acta physica slovaca vol. 45 No. 3, 407 – 412 June 1995

UNPOLARIZED LIGHT AND CORRELATION FUNCTIONS[1]

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Received 26 April 1995, accepted 10 May 1995

A general description of unpolarized light in a classical and a quantum mechanical framework is presented. We classify common properties as well as typical differences distinguishing two types of unpolarized light. These properties are transferred to the correlation functions. Using the formalism a few examples are given.

1. Introduction

Natural light surrounding us every day is unpolarized in general, and for a long time the only light available was unpolarized thermal light [1]. But in the last few decades new techniques were developed to generate light with photon statistics differing from the thermal distribution which also could be used to generate various kinds of unpolarized light. Although unpolarized light is not polarized *a priori* it can have some interesting properties such as polarization correlations. For instance, when a beam of unpolarized photons is sent to a polarization independent beam splitter, the polarizations of the emerging beams can be correlated, as was recently shown [2], [3]. It was suggested that they can be measured in Bell-type experiments similar to those performed with polarized light first by Ou and Mandel [4]. However until now unpolarized light has not been studied systematically, even in classical optics, to consider this and other problems in general. In the literature [1] one speaks of unpolarized light when the coherence matrix is diagonal with the equal mean intensities $\langle E_x^* E_x \rangle = \langle E_y^* E_y \rangle$ of the two orthogonal polarization modes as elements. Equivalently the three Stokes parameters vanish. From our point of view the above mentioned definitions are not sufficient to describe unpolarized light in general. A general approach was proposed in [5]. Here we follow this treatment: We require that if we perform an experiment measuring any physical

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0323-0465/95

property of an unpolarized light beam in a basis system of two orthogonal modes of linear polarization we must obtain the same result after a rotation of the system around the axis of propagation by an arbitrary angle ϕ. (Actually the rotational invariance is also satisfied by circularly polarized light which must be excluded.) In addition we take into account the property of natural light to remain unchanged when passing a phase retarder [1]. The requirement of rotational invariance must be satisfied also by correlation functions of arbitrary order. Further some general properties of correlation functions related with the polarization can be derived from the definition of unpolarized light. This allows a generalization of the coherence matrix for unpolarized light defined by correlations of second order in the field strength, to arbitrary orders.

2. Unpolarized light

Here we present a short summary of the main characteristics of the description of unpolarized light given in [5]. To define unpolarized light a set of requirements must be satisfied in the mathematical description:

(i) We require, as the first necessary condition, all measurable properties of unpolarized light to remain unchanged when the x-, y-basis is rotated by an angle ϕ around the axis of propagation.

(ii) Since (eventually partially) circularly polarized light must be excluded we introduce a second (necessary) condition: The field distribution function or the density operator in quantum mechanics must be symmetric with respect to left- and right-handed circular polarization.

(iii) As an additional requirement we consider the invariance under phase retardation, i.e. with respect to changing the relative phase between the components of the field strenght E_x and E_y.

Fulfilling the (necessary) conditions (i) and (ii) only, we arrive at a form of unpolarized light we classify as *type II*. When condition (iii) is satisfied in addition, we speak of unpolarized light of *type I*.

In classical optics the field properties are determined by the normalized distribution function $f(E_x, E_y)$ for the complex field strengths E_x and E_y. It is advantageous to use the representation in the circular basis which is connected with the basis of linear polarization by the unitary transformation

$$\begin{pmatrix} E_r \\ E_l \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{1}$$

Unpolarized light of type II is described by a distribution function of the form

$$f(E_r, E_l) = f(|E_r|, |E_l|, E_r E_l) = \tilde{f}(|E_x|^2 + |E_y|^2, E_x^* E_y - E_x E_y^*, E_x^2 + E_y^2) \tag{2}$$

with the symmetry required in (ii). Light of type I is characterized by

$$f(E_r, E_l) = f(|E_r|^2 + |E_l|^2) = f(|E_x|^2 + |E_y|^2) = f(E_x, E_y) \tag{3}$$

and so it is a special case of type II.

In quantum optics the requirements for unpolarized light lead to a density operator given in the basis of circularly polarized number states that is of the general form

$$\hat{\rho} = \sum_{N,M=0}^{\infty} \sum_{n,m=0}^{N,M} \rho_{N-n,n,M-m,m} \delta_{N-2n,M-2m} |N-n\rangle_r |n\rangle_l \; {}_r\langle M-m| \; {}_l\langle m| \tag{4}$$

for unpolarized light of type II. The subsciptes r and l denote the right- and left-handed circularly polarized modes, respectively. The coefficients satisfy $\rho_{N-n,n,M-m,m} = \rho_{n,N-n,m,M-m}$. In the special case of light of type I only diagonal elements are different from zero, and the density operator can be written as

$$\hat{\rho} = \sum_{N=0}^{\infty} \rho_N \sum_{n=0}^{N} |N-n\rangle_r |n\rangle_l \; {}_r\langle N-n| \; {}_l\langle n|. \tag{5}$$

This operator has the same form in the orthogonal basis. The ρ_N are arbitrary non-negative coefficients which do not depend on n. In the following we focus on the quantum mechanical treatment, similar consideration can also be performed in the classical framework.

