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PREEQUILIBRIUM REACTION THEORY, STATISTICAL ASSUMPTIONS AND PARAMETERS[1]

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Statistical assumptions for pre-equilibrium reaction theories are examined. Improved method to calculate transmission factor and spreading width for preequilibrium process are presented. For theses evaluation absorptive potentials based on thermal theory are discussed.

1. Introduction

About 30 years ago Griffin [1] originated the exciton model of pre-equilibrium reactions, and extensive and successful phenomenological analyses followed.Feshbach, Kerman and Koonin (FKK) [2] gave a quantum mechanical basis of pre-equilibrium reaction theory, and the distinction between multistep direct (MSD) and multistep compound (MSC) proccesses became clear. Next Tamura, Udagawa, and Lenske (TUL) [3] developed a MSD theory based on DWBA, and many pre-equilibrium reaction data were successfully analyzed using either the FKK or TUL theory [4, 5].

We joined in effort to develop more fundamental understanding of pre-equilibrium reaction by using the random matrix and Grassmann integral, which was very successful in solving the compound reaction problems [6]. We studied both MSD and MSC reactions (NWY) [7, 8, 9, 10, 11]. For the former we adopted TUL approach, but with some modification in statistical assumptions. For MSC reactions random matrix theory is applied and weak and strong coupling approximations are examined. Furthermore connection between MSD and MSC and effect of direct reaction on MSC process was discussed. Koning and Akkermans [12, 13] summarized the development of these theories and classified them according to the way how the statistical assumptions are introduced.

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To formulate pre-equilibrium reaction theories statistical assumptions are introduced to neglect intereference terms by employing the statistical argument of random sign of matrix elements. FKK considered that the leading particle is responsible for random sign, while TUL and NWY the residual nucleus. These two kinds of introduction of statistics are called the leading particle statistics and the residual system statstics respectively by Koning and Akkerman [12, 13].

Application of our theories is still limited at present. For MSD calculations with some approximation are available by Koning [13] and Lenske et al. [14], but full calculation was not yet performed. For MSC, Herman, Reffo and Weidenmüller [15] made a major contribution, but there is still gap between the theory and the practical application.

The purpose of this paper is first to examin the statistical assumptions on which the pre-equilibrium theories are based using a schematic model. The second is to discuss the relation between parameters necessary to theories and the phenomenological ones.

2. Statistical assumptions

In pre-equilibrium reaction theory, equilibration in whole system is not assumed, but equilibration within the states with same exciton number is assumed. In the case of MSC process equilibration time τ_0 of a state with the exciton number $2m$ should be short compared with the decay time τ_+ into exciton states with $2(m+1)$; namely $\tau_0 < \tau_+$. The equilibration time τ_0 is estimated from the spreading widths [16], and approximately given by $\hbar/\Gamma^{\downarrow}(\Delta m = 0)$ where the denominator is the spreading width in which the exciton number does not change. The decay time is also estimated from the spreading width for $\Delta m = 1$. They are proportional to the accessible state density (see Sec.4), which is further approximated just a state density. If we use the Obložinský formula [17], the state density monotonically increases with the exciton number for the energy region we are interested in. Then $\tau_0 > \tau_+$, and equilibration within the states of same exciton number is never accomplished.

However if the state density which includes effects of residual interaction is adopted [18], the situation changes. Residual interaction mixes exciton states, and state density for lower exciton states increases in lower energy region, as shown in Fig. 1. So in some circumstances equilibration within exciton states could be realized, but it is difficult to give any universal conclusion. For MSD process the incident particle, the leading particle, collides with the target nucleus. To proceed in MSD process the leading particle must remain in continuum. Therefore the absorptive potential responsible for this process corresponds to absorption into the P-space, which is denoted by W_P. The average interval of occuring of this collision is called collision time τ_c. For nuclear matter it is given by

$$\tau_c \approx \frac{\hbar}{-2W_P}. \tag{1}$$

Absorptive potentials for P and Q spaces are energy dependent as shown in Fig. 2. The energy of the leading particle after the first collision decreases, and also $-W_P$ decreases.

