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PHASE OPTIMISED STATES OF LIGHT VIA DISCRETE COHERENT STATE SUPERPOSITION[1]

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It is shown that phase optimized quantum states of light can be constructed by superposition of small number of properly chosen coherent states along the positive real semiaxis in phase space.

Recently, much effort has been taken to the problem of generating quantum states that has minimal quantum noise, i.e. the uncertainty of one measured quantity of the state is minimal. In previous papers it was shown that several quantum states can be approximated at arbitrary precision by discrete coherent-state superposition [1, 2]. In this paper we shall discuss the possibility of engineering phase optimized quantum states (POS), having minimal phase uncertainty at a given mean photon number, via discrete superpositions of coherent states. There are only approximating mathematically constructed phase optimized states known in the literature [3, 4]. A direct method to prepare approximate POS experimentally using degenerate parametric interaction was proposed by Bandilla [5].

Nonlinear interaction of the field, being initially in a coherent state, with a Kerr-like medium [6] or in degenerate parametric oscillator [7] leads to superpositions of finite number of coherent states. Back-action evading and quantum nondemolition measurements can also yield such superposition states [8, 9]. An atomic interference method has been developed, which can result in arbitrary superposition of coherent states on a circle in phase space [10]. Implementation of experiments capable of producing arbitrary superpositions of coherent states can be anticipated. Therefore finding coherent-state superpositions approximating given states can be important for experimental realization of the states.

An approximating discrete superposition can be found knowing the one-dimensional coherent-state representation of the state [11]. As there is no such representation of POS known, therefore we have developed a systematic optimizing method for finding the weights and the amplitudes of the constituent coherent states. We will show that even a small number of coherent states can approximate a phase optimized state at a high precision.

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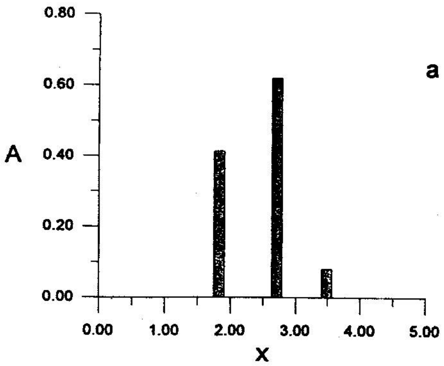

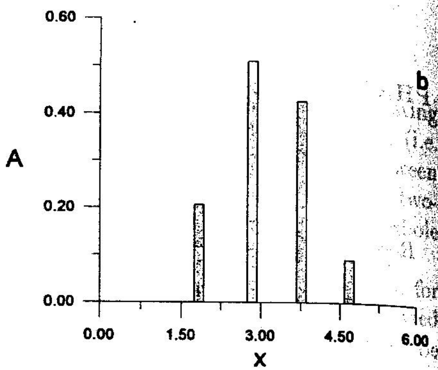

Fig.1. The bar charts show the amplitudes A_j and the positions x_j of the coherent states in the discrete superpositions of CPOS. The mean photon number is $\langle \hat{N} \rangle = 6$ (a) and $\langle \hat{N} \rangle = 10$ (b).

Let us consider the following discrete coherent-state superpositions along the real axis of the phase space

$$| \psi_p \rangle = \sum_{j=0}^{p} A_j \, | \, x_j \rangle_{coh} \, . \tag{1}$$

The amplitudes A_j are chosen to be real which assumption corresponds to general properties of POS [3]. The state $| \psi_p \rangle$ is also required to be normalized:

$$\langle \psi_p \, | \, \psi_p \rangle = \sum_{k,j=1}^{p} A_k A_j e^{-|x_k - x_j|^2/2} = 1 \, . \tag{2}$$

The numerical optimization of the parameters A_j and x_j is accomplished at fixed mean values of the photon number, i.e. at fixed mean energies

$$N = \langle \hat{N} \rangle = \langle \psi_p \, | \, \hat{a}^\dagger \hat{a} \, | \, \psi_p \rangle = \sum_{k,j=1}^{p} A_k A_j x_k x_j e^{-|x_k - x_j|^2/2} \, . \tag{3}$$

In the further calculations we will use the Fock state expansion of $| \psi_p \rangle$

$$| \psi_p \rangle = \sum_{n=0}^{\infty} c_n \, | \, n \rangle , \quad c_n = \frac{1}{\sqrt{j!}} \sum_{j=1}^{p} A_j x_j^n e^{-x_j^2/2} \, . \tag{4}$$

For the investigation of phase properties of the state $| \psi_p \rangle$ we will use the Pegg-Barnett formalism [12]. This formalism is based on a Hermitian phase operator $\hat{\Phi}_\theta$ which exists in a finite dimensional state space. Expectation values and variances of physical quantities are to be calculated in the finite space and the infinite limit in the dimension of the space should be taken only after c-number expressions are obtained.

