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TWO-ATOM COOPERATION IN A MICROLASER [1]

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In this paper we investigate a system of two three-level atoms strongly interacting with a coherent driving field and two damped quantised cavity modes a and b via a radiative cascade. Upon adiabatic elimination of one of these modes the dynamics can lead to a strong entanglement between the internal states of the two atoms. Choosing appropriate operating conditions the two-atom system will preferentially occupy a symmetrical linear combination of internal states. As a consequence the two-atom system behaves very much like a single atom with correspondingly larger dipole-moments, i.e., a superradiant two-atom system.

1. Introduction

One of the fundamental building blocks of standard laser theory is the assumption of statistical independence of the individual atoms making up the gain medium. The socalled *independent atom* model neglects the correlations that can build up between atoms due to their common interaction with a quantised light field inside a resonator. A certain randomness in coupling strength arising from the thermal nature of a gaseous gain medium or a solid state suffices to constantly decorrelate the atoms. In a different guise this is better known as the *private bath* assumption. Assuming a low-density medium and a width of the position distribution larger than a wavelength of the emitted light each atom may be viewed as coupled to its own reservoir all of which are statistically independent. Spontaneous decay will thus take place independently in each atom. This randomization thus froms the physical basis for the independent atom model.

The situation is clearly different for the case of a microlaser. Such a device consists of only very few atoms whose positions are fixed and known to a certain degree as would be the case for trapped ions or a doped fibre placed in the evanescent field of a microsphere. Lately, collective effects have become a topical issue in quantum optics in connection with interesting mechanical light effects [1–3]. Microlasers tie in well with

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this trend as they are ideally suited for developing an understanding as to how atoms interact via a quantised mode [4].

Recently, there has been renewed interest in finding a superradiant laser source [5], a device which thrives upon strong correlations between individual atoms [6]. In an attempt to mimick the behaviour of many well correlated atoms Horak and coworkers [7] have investigated whether some of the predictions derived from the semiclassical model in Ref. [5] can also be obtained from a fully quantum one-atom model with rescaled dipole-moment. While some features could be reproduced, the overall issue of whether substantial correlation between individual atoms can build up could not be addressed.

In this paper we present a fully quantum two-atom model of a cascade microlaser. We investigate if an atom-atom coupling mediated through a strongly damped and coupled mode can give rise to enhanced lasing for the remaining mode of the cascade transition. By a comparision with a noncollective version of our model we will try to assess the importance of atom-atom entanglement for the lasing process.

2. Motivation

It is clear that an ensemble of initially uncorrelated atoms will only display cooperative behaviour if there is a means available through which they can interact with each other. In the well known case of superfluorescence [6] such a means is the free electromagnetic field. Suppose now we were interested in constructing a device that generates spontaneously emitted superradiant fluorescence pulses. Clearly, the requirement of our atoms having to be confined to less than a coherence length ($\geq$ wavelength) of the generated radiation will see only very few atoms participate in the collective emission process.

It thus remains to explore the utility of other schemes which could establish coherence between the individual atoms. Haake *et al* have suggested [5] to utilise a single mode of an optical resonator instead of the whole free electromagnetic field. The strength of the atom-atom interaction could be strongly enhanced by the finesse of the resonator which can be extremely large. Such a device would offer the possibility to tailor the coherence to some extent by manipulating the decay rate and the resonance frequency of the cavity. Furthermore, the periodic spatial mode structure of a resonator allows us to separate the atoms by macroscopic distances while still maintaining the coherence of the interaction. It seems pausible that a single *strongly damped* resonator mode can lead to a substantial dipole-dipole coupling between atoms. The single mode can be adiabatically eliminated by assuming a short autocorrelation time of the mode on the timescales relevant for the atoms. This procedure is similar to what is normally done to remove the reservoir degrees of freedom from the equations of motion of the "system"-variables. We conclude that for two two-level atoms a single strongly damped and coupled mode should be as suited to introduce coherence as are the many weakly coupled modes of the free electromagnetic field. It is, however, not totally straightforward to see whether the same arguments also apply to a more complicated level configuration which is what we are going to address in the remainder of this paper.