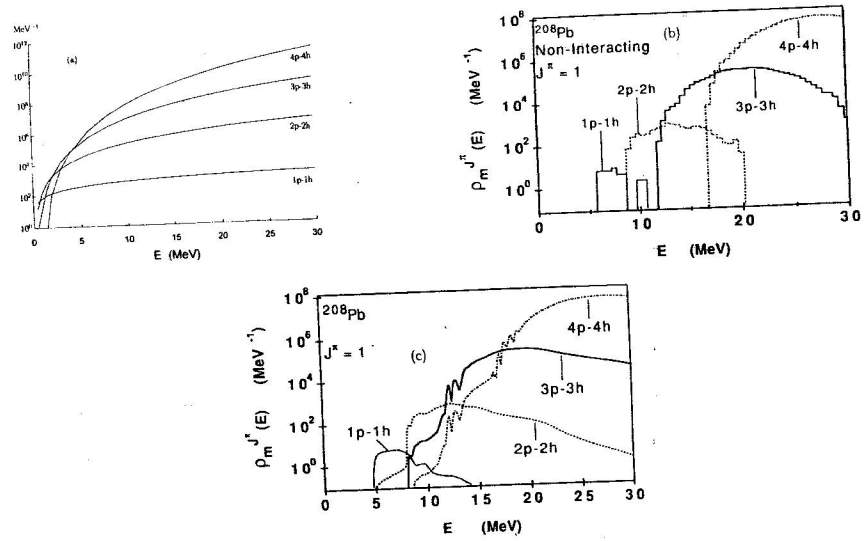

Fig. 1. State densities for ^{208}Pb. (a) Obložinský formula, (b) independent particle model, (c) with random residual interaction.

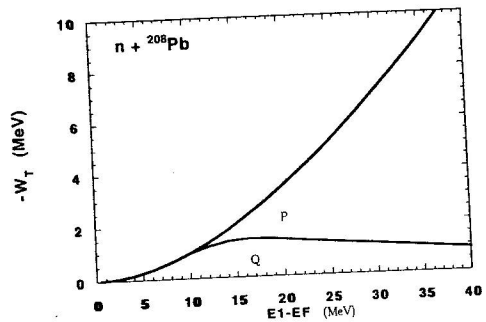

Fig. 2. Absorptive potential for P and Q absorption for $n+^{208}$ Pb.

The collision time for the second collision depends on the incident energy, and in some circumstances $-W_Q(\Delta m = 0) > -W_P$. If the following relation

$$\tau_c > \tau_+ > \tau_0 \tag{2}$$

holds, equilibration is achieved within the target nucleus. However it is difficult to expect such equilibration at high incident energy.

In pre-equilibrium reactions residual nucleus is densely populated and they cannot be observed individually. As the time interval between the reaction and detection

of reaction product is long enough the residual states are well equilibrated and the statistical treatment is justified. A final state wave function $|\chi_\beta(\varepsilon_\beta)\rangle$ is expanded in terms of exciton states $|\chi_{m\mu}\rangle$ as

$$|\chi_\beta(\varepsilon_\beta)\rangle = \sum_{m\mu} C^\beta_{m\mu}|\chi_{m\mu}\rangle, \tag{3}$$

