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THE EFFECT OF SPIN GENERATED TORSION ON THE STABILITY OF COMPACT ASTROPHYSICAL OBJECTS

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By considering a sphere of constant energy density admitting a spin density surrounded by a layer of matter void of spin density, we calculate the binding energy of the sphere and discuss the effects that the spin density has on the stability of the sphere. We also outline how the total binding energy of the two layer configuration can be calculated.

1. Introduction

Theories of the early universe may include elements that are foreign to conventional Big Bang cosmology and particle theory in the GeV range [1]. Among these elements we must admit that higher dimensions [2], non-trivial topologies [3], instanton effects [4], quantum gravitational effects [5] and spin generated effects [6] all could figure into the dynamics of the early universe. Since spin is such an elemental notion with a pure quantum interpretation we might expect that at high densities spin-spin couplings would effect the dynamics of matter and space-time. In this regard the Einstein-Cartan theory was invented to take into account both the effects of energy momentum and spin on the geometry of space time through the presence of a non-symmetrical connection constructed from the metric and torsion [7]. Actually there are numerous torsion theories and a main competitor for the Einstein Cartan theory is the Poincare theory of gravitation which is the logical consequence of gauging the Poincare group [8]. In this theory, the fundamental fields are the tetrad e^{α}_{μ} and spin connection $\omega^{\alpha\beta}_{\mu}$ which in turn generate the metric and torsion.

The effect that torsion has on both local and cosmological dynamics makes the study of it well worthwhile. When adjoined to the Einstein theory it can avert a cosmological singularity [9], and for a dust sphere it will halt its unimpeded collapse [10]. In two important papers, Kerlick [11 - 12] has discussed the effect of spin-generated torsion on singularity prevention through the modification of the Hawking Penrose singularity theorems. A rather remarkable fact is that convective spinning dust opposes singularity formation, while spin generated by a Dirac field enhances singularity formation. More

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recently both Gasperini [13] and Demienski et. al. [14] have shown that spin generated torsion leads to cosmological solutions which are inflationary in character. In black hole physics DeSabbata et. al. have shown that torsion can stabilize a black hole to Hawking radiative decay by generating a zero effective temperature [15]. When torsion interacts with electromagnetism it can lead to the dispersion of E. M. waves [16] and also the rotation of the plane of polarization of E.M. waves coming from cosmological sources [17]. Numerous experimental tests for the presence of torsion have been proposed with no conclusive results reported to date [18 - 20].

In what follows we discuss the effect that spin generated torsion has on a isotropic sphere of matter when it is surrounded by a layer of matter with no spin density. Actually in a previous note [21] we have found the solution for the metric and pressure for such a configuration and in this paper we construct the binding energy of the inner sphere and discuss what effect spin generated torsion has on its stability. If the outer spinless layer is also considered we outline how the total binding energy can be calculated.

2. The Effect of Spin Generated Torsion on the Stability of a Isotropic Sphere of Matter

In (Ref. 21) we discussed the following model for a compact astrophysical object, for

$$0 < r < R_1, \quad \epsilon = \epsilon_{02}, \quad S = S_0 \quad \text{(spin density)}$$

for

$$R_1 < r < R_2, \quad \epsilon = \epsilon_{01}, \quad S = 0 \tag{2.1}$$

for

$$r > R_2, \quad \epsilon = P = S = 0$$

(ϵ = energy density, P = pressure)

For a spherically symmetric distribution of spin (pointing in radial direction) it has been demonstrated that the Einstein Cartan theory of metric and torsion is equivalent to G. R. if we set [22]

$$P \to P - \frac{2\pi G S^2}{C^2}, \quad \epsilon \to \epsilon - \frac{2\pi G}{C^2} S^2$$

(S = spin density)

Without repeating the calculation in (ref. 21) we found the following solution for the metric

$$(\mathrm{d}S)^2 = e^{\nu}(\mathrm{d}x^4)^2 - e^{\lambda}(\mathrm{d}r)^2 - r^2(\mathrm{d}\Theta)^2 - r^2 \sin^2\Theta(\mathrm{d}\Phi)^2$$

in the three regions of Eq. (2.1).

$$r > R_2, \quad e^{\nu} = e^{-\lambda} = 1 - \frac{2GM}{rC^2}, \quad P = \epsilon = S = 0, \tag{2.2}$$

$$R_1 < r < R_2, \quad S = 0, \quad \epsilon = \epsilon_{01}, \quad P_r = 0$$

$$e^{-\lambda} = 1 - \frac{8\pi G}{3c^4}\epsilon_{01} r^2 + \frac{C_1}{r}, \qquad \nu(r) = \ln_e\left(1 - \frac{2GM}{R_2 C^2}\right) - \int_r^{R_2} \frac{e^{\lambda} - 1}{r}\mathrm{d}r \tag{2.3}$$

