acta physica slovaca vol. 46 No. 1, 1 – 8 February 1996

THE EFFECT OF COUNTER-ROTATING TERMS AND CAVITY DAMPING ON QUANTUM FLUCTUATIONS IN THE JAYNES-CUMMINGS MODEL

J. Seke, G. Adam, V. Bužek[1]
Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstrasse 8-10/136, A-1040 Wien, Austria

Received 31 August 1995, accepted 12 December 1995

We study the influence of the counter-rotating terms and the cavity damping on the squeezing phenomena (i.e., the reduction of quantum fluctuations) in the Jaynes-Cummings model. We analyze two particular cases, namely, when the cavity mode is initially prepared in (1) the vacuum state and in (2) the squeezed vacuum state.

1. Introduction

During the last decade micromasers have been studied extensively both experimentally [1-4] and theoretically [5]. Theoretical description of the micromaser [5] is usually based on the model of an interaction of a two-level atom with a damped harmonic oscillator (a monomode EM field). The atoms are supposed to be near resonant with the field. In this case the interaction of a single atom with a monomode EM field can be described by the Jaynes-Cummings model [6,7] within the rotating-wave approximation (RWA). The rotating-wave approximation is perfectly justified for small values of the parameter g/ω, where g is the atom-field coupling constant in the dipole approximation and ω is the atomic transition frequency. Simultaneously, a detailed investigation of the role of counter-rotating terms (CRT) [i.e. the role of those terms which are neglected by the RWA] in the atom-field interaction Hamiltonian can reveal new nontrivial features of the micromaser dynamics.

The Jaynes-Cummings model without the rotating-wave approximation has been studied recently by number of authors. In particular, several aspects of the atom-field dynamics described by the JCM without the RWA has been analyzed by Graham and Höhnerbach [8]. Huan, Peng and Li investigated [9,10] the Lamb shift of the atomic levels and the shift of the field frequency due to the counter-rotating effects. Compagno with coworkers discussed [11,12] the role of virtual photons (which are associated

[1]Permanent address: Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Slovakia

with the CR terms in the Hamiltonian) in processes of the absorption and the spontaneous emission. Rui-hua Xie, Gong-ou Xu, and Dun-huan Liu investigated [13] atomic squeezing, photon antibunching and the effect of virtual-photon field in the JCM without the RWA. Atomic squeezing and quantum statistics of the damped cavity field in the JCM without the RWA has been analyzed by Seke [14-16]. Zaheer and Zubairy used a path-integral approach [17] to include the CR effects in the atom-field dynamics.

In the present paper we will study the effect of the counter-rotating terms and the cavity damping on dynamics of a two-level atom interacting with a cavity field. In particular we will analyze the role of the CRT on the reduction of quantum fluctuations.

2. Recurrence differential equation for density-matrix elements

The Liouvillian of the JCM without RWA in the interaction picture reads as

$$\hat{L}(t) = \hat{L}_{AR}(t) + i\hat{\Lambda}_R, \quad \hat{L}_{AR}(t) = [\hat{H}_{AR}(t), ...], \tag{2.1}$$

with the corresponding atom-field interaction Hamiltonian ($\hbar = 1$) given by the relation

$$\hat{H}_{AR}(t) = g(\hat{\sigma}_- e^{-i\omega t} + \hat{\sigma}_+ e^{i\omega t}) \otimes (\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}), \tag{2.2}$$

and the field-damping Liouvillian [18]

$$\hat{\Lambda}_R(...) = \kappa([\hat{a}(...), \hat{a}^\dagger] + [\hat{a}, (...)\hat{a}^\dagger]), \tag{2.3}$$

which describes dynamics of the cavity-field mode (i.e., a harmonic oscillator) coupled to a zero temperature reservoir (heat bath). Here $\hat{\sigma}_\pm$ are the atomic dipole moment operators, ω is the frequency of the atomic transition which is assumed to be on resonance with the frequency of the resonant cavity field mode, $\hat{a}^\dagger$ and $\hat{a}$ are the photon creation and annihilation operators, g is the atom-field coupling constant, and κ is the cavity-damping factor.