For the expansion coefficients the orthogonality relation

$$\sum_\beta C^\beta_{m\mu} C^\beta_{m'\mu'} = \delta_{mm'}\delta_{\mu\mu'} \tag{4}$$

holds. The detector system measures the energy of the outgoing particle and consequently the excitation energy of residual nucleus ε, so the count is proportional to

$$\sum_\beta C^\beta_{m\mu} C^\beta_{m'\mu'} \delta_I(\varepsilon - \varepsilon_\beta) = \delta_{mm'}\delta_{\mu\mu'} \sum_\beta |C^\beta_{m\mu}|^2 \delta_I(\varepsilon - \varepsilon_\beta), \tag{5}$$

where I is the energy resolution of the detector and $\delta_I(\varepsilon)$ is a distribution function centered at ε with the width I. In the limit of $I \to 0$ it becomes a delta function. If the width I is large enough, the interference terms vanish due to the orthogonal relation (4). To find the lower limit of I for which interference terms vanish is an interesting problem. The diagonal term is expressed as

$$\sum_\beta |C^\beta_{m\mu}|^2 \delta_I(\varepsilon - \varepsilon_\beta) = -\frac{1}{\pi}\mathrm{Im}\sum_\beta \langle\beta|\frac{Q_{m\mu}}{\varepsilon - \varepsilon_\beta + iI}|\beta\rangle = -\frac{1}{\pi}\overline{\mathrm{tr}\langle\beta|\frac{Q_{m\mu}}{\varepsilon - h}|\beta\rangle} = \rho_{m\mu}(\varepsilon). \tag{6}$$

where the energy average is replaced by the ensemble average, $Q_{m\mu}$ is the projection operator onto the exciton state specified by $m\mu$, and $\rho_{m\mu}(\varepsilon)$ is the true partial state density.

We made a simulative calculation using a simple model; for ^{56}Ni, $J^\pi = 3^+$ $1p-1h$ and $2p-2h$ states are diagonalized by using a delta function type interaction, whose strength is increased by a factor of 2 to enhance the mixing effects. Products of expansion coefficients for $1p-1h$ states $C^\beta_{ph}C^\beta_{p'h'}$ are shown in Fig. 3. If they are summed over β the diagonal product must be unity, while non-diagonal ones vanish. Diagonal products are expected to be distributed around the unperturbed energy ε_{ph} with a width of order of the spreading width of $1p-1h$ state. We found that the non-diagonal products are also distributed in the same energy range. Therefore taking the energy resolution comparable to the spreading width is a sufficient condition of residual system statistics. The first order contribution to MSD cross section is DWBA one multiplied by the true state density for $m = 1$, $\rho_{ph}(\varepsilon_\beta)$.

The residual system statistics is sufficient to calculate 1-step MSD reaction cross sections, but for 2-step or higher MSD reactions the collision time argument must be employed again. In the second order process a $1p-1h$ state is created in the target nucleus by the leading particle. The leading particle looses the energy but if it is still

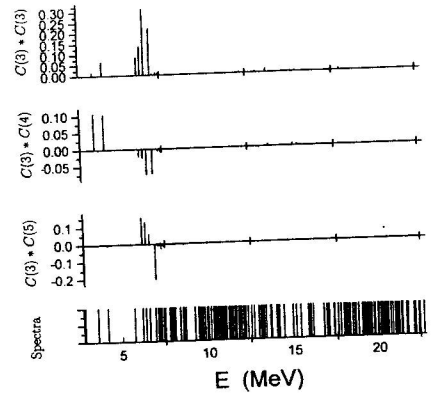

Fig. 3. Products of residual state amplitudes C^β_{ph} plotted against E_β for ^{56}Ni, 3^+ states (147 levels). The numbers 3, 4 and 5 correspond to the neutron particle hole states $(p_{1/2}f^{-1}_{7/2})$, $(p_{3/2}f^{-1}_{7/2})$, $(f_{5/2}f^{-1}_{7/2})$.

high in continuum, the leading particle strikes again the target nucleus before the target nucleus reaches equilibrium. In this case the collision time relation

$$\tau_c < \tau_+ < \tau_0 \tag{7}$$

holds, and equilibration is not achieved. Accordingly statistical treatment at the intermediate state cannot be performed, and intereferences among intermediate states do not vanish. In this situation we proposed the sudden approximation [9]. The 2nd order contribution is proportional to $2p-2h$ state density $\rho_{p_1h_1p_2h_2}(\varepsilon_\beta)$. To get through the $2p-2h$ state we have four choices to select the firstly created $1p-1h$ state, and these terms will interfere. The calculation to implement these effects is difficult and no numerical calculation is yet undertaken.

