cta physica slovaca vol. 45 No. 3, 399 – 402 June 1995

TWO-SLIT COINCIDENCES WITH DOWNCONVERTED BEAMS[1]

J.Řeháček, J.Peřina [2]
Department of Optics, Palacký University, 17.listopadu 50, 772 07 Olomouc, Czech Republic

Received 28 April 1995, accepted 10 May 1995

An interesting experiment has been performed in [1] using downconverted beams in a two-slit arrangement. In this experiment a degree of visibility of Young's fringes behind two-slit screen in signal beam can be controlled by change of width of pinhole in the idler beam, when coincidence measurement scheme is applied. In this communication the experiment is described using the fourth-order quantum correlation function. From the analysis it follows that this fourth-order effect is caused by entanglement of quantum states of downconverted photons. Thus for sufficiently small idler pinhole the fourth-order interference appears regardless of the fact that linear dimensions of the second-order coherence area are smaller than the distance between slits.

1. Introduction

In last years, a number of interesting experiments with downconverted beams were performed [1]. In one of these experiment [2] the signal beam was used to produce interferences of beams diffracted on two slits of the arrangement, whereas the idler beam served for a coincidence measurement between the signal and idler beams to observe the fourth-order interference. Two interesting effects have been observed in this experiment. A "nonlocal" quantum effect was demonstrated [2] giving the possibility to control the degree of visibility of Young's fringes by means of the diameter of a pinhole in the front of idler detector. On the other hand, for sufficiently small idler pinhole, the interference was preserved even if the linear dimensions of the second order coherence area in two-slit screen were smaller than the distance between slits. In the following we use a simple theoretical model of the downconversion process to explain effects which were observed. We also point out some relationship between the fourth- and the second-order interference with the downconverted beams.

[1]Presented at the **3rd central-european workshop on quantum optics**, Budmerice castle, Slovakia, 28 April - 1 May, 1995
[2]E-mail address: perina@risc.upol.cz

2. Fourth-order correlations

We consider the experimental arrangement presented in [2]. A pump beam is producing signal and idler beams in a nonlinear crystal. A two-slit screen is inserted in the signal beam, which can exhibit the interference observable through coincidences of photoelectrons from two detectors exposed to the signal and idler beams. A pinhole having variable diameter is placed in front of the idler detector, so that range of idler beam wave vectors can be changed. We will describe only a two-dimensional arrangement. In an ideal case when the frequency and phase matching conditions are satisfied, the angles α_s and α_i between signal and idler wave vectors and the pump wave vector are determined by their magnitudes and we may write

$$|\omega_s,\alpha_s\rangle = |\omega_s,\alpha_s(\omega_s)\rangle \equiv |\omega_s\rangle \quad , \quad |\omega_i,\alpha_i\rangle = |\omega_i,\alpha_i(\omega_i)\rangle \equiv |\omega_i\rangle .$$

Thus the state of the field produced by nonlinear crystal at time t is given in the interaction picture by [3],

$$|\Psi(t)\rangle = |vac\rangle_{s,i} + \zeta v_p \int d\omega_s \;\Phi(\omega_s,\omega_p-\omega_s;\omega_p)\,|\omega_s\rangle_s\,|\omega_p-\omega_s\rangle_i , \quad (1)$$

where ζ is an efficiency coefficient of the downconversion, Φ is a spectral function of downconversion and v_p is an amplitude of classic pump wave. We suppose that the pump beam is strictly monochromatic plane wave with the frequency ω_p and wave vector $\vec{k_p}$ and the center of crystal is considered to be origin of the coordinate system. The fields incident on the detectors can be written as

$$\begin{aligned} \hat{E}_s^+(P,t) &= a\hat{E}_s^+(\vec{r_1},t) + a\hat{E}_s^+(\vec{r_2},t-\tau_s) \\ &= a\int d\omega\;\hat{a}_s(\omega)e^{-i\omega t}e^{i\vec{k}_s(\omega)\vec{r_1}} + a\int d\omega\;\hat{a}_s(\omega)e^{-i\omega(t-\tau_s)}e^{i\vec{k}_s(\omega)\vec{r_2}} , \quad (2) \\ \hat{E}_i^+(Q,t) &= a\int_{[\omega]} d\omega\;\hat{a}_i(\omega)e^{-i\omega t}e^{i\vec{k}_i(\omega)\vec{r_i}} , \quad (3) \end{aligned}$$

