical expressions have been obtained using simplifying hypotheses concerning the structure of the emitted particles.

The effect of the structure of the emitted particles on the reduced widths using for the intrinsic wave functions of the emitted particles only $^3S_1$ - state functions has been analysed [1].

In the present paper, we have studied the effect of other states than the $^3S_1$ - state on the reduces widths. We have, for instance, considered configuration $^3S_1 +^3 D_1$ for the intrinsic deuteron wave functions.

A good agreement with experimental data is obtained.

Explicit Expression for the Deuteron Overlap Integral

Explicit expression for the amplitude of the reduced widths occurring in the reactions with deuteron has been obtained in Ref. [1].

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$$\gamma_{clj_sL_3} = \left(\frac{h^2}{2\mu}\right)^{\frac{1}{2}} R_c \left(\begin{array}{c} n \\ A_3 \end{array}\right)^{\frac{1}{2}} \sum_{L_bL_4J_4} [(2J_4+1)(2L_b+1)/(2S_1+1)(2l+1)]^{\frac{1}{2}}$$
$$\langle l^n[f_1]L_1S_1T_1|\}l^{n-A_3}[f_2]L_2S_2T_2, l^{A_3}[f_3]L_bS_3T_3, L_1S_1T_1\rangle$$
$$(T_2T_3m_{T_2}m_{T_3}|T_2T_3T_1m_{T_1})\langle L_2L_3(L_4)S_1js|L_2, L_3S_1(J_4)js\rangle$$
$$\langle L_2S_2(J_2)L_3S_3(J_3)js|L_2L_3(L_4)S_2S_3(S_1)js\rangle$$
$$\langle L_2J_4(js)L_bL_3(l)J_1|L_2L_b(L_1)J_4L_3(S_1)J_1\rangle$$
$$\sum_\nu R_{\nu l}(R_c)\,[A_1/(A_1-A_3)]^{\nu+l/2}\,Q_{\nu L_blL_3}$$

In the relation (A), μ is the reduced mass of the neutron-proton system and R_c the channel radius of the nuclear reaction with deuterons. $\binom{n}{A_3}$ is the multiplier resulting from the antisymmetrization of the functions implicated in the reactions with deuterons. The notation $(j_1j_2m_1m_2|j_1j_2jm)$ represents the coefficient of vector addition, while the quantity

$$\langle j_1j_2(j_{12})j_3j_4(j_{34})j|j_1j_3(j_{13})j_2j_4(j_{24})j\rangle =$$
$$\sqrt{(2j_{12}+1)(2j_{34}+1)(2j_{13}+1)(2j_{24}+1)}\left\{\begin{array}{ccc} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{34} \\ j_{13} & j_{24} & j \end{array}\right\}$$

is proportional to the 9-j symbol. The Racah coefficients W are defined by the relation

$$\langle j_1j_2(j_{12})j_3j|j_1j_2j_3(j_{23})j\rangle =$$
$$\sqrt{(2j_{12}+1)(2j_{23}+1)}W(j_1j_2jj_3, j_{12}j_{23}).$$

More details relating to the significance of the notations used in this work can be found in Refs. /1-2/.

The deuteron overlap integral intervening in relation (A) has the following form

$$Q_{\nu L_blL_3} = \sum_{m_{L_3}m} (L_3lm_{L_3}m|L_3lL_bm_{L_b})$$
$$\int \left\langle n_1l_1n_2l_2L_bm_{L_b}|\chi_{L_3m_{L_3}}(r)|\nu lm(\vec{R}_3)\right\rangle d\vec{R}_3 d\vec{r}, \quad (1)$$

where $\chi_{L_3m_{L_3}}$ is the spatial part of the deuteron internal wave function, $|\nu lm(R_3)\rangle$ harmonic oscillator wave function taken in the deuteron centre of mass and

$$|n_1l_1n_2l_2L_bm_{L_b}\rangle = \sum_{m_1m_2} (l_1l_2m_1m_2|l_1l_2L_bm_{L_b})|n_1l_1m_1\rangle\,|\,n_2l_2m_2\rangle. \quad (2)$$