If the equilibration is achieved, then the calculation becomes easy and additional assumption leads to the 2nd order DWBA type expression. This is the work of TUL [3].The target nuclear wave function for the final state is expanded as

$$|\chi_\beta\rangle = \sum C^\beta_{p_2h_2,\gamma} a^\dagger_{p_2} a^\dagger_{h_2}|\chi_\gamma\rangle \tag{8}$$

where χ_γ represents the equilibrated intermediate state of the target nucleus. In the absolute square of the S-matrix, the following sum is included,

$$\sum_\beta C^\beta_{p'_2h'_2\gamma'} C^\beta_{p_2h_2\gamma}\delta(\varepsilon_\beta - \varepsilon) = \delta_{p'_2p_2}\delta_{h'_2h_2}\delta_{\gamma\gamma'}\rho_{p_2h_2}(\varepsilon_\gamma - \varepsilon). \tag{9}$$

Vanishing of the interference terms is a consequence of the residual system statistics, but it also employed the so-called Axel-Brink hypothesis. A $1p-1h$ state is built on the equilibrated intermediate state χ_γ and the state density of this $1p-1h$ state is given by

the state density at energy less by the excitation energy ε_γ. The Axel-Brink hypothesis is expected to be valid for well developed collective states, but for non-collective $1p-1h$ states it must be testified. The residual system statistics is applied also to the intermediate states according to the adiabatic approximation, and the cross section becomes the DWBA one multiplied by two state densities $\rho_{p_2h_2}(\varepsilon_\beta-\varepsilon_\gamma)\rho_{p_1h_1}(\varepsilon_\gamma)$. (We calculated the sum $\sum_\beta |C^\beta_{ph\gamma}|^2 = \langle\gamma|a_h a_p a_p^\dagger a_h^\dagger|\gamma\rangle$ numerically in the case of ^{56}Ni, and found that it ranges from 1 to 0.6, whereas it is 1 if the hypothesis is valid.) This second order formula is easy to calculate and many applications to practical analyses are performed.

FKK [2] used a random sign assumption and lead a convolution type formula which is easy to calculate. Later Akkermans and Koning [12, 13] analyzed this assumption and concluded that this results are obtained through the leading particle statistics which is concerned with the matrix elements between the leading particle and the residual system.

3. Transitions from P to Q spaces

In MSC reactions the initial exciton state is created by the leading particle, and the P space and the Q space are connected by the transmission coefficient,

$$T_{am} = 2\pi\langle\Phi_a^{(+)}|-2W_Q^m|\Phi_a^{(+)}\rangle \cdot \tag{10}$$

where

$$W_Q^m = \mathrm{Im}\left[H_{PQ}\frac{Q_m}{E-H-W}H_{QP}\right] \tag{11}$$

is the absorptive potential corresponding to Q-space $2m$ exciton states. The distorted wave function satisfies

$$(E-T-U-iW_Q-h-V)\Phi_a^{(+)} = 0, \tag{12}$$

where $W_Q = \sum_m W_Q^m$. In Q space all nucleons are in bound states. On the other hand in an usual optical model, P space corresponds to the elastic channel and Q space to all other channels. Therefore in usual optical model the transmission coefficient is calculated by eq.(10), in which W_Q^m is replaced by W_P+W_Q and the wave function satisfies eq.(12), in which the same replacement of W_Q by W_P+W_Q is taken place. The lowest order calculation in which V is neglected is carried out in [19] and the results for Q space absorption and $P+Q$ absorption are compared. The ratio in the absorptive potentials is a decreasing function of the incident energy, as seen from Fig. 2. The ratio in the reaction cross sections is also a decreasing function, but the reduction is not as large as the one for the absorptive potential. This is because the transmission coefficient is not proportional to the absorptive potential.

