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CUBIC BEHAVIOUR OF OPTICAL QUADRATIC NONLINEARITY[1]

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The propagation of light in a lossless quadratic medium is studied using z-dependent perturbation theory in the Heisenberg picture. Cubic behaviour of the fundamental mode in the case of phase mismatch is characterized. A contrast between a nonperiodic progression and the unreduced behaviour is provided in the framework of the Floquet theory.

1. Introduction

The study of propagation of light in the second-order nonlinear medium began with the second-harmonic generation. To achieve a considerable efficiency of this process, the phase-matching condition was assumed. The quantum optics, which considers the modes at frequencies ω_1, $\omega_2 = 2\omega_1$, hesitated to generalize for the phase mismatch. Not even the paper [1], which has considered the phase mismatch, does use the replacement $\omega_j \to ck_j$ explicitly, avoiding so a violation of the frequency condition. It seems that the quantum-field approach may introduce a more convenient language than the coupled oscillators [2–4]. In this paper we describe the optical system in the familiar Heisenberg picture and solve the equations of motion by a perturbation method, which is, of course, in the phase-matching case, quite old-fashioned. But in the phase mismatch case our modification of this method provides interesting results. To obtain a contrast between a trend and oscillations of the statistics, we apply the Floquet theory [5] to this model in the framework of the perturbation theory.

2. Propagation in second-order nonlinear medium

In this paper we assume two coupled modes of radiation propagating along the z-axis in the lossless quadratic medium. These modes have the frequencies ω_1, $\omega_2 = 2\omega_1$ and are described by the annihilation operators $\hat{a}_1(z')$, $\hat{a}_2(z')$ and the creation operators

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$\hat{a}_1^\dagger(z')$, $\hat{a}_2^\dagger(z')$ with $z' = z, 0$ in the Heisenberg and the Schrödinger picture, respectively. We consider the input port at $z = 0$. We use a modified Hamiltonian formalism with the generator of spatial progression

$$\mathcal{G}(z') = \hat{G}_0(z') + \hat{G}_{\rm int}(z'), \tag{1}$$

where

$$\hat{G}_0(z') = \hbar \sum_{j=1}^{2} k_j \hat{a}_j^\dagger(z')\hat{a}_j(z'), \tag{2}$$

with $k_j > 0$ being wave vectors, $k_j \approx \frac{\omega_j}{c}$, c is the speed of light, and

$$\hat{G}_{\rm int}(z') = \hbar[g^* \hat{a}_1^{\dagger 2}(z')\hat{a}_2(z') + \text{H.c.}] \tag{3}$$

with g being coupling constant. Here H.c. denotes the Hermitian conjugate of the previous term.

In the Heisenberg picture we introduce the slowly varying operators $\hat{A}_j(z)$, $j = 1, 2$, and in the Schrödinger picture a "slowly varying" state operator $\hat{\rho}_{\rm int}(z)$ is appropriate, which can be unified in the following form

$$\hat{A}_j(z') = \hat{a}_j(z')\exp(-ik_j z'), \quad \hat{\rho}_{\rm int}(z'') = \exp(-\frac{i}{\hbar}z''\hat{G}_0(z'))\hat{\rho}(z'')\exp(\frac{i}{\hbar}z''\hat{G}_0(z')), \tag{4}$$

where $z'' = 0, z$ in the Heisenberg and the Schrödinger picture, respectively. In the so modified Heisenberg and Schrödinger pictures, we encounter a z-dependent generator of spatial progression

$$\hat{G}(z, z') = \hat{G}_{\rm int}(z, z'), \quad \hat{G}_{\rm int}(z, z') = \hbar[g^* \hat{A}_1^{\dagger 2}(z')\hat{A}_2(z')\exp(i\Delta k\, z) + \text{H.c.}], \tag{5}$$

where the phase mismatch $\Delta k = k_2 - 2k_1$. The slowly varying field operators obey the commutation relations

$$[\hat{A}_j(z), \hat{A}_j^\dagger(z)] = \hat{1}, \quad j = 1, 2, \quad [\hat{A}_1(z), \hat{A}_2(z)] = [\hat{A}_1(z), \hat{A}_2^\dagger(z)] = \hat{0}. \tag{6}$$

