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SCALING INDICES AND INTERMITTENCY AT HIGH ENERGIES

Mikuláš Blažek
Institute of Physics, Slovak Academy of Sciences, 842 28 Bratislava, Slovak Republic

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Presence of intermittency in high energy particle collisions is deduced by means of the scaling properties characterizing factorial moments. Those statistical moments are usually expressed in two forms differing essentially in normalization. In the present contribution rigorous relations between those (full) moments as well as between corresponding scaling characteristics are derived. The last mentioned relations involve besides the scaling indices and corresponding intercepts also the number of bins as well as newly introduced quantities called effective average multiplicities. The quantities characterizing multifractality in terms of the frequency G-moments enter those relations too. It is called here for such an elaboration of the experimental data which allows to obtain all information enabling one to verify the relations published in the present paper. The piece of knowledge extracted so far from the experimental data by several groups of physicists does not allow to perform such a verification. However, the additional information needed for that verification can be extracted from the existing data by those groups without any essential complication.

1. Introduction

In the frame of high energy physics the intermittency [1] as a multifractal phenomenon is characterized by scaling properties which can be deduced from large rapidity density fluctuations observed in real experiments. Such large fluctuations can be represented also by scaling properties of several other statistical moments (compare e.g. [2], [3]). Those scaling properties are usually described in terms of the corresponding sets of scaling indices (e.g. [4], [5], [6]). In this connection a natural question arises, namely, to what extent those individual sets of scaling indices are mutually related. Correct answer to such a question allows to extract from the data only the necessary minimal amount of scaling indices (or scaling characteristics) enabling us to obtain the rest of them by means of the appropriate mutual relations.

In the present contribution we derive relations between two kinds of factorial moments and the frequency G-moments as well as between corresponding scaling indices and intercepts. Especially, in the next Section the fundamental quantities are introduced and we arrive to several types of relations in the third Section. The last Section

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summarizes shortly main results of the present paper and we call there for publication of additional data which are badly needed e.g. for performing the consistency tests between several sets of scaling indices.

2.Scaling indices and intercepts

Large dynamical fluctuations, when analyzed in terms of the intermittency, are treated usually by means of factorial moment,

$$F_q^{(1)} = M^{q-1} \sum_{j=1}^{M} \frac{n_j(n_j-1)\dots(n_j-q+1)}{N^q} \tag{1}$$

or

$$F_q^{(2)} = M^{q-1} \sum_{j=1}^{M} \frac{n_j(n_j-1)\dots(n_j-q+1)}{N(N-1)\dots(N-q+1)} \tag{2}$$

where M is the number of bins (it is understood that they are of equal size), n_j (with j=1,2, ...,M) is the number of particles observed in the j-th bin and $N = \sum_j n_j$ is total number of particles observed in the event under consideration. Presence of intermittency is manifested by validity of the power-law dependence,

$$\langle F_q^{(1)} \rangle \quad \propto \quad f_q^{(1)} M^{a_q^{(1)}} \tag{3}$$

or

$$\langle F_q^{(2)} \rangle \quad \propto \quad f_q^{(2)} M^{a_q^{(2)}} \tag{4}$$

in the range, say,

$$M_{low} \quad \leq \quad M \quad \leq \quad M_{high} \ . \tag{5}$$

The right hand side of both rel.(3) and (4) represents the leading asymptotic term; the scaling indices $a_q^{(1)}$ and $a_q^{(2)}$ as well as the intercepts $f_q^{(1)}$ and $f_q^{(2)}$ are assumed to be independent of the number of bins M as far as the inequalities (5) are satisfied. The angular brackets on the left hand side of rel.(3) and (4) (as well as in the following part of the present contribution) denote averaging over all events.