Next transitions from P to Q is discussed. The initial exciton number in MSC reaction has been discussed by many authors. At the early time it is considered as a parameter, and the preferable value to fit experimental data was searched. FKK also considered the possibility, but it was not treated explicitly. In our paper [11] the

transmission coefficient in the presence of MSD process was discussed formally, and they are numerically evaluated in [19].

The distorted wave satisfying the Eq. (12) is expanded in perturbation series in V as

$$\Phi_a^{(+)} = \Phi_a^{(0)} + \Phi_a^{(1)} + \Phi_a^{(2)} + \cdots \tag{13}$$

Inserting this in eq.(10) the perturbative expansion of transmission factor is obtained,

$$T_{am} = -4\pi\langle\Phi_a^{(0)}|W_Q^m|\Phi_a^{(0)}\rangle - 4\pi\langle\Phi_a^{(1)}|W_Q^m|\Phi_a^{(1)}\rangle\cdots = T_{am}^{(0)} + T_{am}^{(2)} + \cdots \tag{14}$$

According to numerical calculations the direct transition represented by $T_{am}^{(0)}$ is dominant at low energy, but the second order contribution $T_{am}^{(2)}$, which is absorption via the first step MSD proocess, increases with energy and surpasses the zero-th order one. The odd order contribution vanishes due to the residual system statistics.

These transitions from P to Q via MSD process are also investigated by [20] and show that these paths are essential to get agreement with experimental data. Furthermore in [21] these are included in the framework of FKK, which make the calculation tractable.

4. Spreading widths

Knowledge of transition rates within Q space is essential to evaluate the partial states density and MSC reaction cross sections. The spreading width in Q space plays role of transmission factor for $P \to Q$ transition. The spreading widths are evaluated using the dynamical shell model potential, and the results are compared with experimental data. Energy dependence and jl dependence are not known in details, as spreading widths are obtained only at limited number of points.

We have evaluated spreading widths with semiclassical approximation [22], so they are obtained as a continuous function of energy and convenient to our purpose. First quantum mechanical expression is given. The spreading width for nucleon labeled a is given by

$$\Gamma_a^\downarrow = 2\int |\phi(r,E_a)|^2(-W(r,E_a))\mathrm{d}r \tag{15}$$

where $W(r,E_a)$ is the imaginary part of the optical potential, while the radial wave function $\phi(r,E_a)$ is a solution for a real part of the optical potential. This expression is not exact, but gives very good approximation. When the optical potential is non-local, correction factors must be included [23]. In the semiclassical approximation the density is replaced by

$$|\phi(r,E)|^2 \to k^{-1}(r,E) \tag{16}$$

where $k(r,E)$ is the local wave number. It was found that the semiclassical approximation works very well, and the energy and jl dependences of spreading widths are clearly seen.

To obtain the spreading width for an exciton state $\Gamma_a^\downarrow$ must be averaged over the energy E_a [15]. Furthermore the absorptive potential appeared in (15) is partitioned according to the change of the exciton number, and the spreading width from exciton

state m to n $\Gamma^{\downarrow}_{nm}$ is obtained. From this the internal transition rate which appears in the master equation for MSC process is given by

$$2\pi\rho_n\Gamma^{\downarrow}_{nm} = T^{int}_{nm}. \tag{17}$$

On the other hand the internal transition rate is expressed in terms of the second moments of the residual interaction and the the state density without residual interaction $\rho^{(0)}_m$ is given by

$$T^{int}_{nm} = 2\pi\rho^{(0)}_m M_{mn} 2\pi\rho^{(0)}_n \tag{18}$$

in weak coupling approximation. In strong coupling it is replaced by

$$T^{int}_{mn} = -4\ \mathrm{Im}(\sigma_m)(M^{-1})_{mn}\mathrm{Im}(\sigma_n), \tag{19}$$

where σ_m is a Hubbard-Stratonovitch variable (see Eq. (22)). From these relations the second moments M are obtained.