For $L_3 = 0$ the expression (1) has been calculated in /2/ and for $L_3 = 2$ it is to be calculated. Thus, by means of the transformation

$$\vec{R}_3 = (\vec{r}_1 + \vec{r}_2)2^{-1}; \quad \vec{r} = \vec{r}_1 + \vec{r}_2, \quad (3)$$

we shall pass from the wave function (2), expressed in coordinates with respect to the centre of the potential well, to a sum of the wave functions expressed in relative and centre of mass coordinates of the nucleons

$$|n_1l_1n_2l_2L_bm_{L_b}\rangle = \sum_{n_{R_3}l_{R_3}}\sum_{n_rl_r} \langle n_rl_rn_{R_3}l_{R_3}l_b|1,1|n_1l_1n_2l_2L_b\rangle\,|n_rl_rn_{R_3}L_bm_{L_b}\rangle \quad (4)$$

with the selection rule resulting from the energy conservation law

$$2n_1 + l_1 + 2n_2 + l_2 = 2n_r + l_r + 2n_{R_3} + l_{R_3} \quad (4')$$

Inserting the expression (4) into the overlap integral (1) and integrating over the angular variables, one obtains

$$Q_{\nu L_blL_3=2} = \langle N-\nu, L_3 = 2, \nu l, L_b|1,1|n_1l_1n_2l_2L_b\rangle \int_0^\infty R_{N-\nu,L_3=2}(r)f_D(r)r dr, \quad (5)$$

where $R_{nl}(r)$ is the radial part of the oscillator wave function and $f_D(r)$ can be expressed as /3/

$$f_D(r) = N_D exp(-\delta r)\left[1 + 3(\delta r)^{-1} + 3(\delta r)^{-2}\right] \quad \text{for} \quad r_c = 0 \quad (6)$$

and

$$f_D(r) = N_S\left[1 + 6(\delta r_c)^{-1} + 12(\delta r_c)^{-2} + 6(\delta r_c)^{-3}\right]^{-1/2}$$
$$\left[1 + 3(\delta r)^{-1} + 3(\delta r)^{-2}\right]\exp(-\delta r) \quad \text{for} \quad r_c \neq 0 \quad (7)$$

The explicit expression for the matrix element $\langle|\rangle$ can be found in Ref./2/.

For the zero radius of hard core ($r_c = 0$), taking into account the expression of $R_{nl}(r)$ and (6), the overlap integral (5) becomes

$$Q_{\nu L_blL_3=2} = \langle N-\nu, 2, \nu l, L_b|1,1|n_1l_1, n_2l_2, L_b\rangle$$
$$\alpha_r^{3/2}\sqrt{\frac{2(N-\nu)!}{(N-\nu+5/2)!}}\sum_{k=0}^{N-\nu}\left(\begin{array}{c} N-\nu+5/2 \\ N-\nu-k \end{array}\right)\frac{(-1)^k}{k!}F_k \quad (8)$$

where F_k is given by the integral

$$F_k = \int_0^\infty \exp(-\frac{\alpha_r^2 r^2}{2})(\alpha_r r)^{2k+2} f_D(r) r dr. \quad (9)$$

Taking into account the connection between the parabolic cylinder functions and the confluent hypergeometric function, (9) becomes

$$F_k = \sqrt{\pi}\frac{N_D(2k+3)!}{\alpha_r^2 2^{k+2}}$$

$$\left\{ g_{k+1}(z/\sqrt{2}) + \frac{3\sqrt{2}}{z(2k+3)} g_{k+1/2}(z/\sqrt{2}) + \frac{6}{z^2(2k+3)(2k+2)} g_k(z/\sqrt{2}) \right\}, \quad (10)$$

in which the significance of the function $g_k(x)$ is

$$g_k(x) = \frac{1}{(k+1/2)!} {}_1F_1(k+1, 1/2, x^2) - \frac{2x}{k!} {}_1F_1(k+3/2, 3/2, x^2). \quad (11)$$

The notation ${}_1F_1(\alpha, \beta, y)$ means the confluent hypergeometric function. Substituting (10) in (8), the overlap integral becomes

$$Q_{\nu L_b l L_3=2} = \langle N-\nu, 2, \nu l, L_b | 1, 1 | n_1 l_1, n_2 l_2, L_b \frac{4\sqrt{2} N_D}{\sqrt{\alpha_r}} \sqrt{\frac{(N-\nu+5/2)!}{(N-\nu)!}}$$

$$\sum_{k=0}^{N-\nu} \frac{(N-\nu)!(-1)^k 2^k}{(N-\nu-k)!} \frac{k+1}{2k+5} R_k(z/\sqrt{2}) \quad (12)$$

where

$$R_k(x) = g_{k+1}(x) + \frac{3}{x(2k+3)} g_{k+1/2}(x) + \frac{3}{x^2(2k+3)(2k+2)} g_k(x). \quad (13)$$