Several authors investigate the presence of multifractality phenomenon in terms of frequency G-moments introduced in the form, [4], [7],

$$G_q \quad = \quad \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q \Theta(n_j - q) \tag{6}$$

where $\Theta(x)$ is the step function, $\Theta(x<0) = 0$ and $\Theta(x \geq 0) = 1$. With respect to rel.(6) the multifractality appears if the following scaling is observed,

$$\langle G_q \rangle \quad \propto \quad g_q M^{-\tau_q} \ , \tag{7}$$

again with the number of bins M satisfying ineq.(5). In rel.(7), τ_q and g_q denote the scaling indices and intercepts which are associated with G-moments.

For our purpose the following expression is important,

$$E \quad = \quad E_q(n) \quad \equiv \quad n(n-1)\dots(n-q+1) \ . \tag{8}$$

We write (8) in the form,

$$E \quad = \quad \frac{\Gamma(n+1)}{\Gamma(n-t)} \quad = \quad \frac{\Gamma[n(1+\frac{1}{n})]}{\Gamma[n(1-\frac{t}{n})]} \tag{9}$$

where

$$t \quad = \quad q-1 \ . \tag{10}$$

Crucial formula allowing to establish the relations we are looking for can be expressed as it follows (some more details can be found in [8]),

$$E \quad = \quad n^q \sum_{k=0} \frac{A_k}{n^k} \quad \dots \quad n \gg 1 \tag{11}$$

with

$$\begin{aligned}
A_0 &= 1 \ , \\
A_1 &= -\frac{1}{2}t(t+1) \ , \\
A_2 &= \frac{1}{24}t(3t^3+2t^2-3t-2) \ , \\
A_3 &= \frac{1}{48}t^2(-t^4+t^3+3t^2-t-2) \ , \\
A_4 &= \frac{1}{2}t\left(\frac{t^7}{192} - \frac{t^6}{48} - \frac{t^5}{288} + \frac{t^4}{15} - \frac{5t^3}{576} - \frac{t^2}{16} + \frac{t}{144} + \frac{1}{60}\right) \ ,
\end{aligned} \tag{12}$$

etc., t being given by (10).

3. Relations between moments and their scaling characteristics

3.1 Relations between full form of statistical moments (1), (2) and (6)

With respect to rel.(8) and (11) the factorial moments $F_q^{(1)}$ and $F_q^{(2)}$, rel.(1) and rel.(2), respectively, can be expressed in the form,

$$\begin{aligned}
F_q^{(1)} M^{1-q} \quad &= \quad \sum_{j=1}^{M} E_q(n_j)/N^q \\
&= \quad \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q \sum_{k=0} \frac{A_k}{n_j^k}
\end{aligned} \tag{13}$$

and

$$F_q^{(2)} M^{1-q} = \frac{\sum_{j=1}^{M} n_j^q \sum_{k=0} \frac{A_k}{n_j^k}}{N^q \sum_{\ell=0} \frac{A_\ell}{N^\ell}} = \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q \frac{\sum_k \frac{A_k}{n_j^k}}{\sum_\ell \frac{A_\ell}{N^\ell}} = \frac{F_q^{(1)} M^{1-q}}{\sum_\ell \frac{A_\ell}{N^\ell}} , \quad (14)$$

the expression for $E_q(n_j)$ being given by (8) and (11), and the coefficients $A_k, (k = 0, 1, 2, \ldots)$ by (12). The last relation gives,

$$F_q^{(2)} = F_q^{(1)} \left[\sum_\ell \frac{A_\ell}{N^\ell}\right]^{-1} . \quad (15)$$