State density with residual interaction, true state density, may be calculated for given single particle spectra if the residual interaction matrix elements are assumed to be random matrices. Partial state density for the exciton number m is given by

$$\rho_m(\varepsilon) = -\frac{1}{\pi}\lim_{\delta\to 0}\mathrm{Im}\ \mathrm{tr}\overline{[(\varepsilon - h + i\delta)^{-1}Q_m]}. \tag{20}$$

and the relation $\rho(E) = \sum_m \rho_m(E)$ holds. It is calculated from

$$\rho_m(\varepsilon) = -\frac{1}{\pi}\mathrm{Im}\sum_{\mu}[\varepsilon - h_0 - \sigma]^{-1}_{m\mu,m\mu}, \tag{21}$$

where h_0 is the independent particle part of the hamiltonian h, and the Hubbard-Stratonovitch variable σ_m represents the mean field and obtained by solving the saddle point equation

$$\sigma_m = \sum_n M_{mn}\sum_{\nu}[\varepsilon - h_0 - \sigma]^{-1}_{n\nu,n\nu}. \tag{22}$$

We have calculated state density using second moment calculated by a shell model, and results were already shown in Fig. 1. In the shell model calculation not so many states are available, so the second moment are averaged over a large energy range, and energy dependence of the second moment became obscure. However using the method outlined here we can evaluate more realistic density than before.

5. Calculation of absorptive potential

In the last esction, we showed that the absorptive potential plays an essential role in the calculation of pre-equilibrium reaction cross section. So far optical potentials have been studied phenomenologically or microscopically by many authors, but the following conditions should be met in order to apply them in pre-equilibrium reactions.

(1) The target nucleus is not always in its ground state, but excited states specified by

the exciton number.

(2) The compound system produced by absorption of the leading particle is restricted to Q space with fixed exciton number.

To proceed the calculation further, we use the following methods here;

(3)The semiclassical approximation and the Thomas-Fermi model are used for nuclear model with the local momentum approximation.

In this approximation the average over the energy and the mass number are automatically performed and the calculation becomes much easier.

(4) Only the lowest order processes one are considered.

(5) The incident particle is restricted to nucleon and the collective degree of freedom is not taken into account, so only the volume absorption is obtained.

(6) For residual interaction delta type interactions are mainly used, and numerical calculations are restricted to this type.

Under these assumptions the absorptive potential is calculated [24].

The imaginary part of the self-energy of nucleon in the nucleus is given by

$$\begin{aligned}\langle 1|W|1'\rangle &= -\frac{\pi}{2}\sum_{234}\langle 12|V|34\rangle\delta(E_1+E_2-E_3-E_4)\\ &\times\ [n(E_2)\overline{n}(E_3)\overline{n}(E_4)+\overline{n}(E_2)n(E_3)n(E_4)]\langle 43|V|21'\rangle,\end{aligned} \tag{23}$$

where the first term in the square brackets of the right hand side is called the polarization contribution, and the second term the correlation one. In the equation $n(E)$ is the occupation probability of a hole and $\overline{n}(E)$ is of a particle, and they are fixed once the temperature T of the nucleus is given,

$$n(E) = \frac{1}{1+e^{(E-\lambda)/T}}, \qquad \overline{n}(E) = 1 - n(E) = \frac{1}{1+e^{-(E-\lambda)/T}}. \tag{24}$$