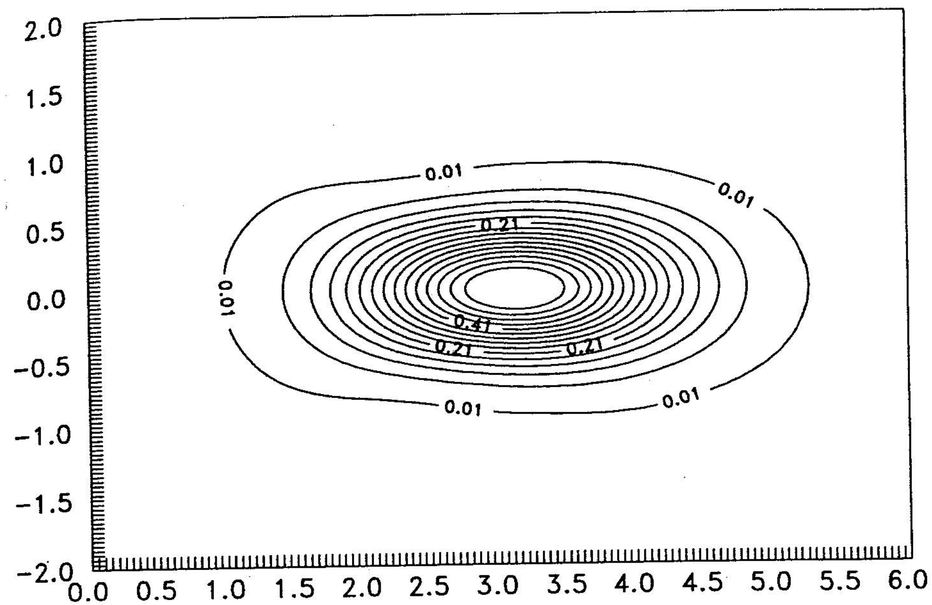

Fig. 2. shows the topological picture of the Wigner function of the CPOS with $\langle \hat{N} \rangle = 10$. It can be clearly seen that the Wigner function has a drop-like shape stretched along the positive real semiaxis ensuring small phase variance.

For the state $| \psi_p \rangle$ it is convenient to choose the reference phase of the formalism to be $-\pi$. As a consequence the mean value of the phase operator $\langle \hat{\Phi} \rangle = 0$. In the end using Eq. (4), the following formula can be evaluated for the phase variance:

$$\langle \Delta \hat{\Phi}^2 \rangle = \langle \hat{\Phi}^2 \rangle = \frac{\pi^2}{3} + 4 \sum_{k>l}^{\infty} \frac{(-1)^{k-l}}{(k-l)^2} \frac{1}{\sqrt{k! l!}} \sum_{r,t=1}^{p} A_r A_t x_r^k x_t^l e^{-(x_r^2 + x_t^2)/2} \, . \tag{5}$$

The algorithm of the numerical optimization starts with large enough number p of constituent coherent states. Then it changes the parameters A_j and x_j systematically to reduce phase variance (Eq. (5)) taking into account the constraint for the mean energy (Eq. (3)).

The result of the optimization shows that even a small number of coherent sates can approximate a phase optimized state at a high accuracy. The number of the constituent coherent states whose amplitudes differs form zero at a fixed computing precision depends on the mean photon number. This number gradually increases as the mean energy rises. Fig. 1 shows the positions x_j and the amplitudes A_j of coherent states in the resulting coherent-state superposition phase optimized state (CPOS) for two different mean photon numbers. The CPOS with mean photon number $\langle \hat{N} \rangle = 6$ (Fig. 1a) and $\langle \hat{N} \rangle = 10$ (Fig. 1b) consist of only 3 and 4 coherent states respectively.

It is interesting to realize that the distance between two adjacent coherent states is approximately 0.9. This is in a good agreement with a former result that due to the quantum interference, superposition of two coherent states can show maximal quadrature squeezing at this distance [13, 14]. It is worth mentioning that the phase squeezing in CPOS is very close to the phase squeezing of the mathematically constructed approximating phase optimized state of Summy and Pegg (SPPOS). For example, for mean photon number $\langle \hat{N} \rangle = 10$ the phase variances are $\langle \Delta\hat{\Phi}^2 \rangle_{CPOS} = 0.126589$ and $\langle \Delta\hat{\Phi}^2 \rangle_{SPPOS} = 0.126487$. The difference is less than one thousandth.

The Wigner quasiprobability function of CPOS can be easily obtained:

$$W(\alpha) = \frac{2}{\pi^2} e^{2|\alpha|^2} \int d^2\beta \, \langle -\beta \mid \psi \rangle \langle \psi \mid \beta \rangle e^{2(\beta^* \alpha - \beta \alpha^*)} = \\ = \frac{2}{\pi} e^{-2|\alpha|^2} \sum_{k,l=1}^{p} A_k A_l e^{-(x_k - x^l)^2} e^{2(\alpha x_l + \alpha^* x_k)} \, . \tag{6}$$

Fig. 2. shows the topological picture of the Wigner function of the CPOS with $\langle \hat{N} \rangle = 10$. It can be clearly seen that the Wigner function has a drop-like shape stretched along the positive real semiaxis ensuring small phase variance.

In conclusion, we have shown that phase optimized quantum states of light can be engineered by superposition of small number of coherent states.

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