Since

$$\left[\sum_{\ell=0} \frac{A_\ell}{N^\ell}\right]^{-1} = \sum_{s=0} \frac{c_s}{N^s} \quad (16)$$

where

$$\begin{aligned} c_0 &= 1, \\ c_1 &= -A_1, \\ c_2 &= A_1^2 - A_2, \\ c_3 &= -A_1^3 + 2A_1A_2 - A_3, \\ c_4 &= A_1^4 - 3A_1^2A_2 + 2A_1A_3 + A_2^2 - A_4, \end{aligned} \quad (17)$$

etc., we obtain from (15),

$$F_q^{(2)} = F_q^{(1)} \sum_{s=0} \frac{c_s}{N^s} . \quad (18)$$

Rel.(18) expresses the fact that both (full) factorial moments (1) and (2) characterizing one event, are related in the following way,

$$F_q^{(2)} = F_q^{(1)} \left(1 - \frac{A_1}{N} + \frac{A_1^2 - A_2}{N^2} + \cdots\right) ; \quad (19)$$

the coefficients $A_k, (k = 0, 1, 2, \ldots)$ are given by (12). Rel.(19) suggests that $F_q^{(2)} \to F_q^{(1)}$ with increasing (total) number of particles, N, in the event under consideration.

Moreover, the factorial moments $F_q^{(1)}$ and $F_q^{(2)}$, rel.(1) and (2), respectively, are related also with the frequency G-moments, rel.(6); namely, rel.(13) gives,

$$F_q^{(1)} M^{1-q} = \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q \left(1 + \frac{A_1}{n_j} + \frac{A_2}{n_j^2} + \cdots\right)$$

$$= \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q + \frac{A_1}{N} \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^{q-1} + \frac{A_2}{N^2} \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^{q-2} + \cdots$$

$$= G_q + \frac{1}{N} A_1 G_{q-1} + \frac{1}{N^2} A_2 G_{q-2} + \cdots \quad (20)$$

and rel.(14) leads to the expression,

$$F_q^{(2)} M^{1-q} = \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q \left(1 + \frac{A_1}{n_j} + \frac{A_2}{n_j^2} + \cdots\right) \left(1 + \frac{A_1}{N} + \frac{A_2}{N^2} + \cdots\right)^{-1} . \quad (21)$$

Bearing in mind rel.(16) and (6), rel.(21) induces

$$\begin{aligned} F_q^{(2)} M^{1-q} = G_q & \\ &+ \frac{1}{N} A_1 (G_{q-1} - G_q) \\ &+ \frac{1}{N^2} (A_2 G_{q-2} - A_1^2 G_{q-1} + c_2 G_q) \\ &+ \frac{1}{N^3} (A_3 G_{q-3} - A_1 A_2 G_{q-2} + c_2 A_1 G_{q-1} + c_3 G_q) \\ &+ \frac{1}{N^4} (A_4 G_{q-4} - A_1 A_3 G_{q-3} + c_2 A_2 G_{q-2} + c_3 A_1 G_{q-1} + c_4 G_q) + \cdots . \end{aligned} \quad (22)$$

Rel.(22) can be expressed also in the form,

$$\begin{aligned} [F_q^{(2)} - F_q^{(1)}] M^{1-q} = & \\ & - \frac{1}{N} A_1 G_q \\ & + \frac{1}{N^2} (-A_1^2 G_{q-1} + c_2 G_q) \\ & + \frac{1}{N^3} (-A_1 A_2 G_{q-2} + c_2 A_1 G_{q-1} + c_3 G_q) \\ & + \frac{1}{N^4} (-A_1 A_3 G_{q-3} + c_2 A_2 G_{q-2} + c_3 A_1 G_{q-1} + c_4 G_q) + \cdots ; \end{aligned} \quad (23)$$

the coefficients c_2, c_3, c_4 are seen in rel.(17). Alternatively, rel.(21) gives also

$$F_q^{(2)} M^{1-q} \left(1 + \frac{A_1}{N} + \frac{A_2}{N^2} + \cdots\right) = \sum_{j=1}^{M} \left(\frac{n_j}{N}\right)^q \left(1 + \frac{A_1}{n_j} + \frac{A_2}{n_j^2} + \cdots\right)$$

$$= G_q + \frac{1}{N} A_1 G_{q-1} + \frac{1}{N^2} A_2 G_{q-2} + \cdots . \quad (24)$$

As it is seen, the full moments

(i) $F_q^{(1)}$ and $F_q^{(2)}$ are related by (19);
(ii) $F_q^{(1)}$ and G_q are related by (20);

(iii) $F_q^{(2)}$ and G_q are related by (22) and (24); and
(iv) $F_q^{(1)}, F_q^{(2)}$ and G_q are related by (23).