The Wigner transform of the absorptive potential is given by

$$\begin{aligned}W_T(E_1,\mathbf{K}_1,\mathbf{R}_1) &= -\frac{\pi}{2}(2\pi)^{-9}\int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4\\ &\times\sum_{s_2,s_3,s_4}\langle s_1\mathbf{K}_1\mathbf{R}_1, s_2\mathbf{K}_2\mathbf{R}_2|VV^{\dagger}|s_3\mathbf{K}_3\mathbf{R}_3, s_4\mathbf{K}_4\mathbf{R}_4\rangle\\ &\times[n(E_2)\overline{n}(E_3)\overline{n}(E_4)+\overline{n}(E_2)n(E_3)n(E_4)]\delta(E_1+E_2-E_3-E_4),\end{aligned} \tag{25}$$

where $\langle|VV^{\dagger}|\rangle$ is the Wigner transform of interaction potentials. If the occupation probabilities are inserted in eq.(25), then absorptive potential for a fixed temperature is obtained, which was already calculated by many authors.

What we need is the exciton-number-fixed absorptive potential, so the relation between the exciton number and the temperature becomes necessary. A temperature fixed state is considered as a mixture of various exciton states, and the distribution function is approximated by a Gaussian function [25].

$$P_T(m) \approx \frac{1}{\sqrt{2\pi\sigma^2}}\exp[-\frac{(m-\overline{m})^2}{2\sigma^2}], \quad \overline{m} = \ln 2\cdot g_F T, \quad \sigma^2 = \overline{(m-\overline{m})^2} = \frac{1}{2}g_F T, \tag{26}$$

where $\overline{m}$ is the average particle or hole number. To obtain the exciton number fixed potentials from the temperature-fixed one the occupation probabilities are decomposed into particle part and hole part at the Fermi energy. The occupation probability $n(E)$ is divided into two partial ones at the Fermi energy E_F, which is the chemical potential corresponding $T = 0$,

$$n(E) = H(E) + p(E), \tag{27}$$

where

$$H(E) = n(E)\Theta(E_F - E), \qquad p(E) = n(E)\Theta(E - E_F) \tag{28}$$

are the hole and particle parts respectively. The particle number $\overline{m}$ is given by

$$\int_{E_F}^{E_0} p(E)g(E)dE = \overline{m} \tag{29}$$

where E_0 is the maximum energy of particle, and is equal to zero in the case of Q space absorption. This equation gives the relation between the temperature and the average exciton number.

For the vacancy probability we divide

$$\overline{n}(E) = 1 - n(E) = h(E) + P(E), \tag{30}$$

where

$$h(E) = \overline{n}(E)\Theta(E_F - E), \qquad P(E) = \overline{n}(E)\Theta(E - E_F). \tag{31}$$

Substituting (27) and (30) into eq.(25), W_T is decomposed into 12 components as

$$W_T(E_1, \mathbf{K}_1, \mathbf{R}_1) = \sum_i W^{(i)}(E_1, \mathbf{K}_1, \mathbf{R}_1; T), \tag{32}$$

where

$$\begin{aligned} W^{(i)}(E_1, \mathbf{K}_1, \mathbf{R}_1; T) &= -\frac{\pi}{2}(2\pi)^{-9} \int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\ &\times \langle |VV^\dagger| \rangle n_2(E_2) n_3(E_3) n_4(E_4) \delta(E_1 + E_2 - E_3 - E_4), \end{aligned} \tag{33}$$

and $n_i(E_i)$ is the appropriate partial occupation probability appeared in eqs.(27) and (30). Each potential given by (32) corresponds to each process shown in Fig. 4. The process (a) of the polarization contribution corresponds to $\Delta m = 1$, the process (b) and (c) $\Delta m = 0$, (d) and (e) $\Delta m = -1$. The process (f) does not contribute to the absorption potential.