Note at $r = R_2$, $\nu(R_2)$ reduces to the value at $r = R_2$ from Eq.(2.2) (the value found in the region $r \geq R_2$). To find C_1 we will match $e^{-\lambda}$ at $r = R_1$ (note for $R_1 < r < R_2$ we choose the radial pressure to be 0). For $o < r < R_1$

$$e^{-\lambda} = 1 - \frac{8\pi G}{3C^4} r^2 \left(\epsilon_{02} - \frac{2\pi G}{C^2} S_0^2\right)$$

$$e^{\nu} = A - B\sqrt{1 - \frac{8\pi G}{3C^4}\left(\epsilon_{02} - \frac{2\pi G}{C^2} S_0^2\right) r^2} \tag{2.4}$$

by matching $e^{-\lambda}$ from Eq. (2.4) to the above expression (Eq. (2.3)) for $e^{-\lambda}$ found for $R_1 \leq r \leq R_2$ at $r = R_1$ we find

$$C_1 = \frac{8\pi G}{3C^4}\epsilon_{01}\left(R_1^3\right) - \frac{8\pi G}{3C^4}\left(\epsilon_{02} - \frac{2\pi G}{C^2} S_0\right) R_1^3 \tag{2.5}$$

also

$$P = \left(\epsilon_{02} - \frac{4\pi G}{C^2} S_0^2\right) \frac{\left(e^{\frac{\nu}{2}}\right)_{R_1}}{\left(e^{\frac{\nu}{2}}\right)_r} - \epsilon_{02} + \frac{4\pi G}{C^2} S_0^2 \tag{2.6}$$

for $0 < r \leq R_1$ which follows from the $\binom{2}{2}$ and $\binom{4}{4}$ Einstein equations upon using $P = 0$ at $r = R_1$. Also A and B in Eq. (2.4) were found by setting $P = 0$ at $r = R_1$ and matching e^{ν} from Eq. (2.4) to e^{ν} from Eq. (2.3) at $r = R_1$ (Ref. 21). By matching Eq. (2.3) (using Eq. (2.5) for C_1) to Eq. (2.2) at $r = R_2$ we find

$$M = \frac{4\pi}{3C^2}\epsilon_{01}\left(R_2^3 - R_1^3\right) + \frac{4\pi R_1^3}{3C^2}\left(\epsilon_{02} - \frac{2\pi G}{C^2} S_0^2\right) \tag{2.7}$$

where we identify

$$M_1 = \frac{4\pi R_1^3}{3C^2}\left(\epsilon_{02} - \frac{2\pi G}{C^2} S_0^2\right)$$

$$M_2 = \frac{4\pi}{3C^2}\epsilon_{01}\left(R_2^3 - R_1^3\right) \tag{2.8}$$

Also in Eq, (2.4) (Ref. 21)

$$\frac{B}{A} = \frac{(1 - \beta/\alpha)}{(3 - \beta/\alpha)\left(1 - 2GM_1/R_1C^2\right)^{1/2}} \tag{2.9}$$

where

$$\beta = \frac{16\pi^2 G^2 S_0^2}{C^6}, \quad \alpha = \frac{2GM_1}{C^2 R_1^3} \tag{2.10}$$

We also assume β/α is small and keep terms in β/α to first order. From Eq. (2.10)

$$\frac{\beta}{\alpha} = \frac{8\pi^2 G S_0^2 R_1^3}{M_1 C^4}\left(M_1 = \frac{4\pi R_1^3}{3C^2}\left(\epsilon_{02} - \frac{2\pi G}{C^2}S_0^2\right)\right) \tag{2.11}$$

The assumption that β/α is small essentially states that

$$\epsilon_{02} \gg \frac{2\pi G S_0^2}{C^2}$$

thus from Eq. (2.11)

$$M_1 = \frac{4\pi R_1^3}{3C^2}\epsilon_{02}$$

and

$$\frac{\beta}{\alpha} \approx \frac{6\pi G S_0^2}{C^2 \epsilon_{02}} \ll 1 \tag{2.12}$$

Using Eq. (2.6) and B/A from Eq. (2.9) we find for $R_1 > r > 0$

$$P = \left(\epsilon_{02} - \frac{4\pi G}{C^2}S_0^2\right)\left[\frac{2/3 + 2\beta/9\alpha}{1 - \left(\frac{1}{3} - \frac{2\beta}{9\alpha}\right)\left(\frac{1-2GM_1r^2/C^2R_1^3}{1-2GM_1/R_1C^2}\right)^{1/2}} - 1\right] \tag{2.13}$$

For the binding energy of the inner sphere ($0 < r < R_1$) we have [23], [24]

$$B.E. = \int_0^{R_1} 4\pi r^2 e^{\frac{\lambda}{2}}\left(\epsilon_{02} - \frac{3}{2}P\right)\mathrm{d}r - M_1C^2 \tag{2.14}$$

where $\epsilon_{02} = \rho_{02}C^2 + \frac{3}{2}P$ and $\rho_0 C^2$ = rest mass energy density and $\frac{3}{2}P$ represents three non-relativistic degrees of freedom of kinetic energy.