Adopting the Heisenberg picture and using (6) and the scheme

$$\frac{d}{dz}\hat{A}_j(z) = \frac{i}{\hbar}[\hat{A}_j(z), \hat{G}(z, z)], \quad j = 1, 2, \tag{7}$$

we derive the equations of motion

$$\frac{d}{dz}\hat{A}_1(z) = i2g^* \hat{A}_1^\dagger(z)\hat{A}_2(z)\exp(i\Delta k\, z), \quad \frac{d}{dz}\hat{A}_2(z) = ig\hat{A}_1^2(z)\exp(-i\Delta k\, z). \tag{8}$$

We assume the initial conditions

$$\hat{A}_1(z)|_{z=0} = \hat{A}_1(0), \quad \hat{A}_2(z)|_{z=0} = \hat{A}_2(0). \tag{9}$$

Although the quasiclassical equations (8) are nonlinear, the quantum theory is linear in principle and the solution to (8) and (9) is described as

$$\hat{A}_j(z) = \hat{U}^\dagger(z)\hat{A}_j(0)\hat{U}(z), \tag{10}$$

where the unitary progression operator is the solution to the initial problem

$$\frac{d}{dz}\hat{U}(z) = \frac{i}{\hbar}\hat{U}(z)\hat{G}(z, z), \quad \hat{U}(z)|_{z=0} = \hat{1}. \tag{11}$$

Using the perturbation theory to obtain short-length solutions, we characterize the orders of approximations in terms of ideals. We denote the operator ideal of formal power series in Δk and z having the property that the degree of Δk is no greater than that of z, this being at least k, as $\hat{J}_k$. For a more complete treatment see [6].

We have derived the field operators

$$\hat{A}_j(z) \equiv \hat{A}_j(0) + z\hat{a}_{j1}(\Delta k, z) + z^2\hat{a}_{j2}(\Delta k, z) \mod \hat{J}_3, \quad j = 1, 2, \tag{12}$$

where

$$z\hat{a}_{11}(\Delta k, z) = 2K_1(z)\hat{A}_1^\dagger(0)\hat{A}_2(0), z^2\hat{a}_{12}(\Delta k, z) = 2K_{12}(z)[-2\hat{A}_1(0)\hat{A}_2^\dagger(0) + \hat{A}_1^\dagger(0)\hat{A}_1^2(0)],$$

$$z\hat{a}_{21}(\Delta k, z) = K_2(z)\hat{A}_1^2(0), z^2\hat{a}_{22}(\Delta k, z) = 4K_{21}(z)[\hat{A}_1^\dagger(0)\hat{A}_1(0)\hat{A}_2(0) + \frac{1}{2}\hat{A}_2(0)] \tag{13}$$

with

$$K_1(z) = \frac{g^*}{\Delta k}[\exp(i\Delta k\, z) - 1], \quad K_2(z) = -\frac{g}{\Delta k}[\exp(-i\Delta k\, z) - 1],$$

$$\begin{aligned} K_{12}(z) &= |g|^2\left\{-i\frac{z}{\Delta k} + \frac{1}{(\Delta k)^2}[\exp(i\Delta k\, z) - 1]\right\}, \\ K_{21}(z) &= |g|^2\left\{i\frac{z}{\Delta k} + \frac{1}{(\Delta k)^2}[\exp(-i\Delta k\, z) - 1]\right\}. \end{aligned} \tag{14}$$

The notation $A \equiv B \mod C$ means that the difference $A - B$ belongs to the ideal C.