The time-evolution of the combined atom-field system is described by the Liouville equation for the atom-field density operator $\hat{\rho}(t)$:

$$\frac{d\hat{\rho}(t)}{dt} = -i\hat{L}(t)\hat{\rho}(t). \tag{2.4}$$

In the present paper where we treat the specific initial condition

$$\hat{\rho}(0) = \hat{\rho}_A(0) \otimes \hat{\rho}_R(0) \tag{2.5}$$

with atoms being in a coherent superposition of the upper and lower states

$$\hat{\rho}_A(0) = |\psi_A(0)\rangle\langle\psi_A(0)|; \tag{2.6}$$

$$|\psi_A(0)\rangle = \sin\theta\,|s = 1/2, m = 1/2\rangle_A + \cos\theta\,|s = 1/2, m = -1/2\rangle_A, \tag{2.7}$$

where the vector $|s = 1/2, m = 1/2\rangle_A$ ($|s = 1/2, m = -1/2\rangle_A$) describes the upper (lower) state of the two-level atom under consideration. The radiation field is assumed to be initially either in the vacuum state

$$|\psi_R(0)\rangle = |0\rangle_R, \tag{2.8}$$

or in the squeezed vacuum state described by the state vector

$$|\psi_R(0)\rangle = |0_{\text{squeezed}}\rangle_R = \sum_{k=1}^{\infty} f(k)|2k\rangle_R; \tag{2.9}$$

with

$$f(k) = \frac{1}{\sqrt{\mu(2k)!}}\left(\frac{\nu}{2\mu}\right)^k H_{2k}(0), \tag{2.10}$$

where H_k are the Hermite polynomials. For the sake of simplicity, we chose the parameters μ and ν such that

$$\mu = \cosh r, \quad \nu = \sinh r, \tag{2.11}$$

where the squeezing parameter r is chosen to be real.

Using basis vectors

$$|n(k)\rangle \equiv |s, s-n\rangle_A \otimes |k\rangle_R, \quad s = \frac{1}{2}, \quad n = 0, 1, \quad k = 0, 1, 2, ... \tag{2.12}$$

we can obtain from the Liouville equation (2.4) the following equation for the density matrix elements $\rho^*_{n(k),l(m)} = \rho_{l(m),n(k)}$:

$$\begin{aligned}\frac{d}{dt}\rho_{n(k),l(m)} = \; & -ig[\sqrt{(1-n)(n+1)(k+1)}\,\rho_{n+1(k+1),l(m)} \\ & -\sqrt{(1-l)(l+1)(m+1)}\,\rho_{n(k),l+1(m+1)} + \sqrt{(2-n)nk}\rho_{n-1(k-1),l(m)} \\ & -\sqrt{(2-l)lm}\,\rho_{n(k),l-1(m-1)} + \sqrt{(2-n)n(k+1)}\,e^{-2i\omega t}\rho_{n-1(k+1),l(m)} \\ & -\sqrt{(2-l)l(m+1)}\,e^{2i\omega t}\rho_{n(k),l-1(m+1)} \\ & +\sqrt{(1-n)(n+1)k}\,e^{2i\omega t}\rho_{n+1(k-1),l(m)} \\ & -\sqrt{(1-l)(l+1)m}\,e^{-2i\omega t}\rho_{n(k),l+1(m-1)}] \\ & +2\kappa\sqrt{(k+1)(m+1)}\,\rho_{n(k+1),l(m+1)} \\ & -\kappa(k+m)\rho_{n(k),l(m)}, \quad n, l = 0, 1, \quad k, m = 0, 1, 2, ...\end{aligned} \tag{2.13}$$

In order to examine the field and the atomic squeezing phenomena we introduce the corresponding squeezing parameters. To be specific, the field quadrature operators $\hat{a}_1$ and $\hat{a}_2$ are defined as

$$\hat{a}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger), \quad \hat{a}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger), \quad [\hat{a}_1, \hat{a}_2] = \frac{i}{2}. \tag{2.14}$$