We recall that **(a)** the low multiplicities ($n_j < q$) don't contribute to relations mentioned above, and **(b)** the terms involving realistic finite values of the total multiplicity, N, might contribute considerably in those relations.

3.2 Relations between scaling characteristics

A. Let us perform the averaging over events. In this case,

(i) rel.(18) or (19) gives,

$$\langle F_q^{(2)}\rangle = \langle F_q^{(1)}\rangle + c_1\langle N^{-1}F_q^{(1)}\rangle + c_2\langle N^{-2}F_q^{(1)}\rangle + \cdots ; \tag{25}$$

(ii) rel.(20) gives,

$$\langle F_q^{(1)}\rangle M^{1-q} = \langle G_q\rangle + A_1\langle N^{-1}G_{q-1}\rangle + A_2\langle N^{-2}G_{q-2}\rangle + \cdots ; \tag{26}$$

(iii) rel.(22) gives,

$$\begin{aligned}\langle F_q^{(2)}\rangle M^{1-q} = \langle G_q\rangle &\\ &+A_1[\langle N^{-1}G_{q-1}\rangle - \langle N^{-1}G_q\rangle]\\ &+[A_2\langle N^{-2}G_{q-2}\rangle - A_1^2\langle N^{-2}G_{q-1}\rangle + c_2\langle N^{-2}G_q\rangle]\\ &+[A_3\langle N^{-3}G_{q-3}\rangle - A_1A_2\langle N^{-3}G_{q-2}\rangle + c_2A_1\langle N^{-3}G_{q-1}\rangle + c_3\langle N^{-3}G_q\rangle]\\ &+[A_4\langle N^{-4}G_{q-4}\rangle - A_1A_3\langle N^{-4}G_{q-3}\rangle + c_2A_2\langle N^{-4}G_{q-2}\rangle\\ &\quad +c_3A_1\langle N^{-4}G_{q-1}\rangle + c_4\langle N^{-4}G_q\rangle] + \cdots ;\end{aligned} \tag{27}$$

(iv) rel.(23) gives,

$$\langle F_q^{(2)}\rangle M^{1-q} = \langle F_q^{(1)}\rangle M^{1-q} - A_1\langle N^{-1}G_q\rangle + [-A_1^2\langle N^{-2}G_{q-1}\rangle + c_2\langle N^{-2}G_q\rangle] + \cdots ; \tag{28}$$

(v) and rel.(24) gives,

$$\begin{aligned}M^{1-q}[\langle F_q^{(2)}\rangle + A_1\langle N^{-1}F_q^{(2)}\rangle + A_2\langle N^{-2}F_q^{(2)}\rangle + \cdots] &=\\ \langle G_q\rangle + A_1\langle N^{-1}G_{q-1}\rangle + A_2\langle N^{-2}G_{q-2}\rangle + \cdots &.\end{aligned} \tag{29}$$

B. To relate the scaling characteristics, i.e. slopes and intercepts, as they are introduced by (3), (4) and (7), let us assume that appearance of fractal structure is characterized also by *effective* average multiplicities, $N_0^{(1)}$ and $N_0^{(2)}$, associated with the factorial moments $F_q^{(1)}$ and $F_q^{(2)}$, respectively,

$$\langle N^{-\lambda}F_q^{(1)}\rangle \propto [N_0^{(1)}]^{-\lambda} f_q^{(1)} M^{a_q^{(1)}} \tag{30}$$

and

$$\langle N^{-\lambda}F_q^{(2)}\rangle \propto [N_0^{(2)}]^{-\lambda} f_q^{(2)} M^{a_q^{(2)}} \tag{31}$$

as well as by the *effective* average multiplicity $N_0^{(G)}$ associated with the frequency G-moments,

$$\langle N^{-\lambda}G_q\rangle \propto [N_0^{(G)}]^{-\lambda} g_q M^{-\tau_q} . \tag{32}$$