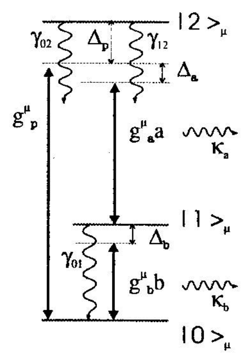

Fig.1. Energy level diagram of the μth atom. The system is damped through spontaneous atomic relaxation as well as through the coupling to the cavity modes a and b. Note that the transition form 0 to 2 should be regarded as a two-photon transition.

3. Model

Let us extend the single-atom cascade laser model introduced in Ref. [7] to a two-atom model. The Hamiltonian (in units of $\hbar$) for the interaction of two distinguishable atoms with two quantised modes a, b of an optical resonator and a coherent driving field is of standard form. As usual, we work in a rotating frame, hence introducing detunings and the following interaction picture Hamiltonian

$$H_I = H_s + H_b + H_R, \tag{1}$$

$$H_s = \Delta_a a^\dagger a + \sum_{\mu=1}^{2}\left(\Delta_p \sigma_{22}^\mu + [g_a^\mu \sigma_{21}^\mu a + \text{H.c.}] + [g_p^\mu \sigma_{20}^\mu + \text{H.c.}]\right), \tag{2}$$

$$H_b = \Delta_b b^\dagger b + [g_b^\mu \sigma_{10}^\mu b + \text{H.c.}], \tag{3}$$

where $\Delta_a = \omega_a + \omega_1 - \omega_p$ (the detuning from two-photon resonance between levels 0 and 1), $\Delta_b = \omega_b - \omega_1$, and $\Delta_p = \omega_2 - \omega_p$. Notice that by superscripting the coupling constants g_a, g_b, and g_p we mean to denote their parametric dependence on the position of the atoms. We assume H_R to contain the dynamics accounting for the system-reservoir interaction. A graphical illustration of our model system is given in Fig. 1. In a real-life system the life time of the atoms in a state other than the ground state as well as the storage time of photons inside the resonator will be limited by dissipation. We thus turn to a quantum stochastic formulation of the system dynamics in which we regard our system as coupled to various reservoirs whose back-action induces irreversible processes [stemming from H_R in Eq. (1)]. This is accomplished most conveniently either by Master equation techniques or by using Itô quantum stochastic calculus which is more convenient for numerical simulations. Assuming spontaneous decay into modes other

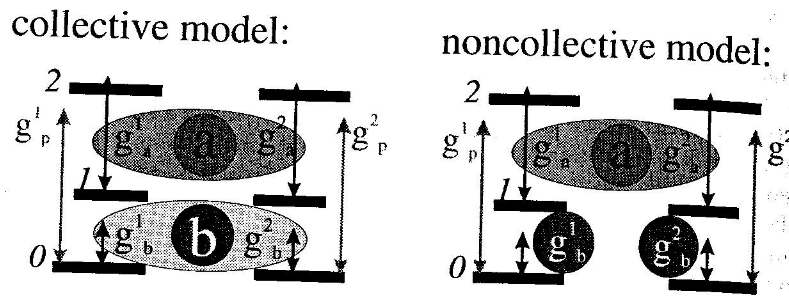

Fig.2. Collective vs. noncollective two-atom model. Instead of being coupled to a single mode b in the noncollective model each atom couples to its own mode b_μ. Any correlations between the atoms in the noncollective model can only be due to their interaction with the laser mode a.

