acta physica slovaca vol. 45 No. 3, 367 – 373 June 1995

ATOM AND FIELD STATISTICS IN A MICROMASER WITH STATIONARY NON-POISSONIAN PUMPING[1]

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Received 21 April 1995, accepted 10 May 1995

By treating the statistics of the arrival times of the individual micromaser pump atoms with the help of the theory of stochastic point processes, a method is developed for investigating the properties of a one-atom micromaser with stationary non-Poissonian pumping. The resulting equations can be simplified when the pump statistics belongs to the class of stationary renewal processes. For this case the photon statistics of the cavity field as well as the level-selective statistics of the atoms leaving the cavity is studied in dependence on the strength of the pump-atom correlations or anticorrelations, respectively, and on the pump-atom correlation decay time.

1. Introduction

A micromaser is pumped by excited Rydberg atoms which, one after the other, interact with the microwave field in a high-Q cavity [1]. Normally the pump atoms are statistically independent thus obeying Poissonian injection statistics. However, one may think of other kinds of pump statistics and investigate the problem as to what extent the micromaser properties are changed. For this purpose in the literature commonly the model of periodic pumping is used where pump atoms are allowed to arrive, with certain probability, only at distinct instants which are located equidistantly in time [2].This model is an intrinsic non-stationary one. In contrast to this, in the present contribution we study a micromaser with stationary non-Poissonian pumping by using the theory of stochastic point processes in order to describe the statistics of the pump-atom arrival times.

The coincidence probability density $P_2^{in}(t_1, t_2)$ for the injection of two pump atoms into the cavity at the time instants t_1 and t_2, notwithstanding the possible injection of

[1]Presented at the **3rd central-european workshop on quantum optics**, Budmerice castle, Slovakia, 28 April - 1 May, 1995

other pump atoms in between, characterizes the pump-atom correlations. For later use we make the specific Ansatz

$$P_2^{in}(t_1,t_2) = r^2(1 + Ce^{-\Gamma|t_1-t_2|}) \tag{1}$$

with r being the pump rate. The parameter C describes the strength of the correlations ($C > 0$) or anticorrelations ($-1 \leq C < 0$) between the pump atoms and Γ is the correlation decay rate. For positive or negative values of C one may speak of pump-atom bunching or antibunching, respectively.

The pump-atom counting statistics with respect to a counting interval of length T is characterized by the factorial moments of the number N of pump atoms counted during T. These moments can be found by integrating the multi-time pump-atom coincidence probability densities [3]. In particular, we obtain

$$\langle N(N-1)\rangle_T = \int_0^T dt_1 \int_0^T dt_2 P_2^{in}(t_1,t_2). \tag{2}$$

The mean number of pump atoms counted during T is given by $\langle N\rangle_T = rT$. When the special relation (1) is valid, pump-atom bunching ($C > 0$) is connected with super-Poissonian counting statistics and antibunching ($C < 0$) yields sub-Poissonian pump statistics. Apart from the case of uncorrelated pump atoms ($C = 0$), or Poissonian pumping, respectively, the relative variance of the pump-atom counting statistics depends on the length of the counting interval.

It is our aim to investigate the influence of the parameters r, C and Γ introduced in Eq.(1) on both the properties of the radiation field in the cavity and the level-selective statistics of the atoms leaving the micromaser. We still mention that for a special kind of stationary super-Poissonian pumping the radiation field has already been studied using a quantum-field model for the injected atomic beam [4].

2. Properties of the cavity field

Over the time between the passage of two consecutive pump atoms the density operator ρ of the field changes due to cavity damping. With the help of the damping operator L this can be expressed in the usual way as $\rho(t+\delta t) = \exp(L\delta t)\rho(t)$. We make the conventional assumption that the damping is negligibly small over the transit time of a single atom. The effect of an atom on the field is described by

$$\rho(t + t_{int}) = M\rho(t) = (D + E)\rho(t) \tag{3}$$

where the Jaynes-Cummings operator M has been formally divided into two parts accounting for the possibilities that the pump atom gets de-excited into the lower level or remains in the upper level, respectively, during the transit. Assuming the initial field to be diagonal in the photon-number representation we obtain with $\rho_{nn} \equiv p_n$

$$(D\rho)_{nn} = \sin^2(gt_{int}\sqrt{n})p_{n-1} \tag{4}$$

and

$$(E\rho)_{nn} = [1 - \sin^2(gt_{int}\sqrt{n+1})]p_n \tag{5}$$

where g denotes the atom-field coupling constant [5]. The action of the damping operator L reads

$$(L\rho)_{nn} = \gamma(n_b+1)[(n+1)p_{n+1} - np_n] - \gamma n_b[(n+1)p_n - np_{n-1}] \tag{6}$$

with γ and n_b being the damping rate and the thermal photon number, respectively.