Relations (30), (31) and (32) with $\lambda = 0, 1, 2, \ldots$ extend the relations (3), (4) and (7), respectively, specifying occurance of the multifractal structure; especially, with $\lambda = 0$ the corresponding intercepts $f_q^{(1)}, f_q^{(2)}$ and g_q (together with the scaling indices τ_q and $a_q^{(1)}, a_q^{(2)}$) can be determined. Moreover, it is assumed again, that rel.(30), (31) and (32) are satisfied by the number of bins, M, which obey the inequality (5).

C. Now, remembering rel.(30), (31) and (32) our main results are formulated as it follows,
(i') with respect to rel.(25),

$$f_q^{(2)} M^{a_q^{(2)}} = f_q^{(1)} \left[1 + \frac{c_1}{N_0^{(1)}} + \frac{c_2}{[N_0^{(1)}]^2} + \cdots\right] M^{a_q^{(1)}} . \tag{33}$$

Rel.(33) suggests that if we take into account the equality

$$a_q^{(1)} = a_q^{(2)} , \tag{34}$$

then the intercepts $f_q^{(1)}$ and $f_q^{(2)}$ are related by,

$$f_q^{(2)} = f_q^{(1)} \left[1 + \frac{c_1}{N_0^{(1)}} + \frac{c_2}{[N_0^{(1)}]^2} + \cdots\right] \tag{35}$$

where the coefficients $c_1, c_2, \ldots$ are given by (17). In this case, eq.(34) as well as (35) are independent of the number of bins, bearing in mind ineq.(5) (compare with the statement under rel.(3) in ref. [9]);
(ii') with respect to rel.(26),

$$\begin{aligned}f_q^{(1)} M^{a_q^{(1)}+1-q} &=\\ g_q M^{-\tau_q} + \frac{A_1}{N_0^{(G)}} g_{q-1} M^{-\tau_{q-1}} + \frac{A_2}{[N_0^{(G)}]^2} g_{q-2} M^{-\tau_{q-2}} &+ \mathcal{O}\left(\frac{1}{[N_0^{(G)}]^3}\right) ,\end{aligned} \tag{36}$$

i.e.

$$\begin{aligned}[a_q^{(1)} + \tau_q - (q-1)] \ln M &=\\ \ln\left\{\frac{g_q}{f_q^{(1)}} + \frac{g_{q-1}}{f_q^{(1)}}\frac{A_1}{N_0^{(G)}} M^{\tau_q - \tau_{q-1}} + \frac{g_{q-2}}{f_q^{(1)}}\frac{A_2}{[N_0^{(G)}]^2} M^{\tau_q-\tau_{q-2}} + \mathcal{O}\left(\frac{1}{[N_0^{(G)}]^3}\right)\right\} &.\end{aligned} \tag{37}$$

With large value of the effective average multiplicity $N_0^{(G)}$ and assuming that in this asymptotics the intercepts g_q and $f_q^{(1)}$ are (at least approximatively) equal, rel.(37) gives the well known result (compare also [7], [8], [10]),

$$a_q^{(1)} + \tau_q \;=\; q-1\;; \tag{38}$$

(iii') with respect to rel.(27),

$$f_q^{(2)} M^{a_q^{(2)}+1-q} \;=\; g_q M^{-\tau_q} + \frac{A_1}{N_0^{(G)}}[g_{q-1}M^{-\tau_{q-1}} - g_q M^{-\tau_q}]$$
$$+\frac{1}{[N_0^{(G)}]^2}\left[A_2 g_{q-2}M^{-\tau_{q-2}} - A_1^2 g_{q-1}M^{-\tau_{q-1}} + c_2 g_q M^{-\tau_q}\right] + \mathcal{O}\left(\frac{1}{[N_0^{(G)}]^3}\right)\;, \tag{39}$$

i.e.