The effect of the field-mode squeezing is associated with the reduction of quadrature fluctuations below the vacuum limit, i.e., when the variance $\langle(\Delta\hat{a}_l)^2\rangle$ ($l = 1, 2$) is smaller than 1/4:

$$\langle(\Delta\hat{a}_l)^2\rangle_t = \langle\psi(t)|(\Delta\hat{a}_l)^2|\psi(t)\rangle = \langle(\hat{a}_l)^2\rangle_t - \langle\hat{a}_l\rangle_t^2 < \frac{1}{4}, \quad l = 1, 2. \tag{2.15}$$

The degree of the field squeezing can be quantified with the help of the squeezing parameter Q defined as

$$Q = 4\langle(\Delta\hat{a}_1)^2\rangle_t - 1, \tag{2.16}$$

where squeezing exists for $-1 \leq Q < 0$, while the maximum (100%) squeezing is obtained for $Q = -1$.

The components of the atomic pseudospin operator are given by relations

$$\hat{\sigma}_x = \frac{1}{2}(\hat{\sigma}_+ + \hat{\sigma}_-), \quad \hat{\sigma}_y = \frac{1}{2i}(\hat{\sigma}_+ - \hat{\sigma}_-), \quad [\hat{\sigma}_x, \hat{\sigma}_y] = i\hat{\sigma}_z, \tag{2.17}$$

where $\hat{\sigma}_z$ is the atomic population-inversion operator. The variances of the operators $\hat{\sigma}_x$ and $\hat{\sigma}_y$ obey the uncertainty relation

$$\langle(\Delta\hat{\sigma}_x)^2\rangle\langle(\Delta\hat{\sigma}_y)^2\rangle \geq \frac{1}{4}|\langle[\hat{\sigma}_x, \hat{\sigma}_y]\rangle|^2 = \frac{1}{4}|\langle\hat{\sigma}_z\rangle|^2. \tag{2.18}$$

The atomic state is said to be squeezed whenever

$$\langle(\Delta\hat{\sigma}_l)^2\rangle_t < \frac{1}{2}|\langle\hat{\sigma}_z\rangle_t|, \quad l = x, y. \tag{2.19}$$

This effect can be measured in the Stern-Gerlach–type experiment which allows to measure mean values of the relevant atomic operators. To quantify the degree of the atomic-dipole squeezing one can introduce the squeezing parameter S_l defined as

$$S_l \equiv \frac{2\langle(\Delta\hat{\sigma}_l)^2\rangle_t}{|\langle\hat{\sigma}_z\rangle_t|}, \quad l = x, y. \tag{2.20}$$

If S_l is smaller than unity then the dipole moment exhibits reduction of quantum fluctuation. The maximum (100%) squeezing is associated with the value of S_l equal to zero.

3. Numerical results

We solve the recurrence equation (2.13) numerically within the RWA and without the RWA. In addition, we consider the case of the lossless as well as the damped cavity with the damping parameter $K = \kappa/g = 0.5$ (given in units of the atom-field coupling parameter g). We consider two cases when the cavity field initially prepared in the vacuum state and in the squeezed vacuum state.

The atom is assumed to be initially prepared in the superposition state (2.7) which exhibits dipole squeezing. The mean values of the atomic operators in the state (2.7) the following expressions

$$\langle(\Delta\hat{\sigma}_x)^2\rangle_0 = \frac{1}{4}(1 - \sin^2 2\theta), \quad \langle\hat{\sigma}_z\rangle_0 = \frac{1}{2} - \cos^2\theta. \tag{3.1}$$

In our paper we consider the phase θ of the initial atomic superposition state to be equal to 0.248π (in this case $\langle\hat{\sigma}_z\rangle_t \neq 0$, so we can use the parameter S_l to quantify the degree of the atomic squeezing).