Eq. (2.14) gives

$$B.E. = 4\pi\int_0^{R_1}\frac{r^2\epsilon_{02}\mathrm{d}r}{\left(1 - \frac{2GM_1r^2}{C^2R_1^3}\right)^{1/2}} - \frac{3}{2}(4\pi)\left(\epsilon_{02} - \frac{4\pi G S_0^2}{C^2}\right)$$
$$\times\int_0^{R_1}\left[\frac{r^2\left[\frac{2}{3} + \frac{2\beta}{9\alpha}\right)}{1 - \left(\frac{1}{3} - \frac{2\beta}{9\alpha}\right)\left(\frac{1-2GM_1r^2/C^2R_1^3}{1-2GM_1/R_1C^2}\right)^{1/2}} - r^2\right]\frac{\mathrm{d}r}{\left(1 - \frac{2GM_1r^2}{C^2R_1^3}\right)^{1/2}}$$
$$-M_1C^2 \tag{2.15}$$

or

$$B.E. = \left[\frac{5}{2}\epsilon_{02}(4\pi) - \frac{3}{2}(4\pi)\left(\frac{4\pi G S_0^2}{C_2}\right)\right]\int_0^{R_1}\frac{r^2\mathrm{d}r}{(1 - 2GM_1r^2/C^2R_1^3)^{1/2}}$$
$$-\frac{3}{2}(4\pi)\left(\epsilon_{02} - \frac{4\pi G S_0^2}{C^2}\right)$$

$$\times\int_0^{R_1}\frac{r^2\left(\frac{2}{3} + \frac{2\beta}{9\alpha}\right)\mathrm{d}r}{\left[1 - \left(\frac{1}{3} - \frac{2\beta}{9\alpha}\right)\left(\frac{1-2GM_1r^2/C^2R_1^3}{1-2GM_1/R_1C^2}\right)^{1/2}\right](1 - 2GM_1r^2/C^2R_1^3)^{1/2}} - M_1C^2 \tag{2.16}$$

The second term in Eq. (2.16) represents an extremely difficult integral, to approximate it we replace $r^2 \to \frac{3}{5}R_1^2$ where

$$\bar{r^2} = \frac{\int_0^{R_1} r^2 4\pi r^2 \mathrm{d}r}{\int_0^{R_1} 4\pi r^2 \mathrm{d}r} = \frac{3}{5}R_1^2 \tag{2.17}$$

represents the average of r^2 over the sphere. Using the substitution

$$kr = \sin\Theta, \qquad k^2 = \frac{2GM_1}{C^2R_1^3}$$

we may evaluate Eq. (2.16) after using Eq. (2.17), the result is

$$B.E. = \left[\begin{array}{c}\frac{5}{2}(\epsilon_{02})(4\pi) - \frac{3}{2}(4\pi)\left(\frac{4\pi G S_0^2}{C^2}\right) \\ -\frac{3}{2}(4\pi)\left(\epsilon_{02} - \frac{4\pi G}{C^2}S_0^2\right)\left(\frac{2/3+2\beta/9\alpha}{1-(1/3-2\beta/9\alpha)\left(\frac{1-6GM_1/5C^2R_1}{1-2GM_1/C^2R_1}\right)^{1/2}}\right)\end{array}\right]\times$$
$$\left(\frac{1}{2}\sin^{-1}\sqrt{\frac{2GM_1}{R_1^3C^2}}R_1 - \frac{1}{2}\left(\frac{2GM_1}{C^2R_1^3}\right)^{\frac{1}{2}}R_1\sqrt{1 - \frac{2GM_1}{R_1C^2}}\right)\times\left(\frac{C^2R_1^3}{2GM_1}\right)^{\frac{3}{2}} - M_1C^2 \tag{2.18}$$

From Eq. (2.18) we see from the second term that since

$$\frac{1 - 6GM_1/5R_1C^2}{1 - 2GM_1/R_2C^2} > 1,$$

the spin density in the second term of Eq. (2.18) will dominate and give an enhanced binding energy due to spin. The spin has a tendency to stabilize compact objects over and above the situation when spin is absent. We see from Eq. (2.16) that if we neglect spin and omit the factor $(1 - 2GM_1r^2/C^2R_1^3)^{-1/2}$ we obtain $B.E. = \frac{4}{3}\pi R_1^3\epsilon_{02} - M_1C^2 = 0$, Neglecting the factor $\left(1 - \frac{2GM_1r^2}{C^2R_1^3}\right)^{-1/2}$ essentially refers to a situation void of gravitation in flat space, that is the binding energy before the Newtonian approximatation. The next terms in the expansion (including all terms containing GM_1) will give the Newtonian, Post-Newtonian, etc. terms in the expansion of the binding energy.