$$[a_q^{(2)} + \tau_q - (q-1)]\ln M =$$
$$\ln\left\{\frac{g_q}{f_q^{(2)}} + \frac{A_1}{N_0^{(G)}}\left[\frac{g_{q-1}}{f_q^{(2)}}M^{\tau_q-\tau_{q-1}} - \frac{g_q}{f_q^{(2)}}\right] + \mathcal{O}\left(\frac{1}{[N_0^{(G)}]^2}\right)\right\}. \tag{40}$$

Again, if the effective average multiplicity $N_0^{(G)}$ is very large and the relation $g_q \approx f_q^{(2)}$ is satisfied, then we obtain from (40),

$$a_q^{(2)} + \tau_q \;\approx\; q-1\;; \tag{41}$$

(iv') with respect to rel.(28),

$$f_q^{(2)} M^{a_q^{(2)}+1-q} \;=\; f_q^{(1)} M^{a_q^{(1)}+1-q} - \frac{A_1}{N_0^{(G)}} g_q M^{-\tau_q}$$
$$+\frac{1}{[N_0^{(G)}]^2}[-A_1^2 g_{q-1}M^{-\tau_{q-1}} + c_2 g_q M^{-\tau_q}] + \mathcal{O}\left(\frac{1}{[N_0^{(G)}]^3}\right). \tag{42}$$

If in the asymptotics $N_0^{(G)} \gg 1$ the following relation is satisfied, $f_q^{(1)} \approx f_q^{(2)}$, then rel.(42) leads to the (trivial) result $a_q^{(1)} \approx a_q^{(2)}$ as it is expressed by rel.(34);

(v') with respect to rel.(29),

$$f_q^{(2)} M^{1-q+a_q^{(2)}}\left(1 + \frac{A_1}{N_0^{(2)}} + \frac{A_2}{[N_0^{(2)}]^2} + \cdots\right) =$$
$$g_q M^{-\tau_q} + \frac{A_1}{N_0^{(G)}} g_{q-1}M^{-\tau_{q-1}} + \frac{A_2}{[N_0^{(G)}]^2} g_{q-2}M^{-\tau_{q-2}} + \cdots. \tag{43}$$

It is perhaps worth to mention the interesting form of rel.(43) arising from the fact that it involves asymptotic series expansion in two effective average multiplicities, namely $N_0^{(2)}$ and $N_0^{(G)}$; the coefficients $A_1, A_2, \ldots$ are given by rel.(12).

D. We add here some few comments: The main results formulated in this Section are represented by several relations between full form of the factorial moments and the frequency G-moments as well as by several asymptotic relations between scaling characteristics of those moments. Especially the last results are presented in the form of asymptotic series expansions in negative integer powers of the respective effective average multiplicities $N_0^{(1)}, N_0^{(2)}$ and $N_0^{(G)}$ introduced by rel.(30), (31) and (32), respectively; those relations extend the conditions which specify the appearance of the corresponding multifractal phenomenon.

In the case when it is convenient to invert some relations mentioned above, the method described by rel.(19), (20) and (21) of ref. [8] might be applied.

4. Conclusions

In the present paper there are derived several relations between (full) factorial moments represented by two forms and the frequency G-moments, as well as between quantities characterizing their scaling properties.

To verify the corresponding relations, besides the scaling indices published usually, there are badly needed also the intercepts (namely for as broad range of the order q as possible) and the effective average multiplicities. When handling the experimental data, the last two kinds of quantities are accessible without any substantial complication. We call therefore for such elaboration of the data which will allow eventually their publication. Of course, all pieces of information should refer to the same set of events elaborated by the same methods. Moreover, the possibility to follow the energy dependence of those quantities will be acknowledge by many groups.

The aforementioned verification might serve also as a consistency test for various sets of scaling quantities describing multifractality. To our best knowledge, in this region of physics there is missing any consistency test of such a kind.

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