To understand dynamics of the model under consideration we study the time evolutions of the atomic population inversion

$$Z(T = gt) = \mathrm{Tr}[\hat{\sigma}_z\hat{\rho}(t)] \equiv \langle\hat{\sigma}_z\rangle_t, \tag{3.2}$$

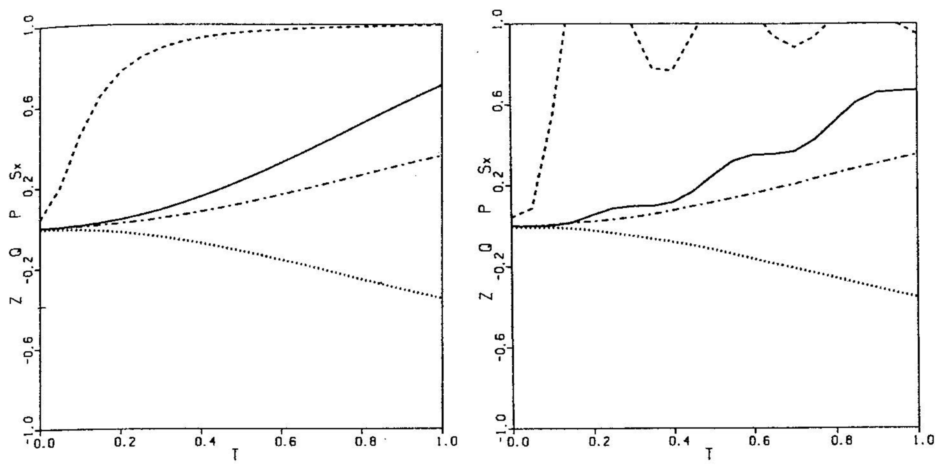

Fig. 1 Time evolutions of the atomic population inversion Z (dotted), the field-squeezing parameter Q (solid), the mean photon number P (dashed-dotted) and the atomic-squeezing parameter S_x (dashed) as functions of the scaled time $T = gt$ for a lossless cavity. The atom is supposed to be initially in the coherent superposition of its upper and lower states ($\theta = 0.248\pi$). The field mode is initially in the vacuum state. Calculations are carried out in (a) the rotating-wave approximation and (b) without the rotating-wave approximation with the coupling constant g given in units of the atomic transition frequency such that $g/\omega = 0.1$.

the mean photon number

$$P(T = gt) = \mathrm{Tr}[\hat{a}^\dagger\hat{a}\hat{\rho}(t)] \equiv \langle\hat{a}^\dagger\hat{a}\rangle_t, \tag{3.3}$$

and the atomic and the field squeezing parameters S_x and Q, respectively. From our numerical calculations it follows that for the chosen phases of the atomic and the field squeezing the parameter S_y is in the present model always larger than unity, i.e., in this quadrature no squeezing can be observed. All plots presented in the paper are given in units of the scaled time $T = gt$.

From Figs.1 and 3 it follows that in the case when the cavity mode is initially prepared in the vacuum state the initial dipole-moment squeezing is rapidly deteriorated during the first instants of the time evolution, i.e., the parameter S_x does increase from its initial value associated with the almost maximum degree of squeezing ($S_x \simeq 0$) to the "steady state" value equal to unity [see Fig.1(a)]. In the case when the rotating-wave approximation is adopted the decrease of the dipole squeezing (i.e., the increase of the parameter S_x) is approximately exponential. It is interesting to note that during the short time interval when the reduction of quantum fluctuations is deteriorated the cavity damping has essentially no influence on the decrease of the atomic squeezing [compare Fig.1(a) and Fig.3(a)]. Moreover, the influence of the cavity damping on the time evolutions of the population inversion and the mean photon number is also not