3. The Binding Energy of the Two Layer System of Matter and Spin Density

In the above calculations we have calculated the binding energy of the inner sphere of matter and spin density, if we choose to calculate the binding energy of the two layer configuration (total sphere from 0toR_2 we have

$$BE = \int_0^{R_1} 4\pi r^2 e^{\frac{\lambda}{2}} \left(\epsilon_{02} - \frac{3}{2}P\right) \mathrm{d}r - M_1 C^2 + \int_{R_1}^{R_2} 4\pi r^2 e^{\frac{\lambda}{2}} (\epsilon_{01} - P_T)\, \mathrm{d}r - M_2 C^2 \quad (2.19)$$

here M_1C^2 = mass energy of inner sphere $(R_1 > r > 0)$
M_2C^2 = mass energy of layer $(R_2 > r > R_1)$
where M_1, M_2 are given by Eq. (2.8).

The second term in Eq. (2.19) contains the term $\epsilon_{01} - P_T$ where $\epsilon_{01} = \rho_0 C^2 + P_T$ is the equation relating the proper energy density $(\rho_0 C^2)$ to the energy density ϵ_{01} and the kinetic energy density P_T. Note P_T is the transverse pressure and since there is no radial pressure, the kinetic energy density has just two degrees of freedom, thus KE (density) = P_T. Of course when the radial pressure vanishes it is difficult to strictly define the kinetic energy density for the vanishing of the normal pressure destroys the isotropy of the configuration. If however, we subscribe to Eq. (2.19) for the definition of the total binding energy we would add the result in Eq. (2.18) to the second term in Eq. (2.19). To evaluate the transverse pressure, we first write down the $\binom{2}{2}$ component of the Einstein equation as

$$e^{-\lambda}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'}{4} + \frac{\nu' - \lambda'}{2r}\right) = -\frac{8\pi G}{C^4}(-P_T) = \frac{8\pi G}{C^4}P_T \quad (2.20)$$

For the $\binom{4}{4}$ equation we have

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{C^4} r^2 \epsilon_{01} \quad (2.21)$$

for $R_2 > r > R_1$. From Eq. (2.20) and Eq. (2.21) we have after much labor with the use of $e^{-\lambda}$, $\nu(r)$ from Eq. (2.3) for $R_1 < r < R_2$

$$\epsilon_{01} - P_T = \frac{C^4}{8\pi G}\left(\frac{1}{2r^2} + \frac{1}{2r^2}e^{-\lambda} - \frac{1}{2r^2}e^{\lambda} + \frac{5}{4r}\left(\frac{8\pi G}{C^4}\epsilon_{01} r\right) + \frac{1}{4r}\left(\frac{8\pi G}{C^4} r\epsilon_{01}\right)e^{\lambda}\right) \quad (2.22)$$

If we substitute $\epsilon_{01} - P_T$ from Eq. (2.22) into the second term of Eq. (2.19) we would find the total binding energy (using $e^{-\lambda}, e^{\lambda}$ from Eq. (2.3)) after integration. That is of course if the definition for the anisotropic binding energy can truly represent a three dimensional gas with just two kinetic degrees of freedom (in the Θ and Φ direction).

4. Conclusion

In Eq. (2.18) we have assumed the spin density is constant. When Eq. (2.18) is expanded to any order in $\frac{GM_1}{R_1 C^2}$ assuming β/α is small, we obtain a formula for the binding energy including spin. Since the spin increases the binding energy it suggests that the gravitational binding triggers a spin-spin coupling which is attractive in nature. IN Ref. (12) it was pointed out that pair production by the gravitational spin-spin interaction becomes significant at much lower densities than pair production by tidal forces. This suggests that the above spin-stabilized compact object might form late in the cosmological time scale creating pairs of particles in the core that later may annihilate to produce high energy X rays or γ rays bursts. Such a mechanism might provide an alternate reason for γ bursts beside the usual dynamo mechanism for γ rays produced in the magnetosphere of a neutron star [25]. If such a phenomena actually existed it would also give us a firm experimental motivation to ascertain the correctness of the Einstein Cartan theory. In Section 3 we have outlined a method to calculate the total binding energy of the two layer configuration provided our definition of the anisotropic kinetic energy correctly represents a three dimensional gas with just two kinetic degrees of freedom. One last curious result of our calculation is that the binding energy of the inner sphere in Section 2 in not way depends on the surrounding shell of spinless matter.

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