than the two cavity modes to take place independently in each atom we find the following equation for any *system* operator X [3]

$$dX = -i[X, H_I]dt - (d\mathcal{L}_s + d\mathcal{L}_b)\,X, \quad (4)$$

where we have split off the damping part of mode b and arranged it in a separate Liouvillian $d\mathcal{L}_b$. We find

$$d\mathcal{L}_b X = \kappa_b(\{X, b^\dagger b\} - 2b^\dagger X b)dt - \sqrt{2\kappa_b}(dB_b^\dagger(t)[X,b] - dB_b[X,b^\dagger]), \quad (5)$$

$$d\mathcal{L}_s X = d\mathcal{L}_a X + \sum_{\mu=1,2}\sum_{0\le i<j\le 2}\Big(\gamma_{ij}(\{\sigma^\mu_{jj}, X\} - 2\sigma^\mu_{ji} X \sigma^\mu_{ij})dt - \sqrt{2\gamma_{ij}}(dB^\dagger_{ij,\mu}(t)[X,\sigma^\mu_{ij}] - dB_{ij,\mu}(t)[X,\sigma^\mu_{ji}])\Big). \quad (6)$$

The rates $\kappa_{a(b)}$ describe cavity decay while the rates $2\gamma_{ij}$ respresent the rates of spontaneous decay from level j to level i.

4. Adiabatic elimination

Eq. (4) proves especially useful for purposes such as adiabatic elimination of the fast mode b an approach useful in trying to understand the way in which correlation can build up between the atoms. An adiabatic model facilitates understanding by removing one parameter thereby greatly simplifying the mathematical treatment. It also allows for a comparison with a quasi independent-atom model in which the mode b is replaced by two independent ones b_1 and b_2, as illustrated in Fig. 2. Both models lead to a relaxation out of state $|1_\mu\rangle$ at a rate $\gamma_b^\mu = 2|g_b^\mu|^2\kappa_b/(\kappa_b^2+\Delta_b^2)$. The collective adiabatic

[3] By *system* we mean to denote the Hilbert space which such an operator is assumed to operate in.

model, however, contains an extra exchange term. It is due to our ignorance as to which atom a photon escaping from the cavity was emitted by. In the case of a finite detuning Δ_b between mode b and the corresponding atomic line we also obtain a beat term between states $|1_1, 0_2\rangle$ and $|0_1, 1_2\rangle$. In our theoretical model these exchange terms can be used as *toggle buttons* with which we can switch on or off the correlating effect of the fast mode b. This will allow us to easily identify the collective effects.

Assuming κ_b and Δ_b to be large and g_b^μ such that γ_b^μ as introduced above is of comparable size with the other system parameters we may eliminate the mode b. It remains to find the adiabatic counterpart of Eq. (4). We now require an operator of our reduced system to have a vanishing commutator with b and $b^\dagger$, whereas any commutator involving atomic coherence operators or creation and annihilation operators of mode a will in general be nonzero. The complete adiabatic system dynamics are thus aptly described by the following equation

$$dX = -i[X, H_s]dt + d\mathcal{L}_s X - \frac{\kappa_b}{\kappa_b^2+\Delta_b^2}\sum_\mu\sum_\nu g_b^\mu g_b^{\nu *}(\{X, \sigma^\mu_{10}\sigma^\nu_{01}\}_+ - 2\sigma^\mu_{10} X \sigma^\nu_{01})\,dt + $$
$$+ \frac{i\Delta_b}{\kappa_b^2+\Delta_b^2}\sum_\mu\sum_\nu g_b^\mu g_b^{\nu *}[X, \sigma^\mu_{10}\sigma^\nu_{01}]dt + $$
$$- \left(\frac{2\kappa_b}{\kappa_b^2+\Delta_b^2}\right)^{\frac{1}{2}}\sum_\mu([X, g_b^\mu\sigma^\mu_{10}]dB + [X, g_b^{\mu *}\sigma^\mu_{01}]dB^\dagger)\,, \quad (7)$$

with $\langle dB dB^\dagger\rangle = dt$ and all other combinations zero. Parts of Eq. (7) can be accomodated in a modified Hamiltonian, which is defined by $H_s \to H_s + V_c$. The remaining terms can be integrated into a modified Liouvillian complemented by a term $d\mathcal{L}_c$. The dipole-dipole potential [4] V_c is given by

$$V_c = -\frac{\Delta_b}{\kappa_b^2+\Delta_b^2}\sum_\mu\sum_\nu g_b^\mu g_b^{\nu *}[X, \sigma^\mu_{10}\sigma^\nu_{01}]. \quad (8)$$

The addition to the Liouvillian can now be obtained from the remaining terms in Eq. (7). The appearance of an effective potential (or alternatively an energy level shift) can be exploited to selectively pump symmetrical and antisymmetrical linear combinations of states as the shift will tend to remove any degeneracies.