For $r_0 \neq 0$ the function (7) should be inserted into (9) and in this case the overlap integral gets the form

$$Q_{\nu L_b l L_3=2} = \langle N-\nu, L_3=2, \nu l, L_b | 1, 1 | n_1 l_1, n_2 l_2, L_b \rangle$$

$$\sum_{n=0}^{N-\nu} \frac{N_D}{\sqrt{\alpha r}} [(N-\nu+5/2)!/(N-\nu)!]^{1/2} \frac{(N-\nu)!(-1)^k}{(N-\nu-k)!}$$

$$\left[\frac{2^{k+5/2}(k+1)}{2^k+5} R_k(z/\sqrt{2}) - \frac{1}{\sqrt{2k!}} \exp(-\alpha_r^2 r_c^2 2^{-1})\right.$$

$$\left. -\delta r_c(1 + 3\delta^{-1} r_o^{-1} + 3\delta^{-2} r_c^{-2})(\alpha_r r)^{2k+4}\right] \quad (14)$$

3. The Deuteron Reduced Width

The explicit expression of the amplitude of the reduced width is given by the relation (A) with (12) and (14).

The expression of the reduced width γ_{cl}^2 is obtained by taking into account the internal wave function (6) /3/. Thus,

$$\gamma_{cl}^2 = \left(\sum_{j_s} \gamma_{clj_s} \right)^2, \quad (15)$$

Nucl.	E (MeV)	J^π	T	Decay mode	l	$^3S_1$ /2/ $r_c = 0$ (fm)	$^3S_1$ /2/ $r_c = 0.43$ (fm)	$^3S_1 + ^3D_1$ $r_c = 0$ (fm)	$^3S_1 + ^3D_1$ $r_c = 0.43$ (fm)	Exp. [2]	R_c (fm)
He^5	16.69	$\frac{3}{2}^+$	$\frac{1}{2}$	H^3+H^2	0	0.63	0.76	0.24	0.71	0.7	7
Li^5	18.81	$\frac{3}{2}^+$	$\frac{1}{2}$	He^3+H^2	0					0.7	

Table 1. The calculated deuteron reduced widths for the case of several simple nuclei.

in which the amplitude of the reduced widths γ_{clj_s} can be written in the form

$$\gamma_{clj_s} = \cos\omega \gamma_{clj_s, L_3=0} + \sin\omega \gamma_{clj_s, L_3=2}. \quad (16)$$

In its expression, the parameter

$$\alpha_r = \left(\frac{m_n m_p}{m_n + m_p} \frac{\omega}{\hbar} \right)^{1/2} \quad (17)$$

appears, where ω is the frequency of the oscillator. In the following calculation, we supposed that the neutron and the proton masses are equal. Thus $\alpha_r = \sqrt{\frac{m\omega}{2\hbar}}$, in which m is the nucleon mass. On the basis of expression (12), (14) and (15), we have calculated the deuteron reduced widths for case of several simple nuclei (Table 1).

4. Discussion

The explicit expression of the deuteron internal wave function depends on the chosen potential. Because it is not yet established which of the potential is the most adequate one, for solving the deuteron problem there are several hypotheses. The functions (14)-(15) have not a good behaviour at the origin [4] and at the same time the function (15) cannot be normalized to unity [5]. If one uses the method (16)-(19) [3] for the determination of the normalization coefficients, it follows that the reduced width is approximately 2.5 times smaller than obtained for the S-state. However, using a hard core potential with functions (20)-(21) [3], the result for the S + D mixture is quite the same as that obtained experimentally. Hence, we can conclude that the effect of the deuteron D-state on the reduced widths depends strongly on the behaviour of the S and D states in the neighbourhood of the origin. For functions (14)-(15) [3] the results with S + D mixture functions are differing very much from the pure S-state wave functions. On the contrary, by introducing a hard core potential we have eliminated the part appearing in the overlap integral which seems to bring for the S + D mixture the largest contribution. These results show that is possible for the considered effect to play a more important part than it seems at the first sight.

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