When the incident energy is low all participating nucleons have momentum close to Fermi momentum, and calculation can be analytically done in the case of $P + Q$ absorption. This approximation is called Fermi liquid approximation. To investigate further eq.(33) is rewritten as

$$W^{(i)} = -\pi \overline{v_{eff}^2} \omega_{eff}^{(i)}(E_1), \tag{34}$$

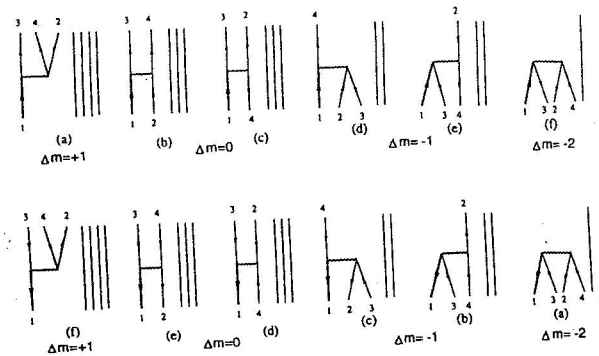

Fig. 4. Diagrams of absorption processes (a)–(f). The upper row shows the case when the incident nucleon is a particle, while the lower row the case of a hole.

where $\overline{v_{eff}^2}$ is the average square of the residual interaction matrix elements given by

$$\overline{v_{eff}^2} = \frac{6}{(2\pi)^4} \left(\frac{m_n^*}{\hbar^2}\right)^3 \frac{{V_0}^2}{{g_F}^3}. \tag{35}$$

where m_n^* is the effective mass of nucleon, V_0 is the strength of the residual interaction and g_F is the single particle state density at Fermi energy. The factor $\omega_{eff}^{(i)}(E_1)$ is called here the accessible state density, which is given by

$$\omega_{eff}^{(i)}(E_1) = \int dE_2 dE_3 dE_4 \delta(E_1 + E_2 - E_3 - E_4) g_F^3 n_2(E_2) n_3(E_3) n_4(E_4). \tag{36}$$

The integral appeard in (36) is analytically performed in the case of $P + Q$ absorption. The results in the limit of $\ln 2\ (E_1 - E_F) g_F \gg m$ are given by

$$\begin{aligned} \omega_{eff}^{\Delta m=1}(E_1) &= g_F^3 \left[\frac{1}{2}(E_1 - E_F)^2 - 3(E_1 - E_F)\frac{m}{g_F} + \left(\frac{\pi^2}{4} + 3(\ln 2)^2\right)\left(\frac{m}{\ln 2 g_F}\right)^2 \right] \\ \omega_{eff}^{\Delta m=0}(E_1) &= g_F^3 \left[3(E_1 - E_F)\frac{m}{g_F} + \left(\frac{\pi^2}{4} - 6(\ln 2)^2\right)\left(\frac{m}{\ln 2 g_F}\right)^2 \right] \\ \omega_{eff}^{\Delta m=-1}(E_1) &= g_F^3 \left[3(\ln 2)^2 \left(\frac{m}{\ln 2 g_F}\right)^2 \right] \end{aligned} \tag{37}$$

In this calculation it was found that Pauli correction term in process (a) appears in (b) and (c) processes so as to cancell each other if all of them are summed over. In the case of delta function residual interaction numerical calculation without Fermi-liquid approximation is performed, and results are compared with those of Fermi-liquid approximation. They agree well in the case of $P + Q$ absorption, but if the absorption is restricted to Q space the agreement is limited at low incident energy.

6. Conclusions

n Sec. 2 the statistical assumptions, on which pre-equilibrium theories are based, are examined. Whether these conditions are realized or not depends especially on the energy. We relied only on the residual system statistics, and the leading paticle statistics was not employed. If both statistics are employed the situation might be improved.

The second topics are evaluations of parameters appeared in MSC theories and state densities. Using the semiclassical approximation and thermal theoretical approach the absorptive potentials are estimated with good accuracy, and from which the transmission coefficients, spreading widths, and the second moments for resdidual interaction are obtained. If we use experimental data as constraint, we can improve the evaluation of pre-equiliribrium cross section and state densities. Furthermore surface absorption effects should be also included, as it is known that both surface and volume absorption are neccessary to fit optical potentials with experimental data.

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