We have also obtained the unitary progression operator $\hat{U}(z)$ up to second order in z. The generator (5) resembles a Hamiltonian of periodically driven system with the period $\frac{2\pi}{\Delta k}$. The Floquet operator $\hat{U}\left(\frac{2\pi}{\Delta k}\right)$ can serve for the definition of the unitary operator

$$\hat{U}_{\rm nonpri}(z) = \left[\hat{U}\left(\frac{2\pi}{\Delta k}\right)\right]^{\frac{\Delta k\, z}{2\pi}}. \tag{15}$$

Here the subscript nonpri stands for nonperiodic. According to the Floquet theory [5] there exists a decomposition

$$\hat{U}(z) = \hat{U}_{\rm nonpri}(z)\hat{U}_{\rm pri}(z), \quad \hat{U}_{\rm nonpri}(z) \equiv \hat{1} + z^2\hat{u}_{2\rm nonpri}(\Delta k\, z) \mod \hat{J}_3, \tag{16}$$

where

$$z^2\hat{u}_{2\rm nonpri}(\Delta k\, z) = i|g|^2\frac{z}{\Delta k}[\hat{A}_1^{\dagger 2}(0)\hat{A}_2^\dagger(0), \hat{A}_1^2(0)\hat{A}_2(0)]. \tag{17}$$

Here $\hat{U}_{\rm pri}(z)$ is a $\frac{2\pi}{\Delta k}$-periodic unitary operator. The occurrence of the commutator can be expected as another description of cascading second-order nonlinearities [7]. Also the unitary operator $\hat{U}_{\rm pri}(z)$ can be determined up to second order in z.

3. Cubic behaviour of the fundamental mode

To illustrate the squeezing properties of the fundamental mode, we concentrate on the principal squeezing variance [8]

$$\langle(\Delta\hat{Q}_1^{(p)}(z))^2\rangle = -1 + 2[\langle\Delta\hat{A}_1(z)\Delta\hat{A}_1^\dagger(z)\rangle - |\langle(\Delta\hat{A}_1(z))^2\rangle|]. \qquad (18)$$

We have investigated this quantity for initial coherent states $|\xi_1\rangle|\xi_2\rangle$, $\xi_1 \in [2, 10]$, $\xi_2 = 1$, the coupling constant $g = 1$, the phase mismatch $\Delta k = -150$, and the interaction lengths $z \in [0, 0.1]$. Applying the Floquet theory, we have found that the constructed nonperiodic behaviour exhibits squeezing when the assumption for the second-harmonic generation is fulfilled, i.e. $|\xi_1| \gg |\xi_2|$. The amount of squeezing increases with the increasing initial number of coherent photons. But this tendency should not be derived from the depletion of the fundamental mode, because this is degraded by the phase mismatch. This behaviour is due rather to the phase mismatch itself than to the fundamental mode depletion. The oscillations contribute to the squeezing when $\xi_1 \approx \xi_2$, i.e. when the trend is absent, but they also deteriorate the squeezing, when the trend is present.

Studying the phase properties of the states generated according to the Schrödinger picture $|\psi\rangle$, $|\psi(z)\rangle = \hat{U}^\dagger(z)|\xi_1\rangle|\xi_2\rangle$, we resort to the argument of the mean complex amplitude $\arg\langle\hat{A}_1(z)\rangle = {\rm Im}(\ln\langle\hat{A}_1(z)\rangle)$. For the above choice of parameters and starting with the Floquet theory, we find again that a phase shift in the counterclockwise direction occurs, but this shift is not present for $\xi_1 \approx \xi_2$. In the unreduced progression there is a phase shift, which collapses and revives repeatedly. Unlike the squeezing analysis, the oscillations attenuate with increasing ξ_1. The phase shift is due only to the phase-mismatch itself, not to the fundamental mode depletion, and this observation has led to the discovery of the cubic or Kerr-like behaviour in this situation [7].

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