3. Correlation functions

To characterize unpolarized light we consider correlations between the two orthogonal modes of linear polarization. The correlation function of the 2k-*th* order related to the two modes of linear polarization is defined as

$$G^{(k;\alpha,\beta)} = \langle (E_x^*)^{k-\alpha} (E_y^*)^{\alpha} (E_x)^{k-\beta} (E_y)^{\beta} \rangle. \tag{6}$$

The brackets denote the classical average or the quantum mechanical expectation value. The transition to quantum mechanics is readily carried out by replacing E_x^*, E_x etc. - apart from a common normalization factor which we will omit in the following - by the photon creation and annihilation operators $\hat{a}_x^\dagger$ and $\hat{a}_x$ etc. written in normal ordering. Because the density operator is given in the basis of circularly polarized states, we have to perform the following procedure:

1) We start from the most general form (4) of a density operator for unpolarized light and determine all correlation functions of the order $2k$ in the circular basis

$$B_c^{(k)} = \langle (\hat{a}_r^\dagger)^k (\hat{a}_r)^k \rangle = \langle (\hat{a}_l^\dagger)^k (\hat{a}_l)^k \rangle = k! \sum_{N=0}^{\infty} \sum_{n=0}^{N} \rho_{N-n,n} \binom{n}{k}, \tag{7}$$

$$C_c^{(k;\gamma)} = \langle (\hat{a}_r^\dagger)^{k-\gamma} (\hat{a}_l^\dagger)^{\gamma} (\hat{a}_r)^{k-\gamma} (\hat{a}_l)^{\gamma} \rangle = k! \binom{k}{\gamma}^{-1} \sum_{N=0}^{\infty} \sum_{n=0}^{N} \rho_{N-n,n} \binom{N-n}{k-\gamma} \binom{n}{\gamma}. \tag{8}$$

All other correlations of the order $2k$ vanish due to the required rotational invariance. The remaining off-diagonal elements of the density operator for unpolarized light of type II give rise to additional expectation values $\langle(\hat a_r^\dagger)^k(\hat a_l^\dagger)^k(\hat a_r)^l(\hat a_l)^l\rangle$ where $k \neq l$. With respect to correlations of 2k-*th* order we do not have to take into account these quantities.

2) In a second step we express the correlation functions defined in Eq. (6) in the circular basis using a transformation equivalent to that in Eq. (1) for the annihilation operators. Only terms of the form (7) and (8) are different from zero. Thus the general form of the correlation functions in the orthogonal basis is

$$G^{(k;\alpha,\beta)} = 2^{-k}(i)^{\alpha-\beta}\left[(1+(-1)^{\alpha+\beta})B_c^{(k)} + \sum_{\gamma=1}^{k-1} P(\gamma;k,\alpha,\beta)\,C_c^{(k;\gamma)}\right], \tag{9}$$

where

$$P(\gamma;k,\alpha,\beta) = \sum_{\mu_1=0}^{k-\alpha}\sum_{\mu_2=0}^{\alpha}\sum_{\nu_1=0}^{k-\beta}\sum_{\nu_2=0}^{\beta}\binom{k-\alpha}{\mu_1}\binom{\alpha}{\mu_2}\binom{k-\beta}{\nu_1}\binom{\beta}{\nu_1}(-1)^{\mu_1+\nu_1}$$
$$\delta(\mu_1+\mu_2-\gamma)\delta(\nu_1+\nu_2-\gamma) = P(\gamma;k,\beta,\alpha). \tag{10}$$

Depending on the values of α,β and γ the expression $P(\gamma;k,\alpha,\beta)$ reduces to a product of Jacobian polynomials eventually multiplied by a prefactor. Under the condition that $\alpha+\beta$ is an odd number the following relation holds

$$P(\gamma;k,\alpha,\beta) = -P(k-\gamma;k,\alpha,\beta). \tag{11}$$

In the orthogonal basis now we specify in analogy to the Eqs. (7), (8)

$$B^{(k)} = G^{(k;0,0)} = 2^{-k}\left[2B_c^{(k)} + \sum_{\gamma=1}^{k-1}\binom{k}{\gamma}^2 C_c^{(k;\gamma)}\right], \tag{12}$$

$$C^{(k;\alpha)} = G^{(k;\alpha,\alpha)} = 2^{-k}\left[2B_c^{(k)} + \sum_{\gamma=1}^{k-1} P(\gamma;k,\alpha,\alpha)\,C_c^{(k;\gamma)}\right], \tag{13}$$