where $\vec{r_1}$ and $\vec{r_2}$ are position vectors of Young's slits, $\vec{r_i}$ is a position vector of the idler detector, τ_s is difference between propagation times from the signal slits to the signal detector and a is a propagator of the signal waves from signal slits to the signal detector. A selection effect of the idler pinhole is expressed in (3) by restricting detector response only to certain range of wave vectors $\vec{k_i}$ with frequencies $[\omega] \equiv \langle\omega_{i0}-\Delta\omega,\omega_{i0}+\Delta\omega\rangle$. We now calculate the fourth-order correlation function between signal and idler detectors considered at times $t+\tau$ and t, which may be used for calculating the coincidence counting rate. It is defined as

$$\Gamma_\tau^{(2,2)} = \langle \hat{E}_s^-(P,t+\tau)\hat{E}_i^-(Q,t)\hat{E}_i^+(Q,t)\hat{E}_s^+(P,t+\tau)\rangle . \quad (4)$$

After substituting from (1), (2) and (3) to (4), we obtain

$$\Gamma_\tau^{(2,2)} = \Big\langle |\,\eta v_p \int_{[\omega_s]} d\omega_s\;\Phi(\omega_s,\omega_p-\omega_s;\omega_p)e^{-i(\omega_p-\omega_s)t}e^{i\vec{k}_i(\omega_p-\omega_s)\vec{r_i}}e^{-i\omega_s(t+\tau)}e^{i\vec{k}_s(\omega_s)\vec{r_1}} \quad (5)$$

$$+\;\eta v_p \int_{[\omega_s]} d\omega_s\;\Phi(\omega_s,\omega_p-\omega_s;\omega_p)e^{-i(\omega_p-\omega_s)t}e^{i\vec{k}_i(\omega_p-\omega_s)\vec{r_i}}e^{-i\omega_s(t+\tau-\tau_s)}e^{i\vec{k}_s(\omega_s)\vec{r_2}}\;|^2\Big\rangle ,$$

where $[\omega_s] \equiv \langle\omega_{s0}-\Delta\omega,\omega_{s0}+\Delta\omega\rangle, \omega_{s0} = \omega_p - \omega_{i0}$. If the signal and idler detectors have quantum efficiencies β_s and β_i, then the coincidence counting rate is given by

$$R_{si} = \beta_s\beta_i \int_{-T_R/2}^{+T_R/2} d\tau \;\; \Gamma_{si}^{(2,2)}(\tau). \quad (6)$$

T_R in (6) represents a resolving time of the coincidence counter. If we consider that T_R is much larger than the reciprocal bandwidth $1/\Delta\omega$ selected by the idler detector, we can take $T_R \to \infty$. After substituting (5) to (6) and if we assume that spectral function $\Phi(\omega_s)$ is practically constant within the range $\Delta\omega$ of selected frequencies and the time difference $\tau_s << 1/\Delta\omega$ (the downconverted light is quasimonochromatic with respect to the propagation time difference $(s_1-s_2)/c$ of the signal wave from both slits to the signal detector), we obtain

$$R_{si} = 2\pi\beta_s\beta_i\,|v_p|^2|\eta|^2\,\Phi(\omega_{s0})\Delta\omega\,(1+\mu\cos\omega_{s0}\tau_s) , \quad (7)$$

where we simply write $\Phi(\omega_{s0})$ instead of $\Phi(\omega_{s0},\omega_{i0};\omega_p)$. The visibility μ of coincidence fringes is given by

$$\mu = \frac{1}{2\Delta\omega}\int_{\omega_{s0}-\Delta\omega}^{\omega_{s0}+\Delta\omega} d\omega\;e^{i\vec{k}_s(\omega)(\vec{r_1}-\vec{r_2})} = \frac{1}{2\Delta\omega}\int_{-\Delta\omega}^{\Delta\omega} d\omega'\;e^{i\frac{\omega_{s0}+\omega'}{c}d\cos(\vec{k}_s,\vec{r_1}-\vec{r_2})} , \quad (8)$$