It is a pleasant feature of the adiabatic model that we may find families of states labelled by an integer n which are closed under the coherent system dynamics. The incoherent dynamics solely cause spontaneous transitions between adjacent families. In Fig. 3 we depict one such family of states $\mathcal{F}_n$ where n stands for the number of photons in mode a when both atoms are in their ground states $|0>$. We realize that the pump dynamics (represented by H_p) and the mode dynamics (H_a) give rise to symmetrical looking transitions between the states within a family $\mathcal{F}_n$. Transitions from $\mathcal{F}_n$ to $\mathcal{F}_{n+1}$ occur through a spontaneous transition of an atom in level 1 to level 0 or by a collective transition mediated by the fast mode b. Similarly, downward transitions can only occur

[4] Note that V_c is effectively proportional to products of the dipole-moments of our atoms.

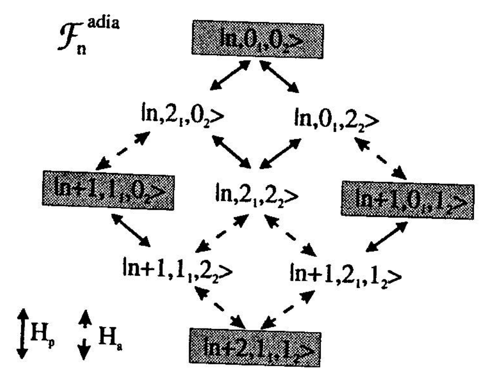

Fig.3. Schematical representation of the couplings introduced by the coherent part of the dynamics between the individual states of a family $\mathcal{F}_n$.

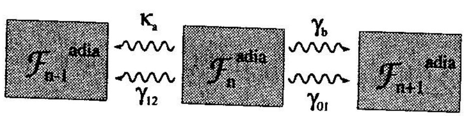

Fig.4. Jump processes occur only between adjacent families. Since the driving field is assumed classical, spontaneous decay from level 2 to level 0 does not lead out of $\mathcal{F}_n$.

via spontaneous emission on the lasing transition or by loss of a photon from the laser mode a. Spontaneous processes between levels 0 and 2 do not lead out of a family. A diagrammatic representation of the various jump processes is given in Fig. 4.

5. Results

Let us now proceed with a discussion of the laser mode intensity and statistics obtained from the two models shown in Fig. 2. We may expect that at least in the limit of a weak laser field our *noncollective* model will yield the same results as a one-atom model with the standard rescaling assumptions (i.e., a simple proportionality of the intensity to the number of atoms). Clearly, an indication of superradiance will then be a significant increase of the mean intensity obtained from the *collective* model over the one obtained from the noncollective model. Likewise we expect to see an increase of the photon number variance as a result of more significant number fluctuations.

For simplicity let us assume in the following that the respective coupling constants g_α^μ to the fields are independent of the atom qualifier μ. This means that from the two-atom ground state only symmetrical two-atom states can be pumped by the coherent dynamics, cf. Appendix .