According to the theory of stochastic point processes, the pump statistics is uniquely described by the complete set of all exclusive probability densities $Q_k^{in}(t_1,t_2,...,t_k)$ where $k = 1,2,...$. These quantities determine the probability densities for the arrival of k pump atoms at the time instants $t_1,...,t_k$ without any other pump atom arriving in between. Especially, the pump-atom waiting-time distribution is given by $f(\tau) = r^{-1}Q_2^{in}(0,\tau)$.

First we want to consider an injection-time conditioned density operator ρ^c which refers to the state of the field immediately before the transit of an atom. For example, if the photon number in the cavity could be measured in the stationary micromaser regime always immediately before a pump atom arrives, one would obtain the mean value $\bar{n}^c = \sum_n n\bar{\rho}^c_{nn}$. On the other hand, by determining the photon number always immediately after the transit of a pump atom, one would find the mean photon number $\sum_n n(M\bar{\rho}^c)_{nn}$. As will be shown later, it is just the evolution of the operator $\rho^c(\tau)$ which is needed for calculating the statistics of the de-excited atoms leaving the micromaser.

For the moment we suppose that an atom has traversed the cavity immediately prior to the initial time $t = 0$. The injection-time conditioned density-operator $\rho^c(\tau)$ then follows from the initial density operator $\rho(0)$ with the help of the equation

$$\rho^c(\tau) = \frac{r^2}{P_2^{in}(0,\tau)}U_c(\tau)\rho(0) \tag{7}$$

where

$$U_c(\tau) = \frac{1}{r^2}\left\{Q_2^{in}(0,\tau)e^{L\tau} + \sum_{k=1}^{\infty}\int_0^{\tau} dt_k \int_0^{t_k} dt_{k-1}\cdots \right.$$
$$\left.\cdots\int_0^{t_2} dt_1 Q_{k+2}^{in}(0,t_1,...,t_k,\tau)e^{L(\tau-t_k)}Me^{L(t_k-t_{k-1})}...Me^{Lt_1}\right\}. \tag{8}$$

On the right-hand side of the above equation we took into account , with proper probability, all possibilities that exactly k atoms ($k = 1,2,\ldots$) may have entered the cavity in the interval $[0,\tau]$ where (for $k \geq 1$) this has occurred at the time instants $t_1,t_2,\ldots,t_k$ which are randomly distributed according to the corresponding exclusive probability densities $Q_{k+2}^{in}(0,t_1,\ldots,t_k,\tau)$ of the injected pump atoms. A considerable simplification can be achieved when the pump statistics is described by a renewal process where the exclusive probability densities factorize into products of waiting-time distributions between neighboring atoms according to

$$Q_k^{in}(t_1,...,t_k) = r\prod_{i=1}^{k} f(t_i - t_{i-1}). \tag{9}$$

Applying the convolution theorem of the Laplace transformation we obtain from Eqs.(8) and (9)

$$U_c(\tau) = \frac{1}{r} f(\tau) e^{L\tau} + \int_0^\tau dt f(\tau - t) e^{L(\tau - t)} M U_c(t). \tag{10}$$

Taking into account the relation $P_2^{in}(0,\infty) = r^2$, the steady-state solution for the injection-time conditioned density operator ρ^c can be shown with the help of the Eqs.(7) and (10) to obey the mapping condition [7]

$$\lim_{\tau\to\infty} \rho^c(\tau) = \bar{\rho}^c = \int_0^\infty dt f(t) e^{Lt} M \bar{\rho}^c \tag{11}$$

which expresses the fact that $\bar{\rho}^c$ is reproduced by the interaction with a single pump atom and subsequent damping over the average waiting time between consecutive pump atoms.