where d in (8) is the distance between slits. To simplify this expression we suppose, that within a narrow range of $\Delta\omega$, the angle α_s' between signal wave vector $\vec{k}_s$ and the central signal wave vector $\vec{k}_{s0}$ is a linear function of their frequency difference $\omega' = \omega_s - \omega_{s0}$ and the same is valid for the idler angle α_i'. Thus we can write

$$\alpha_s' = \kappa_s\;\omega' \quad , \quad \alpha_i' = \kappa_i\;\omega' \; , \quad (9)$$

where κ_s and κ_i are proportionality constants, which can be found from the frequency and phase matching conditions. The argument of exponential function in (8) can be simplified with the help of (9) as follows

$$\frac{\omega_{s0}+\omega'}{c}d\cos(\vec{k}_s,\vec{r_1}-\vec{r_2}) = \frac{\omega_{s0}+\omega'}{c}d\sin\alpha_s'(\omega') \approx k_{s0}d\kappa_s\omega' , \quad (10)$$

where $k_{s0} = \omega_{s0}/c$. In the last step we have approximated $\sin\alpha'$ by α' and neglected term containing ω'^2. After integrating over ω', we obtain finally

$$\mu = \frac{1}{2\Delta\omega}\int_{-\Delta\omega}^{\Delta\omega} d\omega'\;e^{ik_{s0}d\kappa_s\omega'} = \frac{\sin(k_{s0}d\kappa\Delta\alpha_i)}{k_{s0}d\kappa\Delta\alpha_i} , \quad (11)$$

where we have introduced a constant $\kappa \equiv \kappa_i/\kappa_s$. Usually $|\kappa|$ is not much different from unity.

3. Discussion

From expression (11) for the fourth-order visibility we can deduce two interesting effects, which were observed in [2]. On one hand, for fixed idler pinhole diameter (i.e. fixed idler beam divergence), the visibility does not depend on the distance between two-slits screen and the source. On the other hand, for fixed distance between two-slits screen and the source, the visibility can be controlled by changing idler pinhole diameter.

We may also compare the fourth-order visibility (11) with a similar expression for the visibility in the second-order interference experiment with one of the downconverted beams [4]. From this experiment it follows that a degree of spatial coherence μ_{12} can be expressed in this case by the well-known Van Cittert-Zernike theorem [5] (for higher-order generalization, see [6]; the quantum generalization is straightforward [7], Sec.2.2). In the Fraunhofer approximation and for the uniform source we may write

$$\mu_{12} = \frac{\sin(k_{s0}a_0 d/r_s)}{k_{s0}a_0 d/r_s}, \tag{12}$$

where we have omitted a constant phase factor. We can see that the fourth-order visibility (11) has a similar form as the second-order visibility (12). If the idler beam with small divergence is selected using of the idler pinhole, so that the diameter of the source $a_0 > \kappa r_s \Delta\alpha_i$, then the fourth-order visibility becomes larger than the second-order visibility and we can observe interference fringes regardless of the fact that the linear dimensions of the second-order coherence area are smaller than the distance between signal slits. Hence a quantity $\kappa r_s \Delta\alpha_i$ can be considered as the fourth-order "effective" source diameter in the sense that the source with this diameter has approximately the same fourth- and the second-order visibility.

Acknowledgements We acknowledge the support by grant no 202/94/0789 of Czech Grant Agency.

References

[1] J. Peřina, Z. Hradil, B. Jurčo : *Quantum Optics and Fundamentals of Physics* (Kluwer, Dordrecht-Boston, 1994) Sec. 8.6;

[2] P.H.S. Ribeiro, S. Pádua, J.C. Machado da Silva, G.A. Barbosa: *Phys. Rev. A* **49** (1994) 4176;

[3] L.J. Wang, X.Y. Zou, L. Mandel: *Phys. Rev. A* **44** (1991) 4614;

[4] P.H.S. Ribeiro, C.H. Monken, G.A. Barbosa: *Appl. Opt.* **33** (1994) 352;

[5] M.Born, E.Wolf: *Principles of Optics* (Pergamon, Oxford, 1993);

[6] J. Peřina: *Opt. Acta* **16** (1969) 289;

[7] J. Peřina: *Coherence of Light* (D.Reidel, Dordrecht-Boston, 1985);