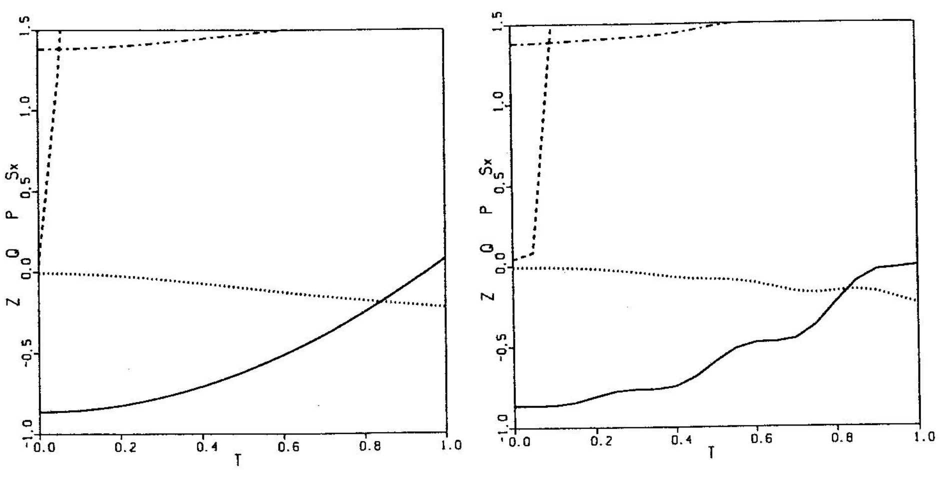

Fig. 2 The same as Fig.1 but the field mode is initially assumed to be in the squeezed vacuum state with the squeezing parameter $r = 1$.

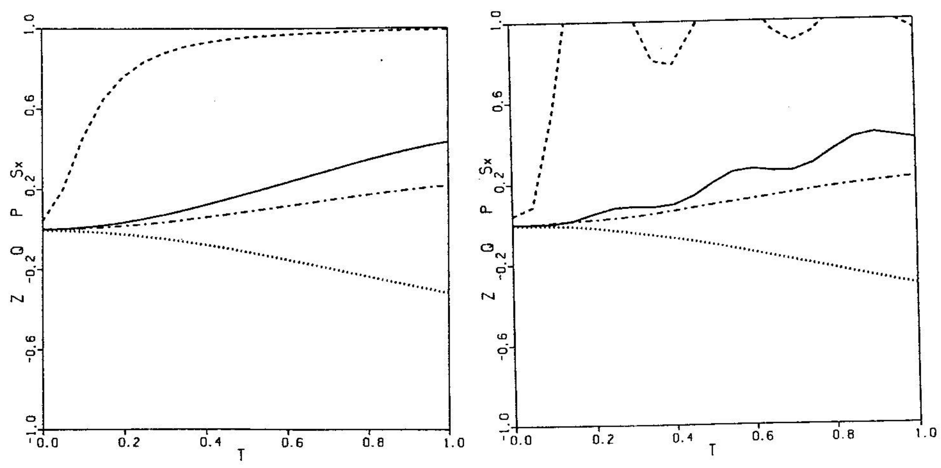

Fig. 3 The same as Fig.1 for a damped cavity with $K = 0.5$.

significant. With the initial atomic state under consideration the field squeezing does not occur in the present model.

When the counter-rotating terms are considered in the Hamiltonian an approximately exponential decay of the atomic squeezing is substituted with the damped

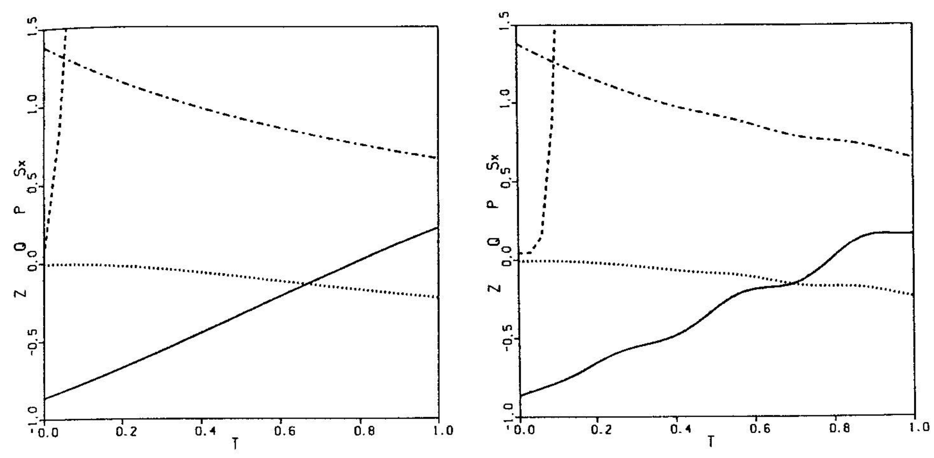

Fig. 4 The same as Fig.2 for a damped cavity with $K = 0.5$.

oscillatory decay of the reduction of quantum fluctuations of the dipole operator [cf. Figs.1(b) and 3(b)]. In this case we see again essentially no influence of the cavity damping on the dipole-moment squeezing. A minimal influence on the population inversion and the mean photon number can be observed.