In order to enable a quantitative treatment we now assume that the waiting-time distribution is given by

$$f(\tau) = \frac{\lambda_1\lambda_2}{\lambda_2 + \alpha\lambda_1}(e^{-\lambda_1\tau} + \alpha e^{-\lambda_2\tau}) \tag{12}$$

$(\lambda_1, \lambda_2 > 0, \alpha \geq -1)$. The coincidence probability density $P_2^{in}(0,\tau)$ can be calculated from the whole set of all exclusive probability densities $Q_{k+2}^{in}(0, t_1, \ldots, t_k, \tau)$ [3]. Making use of Eqs.(9) and (12) we find that the resulting expression is equal to Eq.(1). The parameters λ_1, λ_2 and α are related to r, C and Γ of Eq.(1) by the equations

$$\lambda_{1/2} = \frac{1}{2}[\Gamma + r(1+C)] \pm \frac{1}{2}\sqrt{\Gamma^2 + 2r\Gamma(C-1) + r^2(1+C)^2} \tag{13}$$

and

$$\alpha = -\frac{\Gamma - \lambda_2(1+C)}{\Gamma - \lambda_1(1+C)} \tag{14}$$

[7]. For antibunched injection of pump atoms where $-1 \leq C < 0$ the expression (1) is only compatible with the properties of a renewal process when the condition $\Gamma \geq r(1+\sqrt{|C|})^2$ is fulfilled.

It is interesting to note at this point that there exists a specific physical example where Eqs.(9) and (12) are valid. Indeed, the statistics of the de-excited atoms leaving a micromaser with Poissonian pumping which is in the one-photon trapped state is described by a renewal process exhibiting complete antibunching and obeying Eq.(12) with $\alpha = -1$ [8]. These atoms could be used to pump a second micromaser which works on an adjacent lower microwave transition.

The advantage of the Ansatz (12) consists in the fact that it reduces the integral operator equation (10) to a system of two coupled linear differential operator equations. We obtain

$$U_c(\tau) = U_1(\tau) + U_2(\tau) \tag{15}$$

where

$$\dot{U}_1 = LU_1 + \frac{\lambda_1\lambda_2}{\lambda_2 + \alpha\lambda_1} M(U_1 + U_2) - \lambda_1 U_1, \tag{16}$$

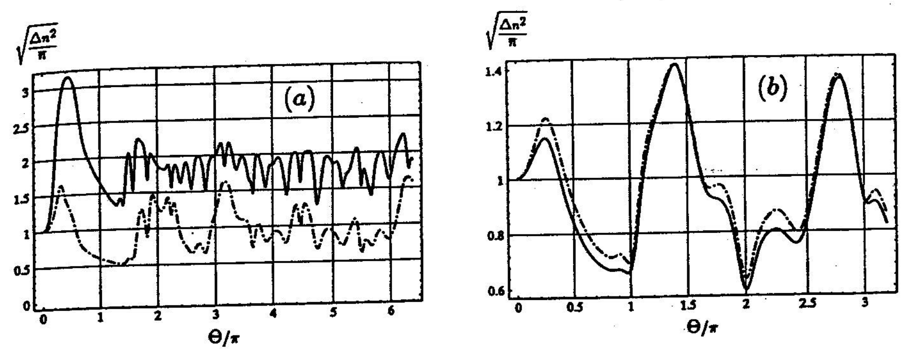

Fig.1: Relative standard deviation $\sqrt{\Delta n^2/\bar{n}}$ of the cavity photon number (full line) for super-Poissonian pumping with $r/\gamma = 10, C = 1$ and $\Gamma = 0.2\gamma$ (a) for sub- Poissonan pumping with $r/\gamma = 2, C = -1$ and $\Gamma = 8\gamma$ (b)in dependence on the pump parameter $\Theta = gt_{int}$ .The thermal photon number is $n_b = .01$, and for comparison the corresponding curves are also plotted for Poissonian pumping (dashed line).

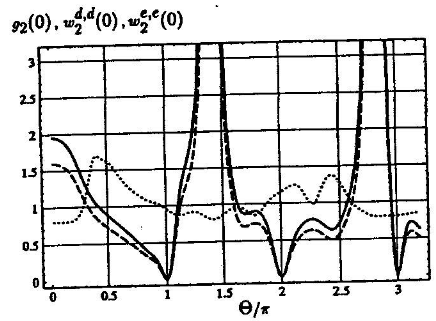

Fig. 2: Normalized coincidence probability densities $w_2^{d,d}(0)$ for the outgoing de-excited atoms (dashed line) and $w_2^{e,e}(0)$ for the outgoing excited atoms (dotted line) in comparison to the normalized intensity correlation function $g_2(0)$ of the cavity field (full line) at $n_b = 10^{-6}$ for pumping with $r/\gamma = 10, C = -0.2$ and $\Gamma = 8\gamma$.

$$\dot{U}_2 = LU_2 + \frac{\lambda_1\lambda_2}{\lambda_2 + \alpha\lambda_1} M(U_1 + U_2) - \lambda_2 U_2 \tag{17}$$

with the initial conditions $U_1(0) = (\lambda_2^2 + \alpha\lambda_1^2)/(\lambda_2 + \alpha\lambda_1)^2 \underline{1}$ and $U_2(0) = \alpha U_1(0)$.The above system can be easily solved numerically and, for simple special cases, even analytically.