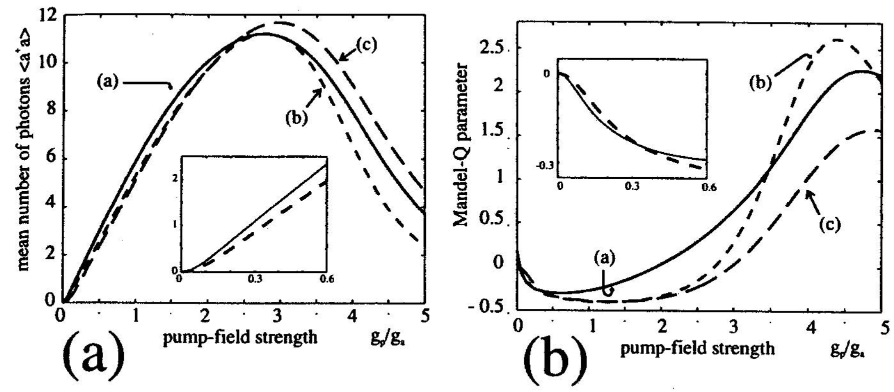

Fig.5a. Mean number of laser photons $\langle a^\dagger a\rangle$ as a function of the coherent pump field g_p. b. Mandel Q-Parameter vs. g_p. The parameters are (all in units of g_a) $\gamma_b = 1.8$, $\kappa_a = 0.1$, $\gamma_{01} = 0.3$, and $\gamma_{12} = \gamma_{02} = 0.01$. Curve (a) was obtained from the collective two-atom model, curve (b) from the noncollective two-atom model. Curve (c) represents a one-atom model with rescaled dipole-coupling constant $g_a \to \sqrt{2}g_a$.

5.1. The influence of pumping

In Fig. 5a we depict the steady state mean number of laser photons vs. the coherent pump strength $|g_p|$.

We realize that given sufficiently weak pumping there is nearly no difference between the predictions for $\langle a^\dagger a\rangle$ from a noncollective two-atom model and a rescaled one-atom theory. Clearly, as soon as saturation effects become important there are more substantial discrepancies. We also find that the collective model predicts a larger photon number than the noncollective one this being an indication of the beneficial influence of the mode b. The effect, however, appears to be fairly small except for around zero pumping, as illustrated by the inset in Fig. 5a. The ratio of the two photon numbers approaches a value of roughly 1.7 in the limit of g_p tending to zero.

It is also interesting to have a look at photon number fluctuations for which the intracavity Mandel Q-parameter is a measure, $Q = (\langle(a^\dagger a)^2\rangle - \langle a^\dagger a\rangle^2)/\langle a^\dagger a\rangle - 1$. We realise from Fig. 5b that the number fluctuations are smaller in the noncollective model except for very weak coherent pumping, cf. the inset in Fig. 5b. This can be explained by recalling that in the collective model there are long lived antisymmetrical two-atom states which do not contribute to the gain. This gives rise to a broadening of the photon number distribution. In the limit of weak pumping the pumping cycle is slower than the life-time of the antisymmetrical states and the broadening will disappear.

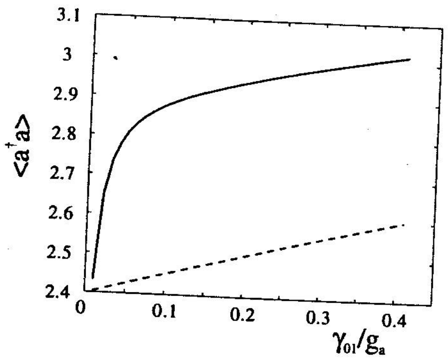

Fig.6. Average photon number as a function of the spontaneous rate γ_{01}. The parameters are $\gamma_{02} = \gamma_{12} = 0.01$, $\gamma_b = 1.8$, $g_p = 0.5$, and $\kappa_a = 0.1$. The solid (dashed) curves represent the results from the collective (noncollective) models, respectively

5.2. Two-atom gray states and spontaneous decay

It is obvious that spontaneous emission plays a tremendously important role in our two-atom system. Independent spontaneous emission tends to decorrelate the atoms precluding them from interacting with the light fields in unison. What complicates the situation further is that in our idealized model the presence of spontaneous decay on the laser transition suffices to trap the atoms in a long-lived antisymmetrical state which does not contribute to the amplification process. We therefore realize how important spontaneous decay is on the transition $1 \to 0$ for the depletion of this subradiant state. We have thus again made a comparison of the average photon number and Mandel-Q parameter predicted by our model and its noncollective counterpart We realize from Fig. 6 that in the collective two-atom model the photon number can be highly sensitive to changes in the rate γ_{01} while for the noncollective model we observe a linear increase in laser intensity due to an enhanced recycling of electrons back to the ground state. Next we have investigated the dependence of both the photon number and the Mandel Q-parameter on the rate of spontaneous decay on the lasing transition 2–1. From Fig. 7 we find that large spontaneous decay (γ_{12}) can cause the performance of the collective model to fall behind that of the noncollective one. An explanation for this could be the fact that photons can be absorbed from the laser field by a laser mediated transition between two antisymmetrical two-atom states and a subsequent spontaneous decay.