From Figs. 2 and 4 we can conclude that when the cavity mode is initially prepared in the squeezed vacuum state then the atomic squeezing is deteriorated even more rapidly than in the case with the field mode initially prepared in the vacuum state. In the RWA case, the atomic squeezing is deteriorated almost linearly within an extremely short time interval, whereas the decay of the field squeezing is slower. The CRT enlarge somewhat the deterioration time of the atomic squeezing and leads to an oscillatory increase of fluctuations of the field mode. Simultaneously, the CTR does not cause significant effects regarding the mean photon number and the population inversion during the observation time interval.

We see from Fig.4 that the damping leads to the decay of the mean photon number and to a more rapid (almost linear) deterioration of the field squeezing, but does not influence the decrease of the atomic squeezing. The time evolution of the population inversion is not affected significantly during the short observation time interval under consideration.

4. Conclusion

In the present paper we have derived an exact recurrence equation for density matrix elements of the JCM without the RWA. We have solved this equation numerically for two initial states of the cavity mode. Namely, we have assumed to field mode to be initially prepared in the vacuum state and the squeezed-vacuum state. We have considered both the lossless and the damped cavity.

We have found that the influence of the cavity damping in the chosen observation time interval, during which both the initial atomic squeezing and the field squeezing decrease to zero, is not very significant. While the cavity damping has no influence on the atomic squeezing and accelerates the deterioration of the field squeezing, the CRT leads to the oscillatory decay of the initial squeezing.

Furthermore, we have shown that deterioration of the atomic squeezing depends on the squeezing of the field mode. In particular, for chosen phases of the field and the atomic squeezing the deterioration of the atomic squeezing is enhanced by the presence of the initial field squeezing.

Acknowledgements This work was supported by the East-West Programme of the Austrian Academy of Sciences under Contract No. 45.367/1-IV/6a/94 of the Österreichisches Bundesministerium für Wissenschaft und Forschung.

References

[1] D. Meschede, H. Walther, G. Müller: *Phys. Rev. Lett.* **54** (1985) 551
[2] G. Rempe, H. Walther, N. Klein: *Phys. Rev. Lett.* **58** (1987) 353
[3] G.Rempe, F.Schmidt-Kaler, H.Walther: *Phys. Rev. Lett.* **64** (1990) 2783
[4] S.Haroche, J.M.Raimond: in *Advances in Atomic and Molecular Physics*, edited by D. Bates and B. Bederson (Academic, New York, 1985), Vol. 20, p. 347;
[5] P.Filipowicz, J.Javanainen, P.Meystre: *Phys. Rev. A* **34** (1986) 3077
[6] E. T. Jaynes, F. W. Cummings: *Proc. IEEE* **51** (1963) 89
[7] B.W.Shore, P.L.Knight: *J. Mod. Opt.* **40** (1993) 1195
[8] R.Graham, M.Höhnerbach: *Phys. Lett. A* **101** (1984) 61
[9] X.Y.Huan, J.S.Peng: Phys. Scr. **T21** (1988) 100
[10] J.S.Peng, G.X.Li: *Phys. Rev. A* **45** (1992) 3289
[11] G.Compagno, G.M.Palma, R.Passante, F.Persico: *Europhys. Lett.* **9** (1989) 215
[12] G.Compagno, R.Passante, F.Persico: *J. Mod. Opt.* **37** (1990) 1377
[13] Rui-hua Xie, Gong-ou Xu, Dun-huan Liu: *Phys. Lett. A* **202** (1995) 28
[14] J. Seke: *Quantum Opt* **3** (1991) 127
[15] J. Seke: *Quantum Opt* **4** (1992) 151
[16] J. Seke: *Physica A* **193** (1993) 587
[17] K.Zaheer, M.S.Zubairy: *Phys. Rev. A* **37** (1988) 1628
[18] C. W. Gardiner: *Quantum Noise* (Springer-Verlag, Berlin, 1991);