When we are interested in the photon statistics of the cavity field we have to know the field density operator ρ that is not injection-time conditioned. The evolution of the latter from an initial time $t = 0$ to a final time $t = \tau$, which are both arbitrarily located with respect to the pump-atom arrival times, is described in analogy to Eq.(8) by [7]

$$\rho(\tau) = \left\{ \int_\tau^\infty dt_1 \int_{-\infty}^0 dt_0 Q_2^{in}(t_0,t_1) e^{L\tau} + \sum_{k=1}^\infty \int_\tau^\infty dt_{k+1} \int_0^\tau dt_k \int_0^{t_k} dt_{k-1} \ldots \right.$$
$$\left. \ldots \int_0^{t_2} dt_1 \int_{-\infty}^0 dt_0 Q_{k+2}^{in}(t_0,\ldots,t_{k+1}) e^{L(\tau-t_k)} M \ldots M e^{Lt_1} \right\} \rho(0). \qquad (18)$$

The steady-state solution $\bar{\rho} = \lim_{\tau\to\infty} \rho(\tau)$ yields expectation values for the field quantities which are equal to their time-averaged values in the stationary micromaser regime. For pumping according to a renewal process, $\bar{\rho}$ and the injection-time conditioned operator $\bar{\rho}^c$ are related by the equation [7]

$$r(M-1)\bar{\rho}^c + L\bar{\rho} = 0. \qquad (19)$$

It turns out that for Poissonian pumping ($C = 0$) both operators are equal. For antibunching of the pump atoms ($C < 0$ in Eq.(1)) the difference between $\bar{\rho}$ and $\bar{\rho}^c$ leads to the inequality $\bar{n} > \bar{n}^c$ for the time-averaged mean value $\bar{n}$ of the cavity photon number and for its injection-time conditioned average value $\bar{n}_c$, whereas for bunching ($C > 0$) the opposite inequality is valid. The relative standard deviation of the photon-number distribution calculated numerically from the steady-state solution of Eqs.(15)–(17) and from Eq.(19) is shown in Fig.1. In comparison to Poisssonian pumping, it is increased for super-Poissonian pump statistics (bunching of the pump atoms) and decreased for sub-Poissonian pumping (antibunching).The deviation from the results obtained for Poissonian pumping is diminished with growing decay rate Γ of the pump-atom correlations or anticorrelations, respectively.

3. Statistics of the de-excited atoms

In analogy to the treatment of a micromaser with Poissonian pumpinqg [9] we can calculate both the probability density W_1^d for detecting a de-excited atom at the exit of the micromaser in the stationary regime and the coincidence probability density $W_2^{d,d}(\tau)$ for detecting two de-excited atoms with time difference τ . For the corresponding normalized coincidence probability density we find the expression [7]

$$w_2^{d,d}(\tau) = \frac{W_2^{d,d}(\tau)}{(W_1^d)^2} = \frac{[\mathrm{Tr} DU_c(\tau)D\bar{\rho}^c]}{[\mathrm{Tr}(D\bar{\rho}^c)]^2}, \qquad (20)$$

which can be easily evaluated numerically with the help of Eq.(4) and of the solution of Eqs.(15)-(17). In order to obtain the normalized coincidence probability density $w_2^{e,e}(\tau)$ for the detection of outgoing atoms in the upper micromaser level one simply has to replace the operator D in the above equation by the operator E (see Eq.(5)). It is interesting to compare these level-selective coincidence probability densities with the normalized cavity-field intensity correlation function $g_2(\tau) = \langle a^\dagger(0)a^\dagger(\tau)a(\tau)a(0)\rangle/\bar{n}^2$ with a and $a^\dagger$ being the photon annihilation and creation operators. For Poissonian pumping $w_2^{d,d}(\tau)$ and $g_2(\tau)$ have shown to be equal at zero thermal photon number [9]. For non-Poissonian pumping, however, this is no longer true, as illustrated in Fig.2.

4. Summary

For a micromaser with stationary non-Poissonian pumping a method has been developed for investigating the photon statistics of the cavity field and the level-selective statistics of the outgoing atoms in dependence on the strength of the pump-atom correlations or anticorrelations, respectively, and on the correlation decay rate.

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