6. Conclusions

In brief our findings could be summarized as follows: the main benefit of using an auxiliary cavity field, i.e., mode b, is that collecive recycling to the ground state is more efficient than in the independent atom model. As a result we have found

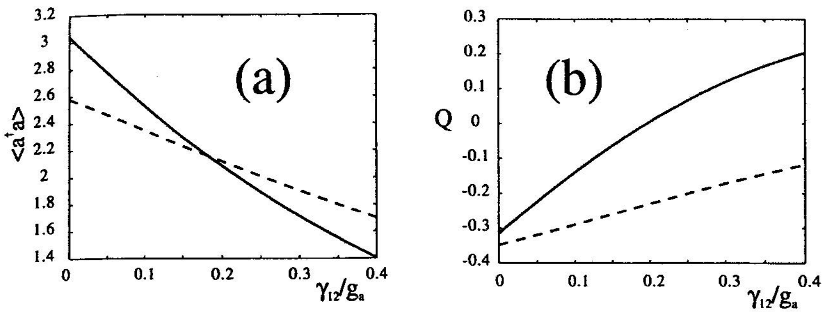

Fig.7a. Average photon number as a function of the spontaneous rate γ_{12}. b. Mandel Q-parameter as a function of γ_{12}. The parameters are $\gamma_{02} = \gamma_{01} = 0.3$, $\gamma_b = 1.8$, $g_p = 0.5$, and $\kappa_a = 0.1$. The solid (dashed) curves represent the results from the collective (noncollective) models, respectively.

that the collective model will in general yield a larger intracavity field in steady state. Clearly, the coherent dynamics will favour symmetrical two-atom states by exclusively coupling the ground state $|0_1, 0_2\rangle$ to a symmetrically entangled two-atom state. The existence of subradiant states in the collective model which cannot relax efficiently to the ground state will give rise to larger photon number fluctuations in mode a and a larger occupation of antisymmetrical subradiant states than in the noncollective model.

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Appendix: Symmetrical and antisymmetrical states

It is useful to separate the state space of the two atoms into a symmetrical and an antisymmetrical part. To this end we will introduce new coupling constants

$$\bar{g}_\alpha = \frac{1}{2}(g^1_\alpha + g^2_\alpha), \qquad h_\alpha = \frac{1}{2}(g^1_\alpha - g^2_\alpha), \tag{9}$$

with $\alpha = a, b, p$. In a similar vein we may now introduce the following Schrödinger coherence operators

$$S_{ij} = \sigma^1_{ij} + \sigma^2_{ij}, \quad \text{and} \quad A_{ij} = \sigma^1_{ij} - \sigma^2_{ij}, \tag{10}$$

for $j > i$. This implies that a Jaynes-Cummings-type coupling between the atoms and a light field can be expressed as the sum of two contributions. For example we find for the coupling to the laser mode a:

$$H_a = \sum_{\mu} (g_a^{\mu} \sigma_{21} a + \text{H.c}) = (\bar{g}_a S_{12}^{\dagger} + h_a A_{12}^{\dagger}) a + \text{H.c.} \quad (11)$$

We realise that for almost equal coupling constants g_a^{μ}, h_a is small and antisymmetrical states couple only weakly to the light fields. Antisymmetrical two-atom states are, however, populated by spontaneous emission which we assume to take place independently in each atom. We may understand this by recalling that any coherence operator may be written as an equally weighted sum of operators A and S. This is important insofar as even in the limit of equal coupling constants it does not suffice to consider only symmetrical two-atom states. Even small spontaneous rates will in this system give rise to what has been referred to as *symmetry breaking*.

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