and if $\alpha \neq \beta$

$$D^{(k;\alpha,\beta)} = G^{(k;\alpha,\beta)}. \tag{14}$$

The reqirements for unpolarized light are satisfied by the correlation functions, too! The demanded rotational invariance leads to a symmetry with respect to an exchange of the x- and y-mode: $G^{(k;\alpha,\beta)} = G^{(k;k-\alpha,k-\beta)}$. Futhermore from Eqs. (9) and (10) one obtains the property $G^{(k;\alpha,\beta)} = G^{(k;\beta,\alpha)}$. Taking into account the symmetry with respect to left- and right-handed circular polarization one finds that Eq. (11) leads to $D^{(k;\alpha,\beta)} = 0$ if $\alpha+\beta$ is an odd number.

With the help of the general expressions for the correlation functions unpolarized light of type I can be easily distinguished from the wider class of unpolarized light of type II. The general form given in Eq. (5) implies $B_c^{(k)} = B^{(k)}$ and $C_c^{(k;\gamma)} = C^{(k;\alpha)}$. The correlations (14) vanish and we find the relations

$$B^{(k)} = \binom{k}{\alpha}C^{(k;\alpha)} = (2^k-2)^{-1}\sum_{\gamma=1}^{k-1}\binom{k}{\gamma}^2 C_c^{(k;\gamma)}. \tag{15}$$

which are valid only for unpolarized light of type I.

The well determined structure of the density operator for unpolarized light allows us to establish relations between the correlation functions of 2k-*th* order also for light of type II depending in addition on α and β. For simplicity we restrict ourselves to the fourth order where we have only $B^{(2)}, C^{(2;1)}$ and $D^{(2;2,0)} = D^{(2;0,2)}$ which satisfy

$$B^{(2)} = 2\,C^{(2;1)} + D^{(2;2,0)}. \tag{16}$$

We now collect all properties of the correlation functions in a matrix consisting of all possible correlation functions of the order $2k$ representing the characteristics of unpolarized light. If we speak about unpolarized light of type I the matrix $M_I^{(k)}$ has a diagonal form. Using the relation (15) the diagonal elements are determined by

$$M_I^{(k)} = \left(B^{(k)}, \binom{k}{1}^{-1}B^{(k)}, \binom{k}{2}^{-1}B^{(k)}, \ldots, \binom{k}{k-1}^{-1}B^{(k)}, B^{(k)}\right)\mathbf{1}. \tag{17}$$

where $\mathbf{1}$ is the unit matrix. The most prominent representative is thermal light where the coefficients in Eq. (5) are $\rho_N = (1-p)^2p^N$. Thus we obtain $B^{(k)} = k!p^k(1-p)^{-k}$ and the matrix $M_I^{(k)}$ is fully determined. The generalized matrix of correlation functions for unpolarized light of type II is given by

$$M_{II}^{(k)} = \begin{pmatrix} B^{(k)} & 0 & D^{(k;2,0)} & \cdot & \cdot & \cdot \\ 0 & C^{(k;1)} & 0 & \cdot & \cdot & \cdot \\ D^{(k;0,2)} & 0 & C^{(k;2)} & \cdot & \cdot & \cdot \\ 0 & D^{(k;1,3)} & 0 & \cdot & \cdot & \cdot \\ D^{(k;0,4)} & 0 & D^{(k;2,4)} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & C^{(k;k-2)} & 0 & D^{(k;k,k-2)} \\ \cdot & \cdot & \cdot & 0 & C^{(k;k-1)} & 0 \\ \cdot & \cdot & \cdot & D^{(k;k-2,k)} & 0 & B^{(k)} \end{pmatrix} \tag{18}$$

where the $B^{(k)}, C^{(k;\alpha)}, D^{(k;\alpha,\beta)}$ are defined in Eq. (12) - (14). An example is the incoherent mixture of circularly polarized Glauber states where the coefficients in Eq. (4) take the values $\rho_{N-n,n,M-m,m} = \exp\{-|\alpha|^2\}|\alpha|^{2N}\delta_{N,M}[\delta_{n,N}+\delta_{n,0}]/(2N!)$. Determining the correlation functions we arrive at $B^{(k)} = C^{(k;\alpha)} = (i)^{\alpha-\beta}D^{(k;\alpha,\beta)} = (|\alpha|^2/2)^k$. In case of $k=1$ we find the well known coherence matrix. The matrix $M^{(k)}$ is rotationally invariant and satisfies the symmetry between left- and right-handed circular polarization. The relations between the correlation functions cause symmetries with respect to both diagonals.

In summary, starting from a general definition based on invariance properties we obtained common characteristics of correlations between the two polarization modes for unpolarized light. This enabled us to generalize the coherence matrix for unpolarized light of type I as well as of type II to arbitrary orders.

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