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SUCCESSIVE CLICKS OF THE SAME KIND IN ONE-ATOM-MASER EXPERIMENTS[1]

B.-G. Englert, Ts.Gantsog, A.Schenzle, Ch.Wagner
Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany

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We derive an analytical expression for the mean number of successive clicks of the same kind in one-atom-maser experiments.

In one-atom-maser (OAM) measurements [1] the only reproducible experimental data concern the statistical properties of the atom-detector clicks. Of special interest is the mean number of successive clicks of the same kind because this quantity is easily accessible by experiment. In the past, this quantity has been computed with the aid of a Monte-Carlo method [2]. It is the objective of the present contribution to derive an analytical expression for this mean number.

In OAM experiments, the atom enters the resonator in the upper one of the two Rydberg states of the maser transition, interacts with the photon field it encounters, and is then probed in which one of two orthogonal states, $|A\rangle$ or $|B\rangle$, the atom emerged from the cavity. These final states can be either pure Rydberg states themselves or their coherent superpositions depending on the specific experimental setup.

Let us consider an arbitrary sequence of events of two kinds: the clicks of the $|A\rangle$ and $|B\rangle$ detectors. It does not matter at all whether we are dealing with random events or with ones that are strongly correlated.

We denote the probability for having exactly n events of one kind between two successive events of the other kind by p_n with $n = 0, 1, 2, 3, \ldots$. For each count of n events of one kind there are $n-1$ counts of zero events of the other kind, so that the sum rule

$$p_0 = \sum_{n=1}^{\infty} (n-1)\, p_n \tag{1}$$

holds. We combine it with the normalization condition $\sum_{n=0}^{\infty} p_n = 1$, to arrive at the statement

$$\sum_{n=1}^{\infty} n\, p_n = 1. \tag{2}$$

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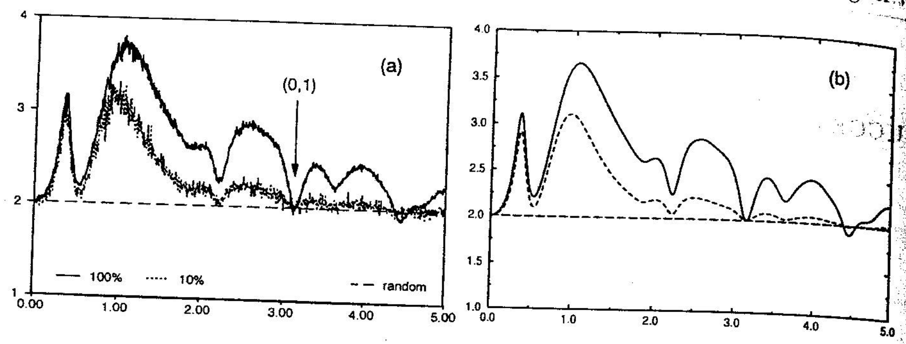

Fig.1. Mean number $\bar{n}$ of successive detector clicks of the same kind in the phase sensitive setup of Ref. [2] vs φ for different detector efficiencies. The pump rate is 8.33 atoms per cavity decay time. The curves are for detector efficiencies of $p = 100\%$ (solid line) and $p = 10\%$ (short dashed line). (a) The results of a Monte Carlo simulation. (b) Analytical results produced by eq. (11).

Let us now ask a slightly different question. How large are the probabilities P_n for getting n events in succession? The way of counting is different here because n cannot equal zero for conceptual reasons. The probabilities P_n are thus normalized according to $\sum_{n=1}^{\infty} P_n = 1$. Except for discarding the $n = 0$ possibility, there is no essential difference between the probabilities p_n and P_n, so that

$$P_n = \frac{p_n}{\sum_{m=1}^{\infty} p_m} = \frac{p_n}{1-p_0} \tag{3}$$

relates them to each other.

The quantity we are interested in is $\bar{n}$, the average number of successive events of the same kind,

$$\bar{n} \equiv \sum_{n=1}^{\infty} n\,P_n. \tag{4}$$

As an immediate consequence of (3) and (2) this number is given by

$$\bar{n} = \frac{1}{1-p_0}, \tag{5}$$

so that we simply need to calculate p_0, that is the probability that there are no events of one kind between two successive events of the other kind.

Let us now calculate the probability p_0, and consequently the mean number $\bar{n}$, for OAM experiments. Here we shall make extensive use of the methods, results and notations of Ref. [3]. We treat the case of "two successive A clicks with no B clicks in between" in detail, the reverse case is handled by exchanging the labels A and B consistently. For simplicity we assume that the detector efficiencies are the same: $p_A = p_B = p$.

The a priori rates for the clicks of the $|A\rangle$ and $|B\rangle$ detectors are

$$r_A = rp\,\mathrm{tr}\{\mathcal{A}\,\rho^{(SS)}\}, \qquad r_B = rp\,\mathrm{tr}\{\mathcal{B}\,\rho^{(SS)}\}, \qquad r_A + r_B = rp, \tag{6}$$

with the symbols introduced in Ref. [3]. The probability that the first click is of type A is given by the relative rate $r_A/(r_A + r_B) = \mathrm{tr}\{\mathcal{A}\,\rho^{(SS)}\}$. After this first A click the photon state is reduced to

$$\rho_A(0) = \frac{\mathcal{A}\,\rho^{(SS)}}{\mathrm{tr}\{\mathcal{A}\,\rho^{(SS)}\}}. \tag{7}$$

Until the next click happens this state evolves according to (see Eq. (2.26) of Ref. [3])

$$\rho_A(t) = \frac{\exp(\mathcal{L}^{(p)}t)\,\rho_A(0)}{\mathrm{tr}\{\exp(\mathcal{L}^{(p)}t)\,\rho_A(0)\}}, \tag{8}$$

with the Liouville operator

$$\mathcal{L}^{(p)} = \mathcal{L} + (1-p)\,r\,(\mathcal{A} + \mathcal{B} - 1), \tag{9}$$

where $\mathcal{L}$ describes the free decay of the photon field inside the resonator. The second term in eq. (9) accounts for the change in the photon state caused by undetected atoms. The probability for having an A click at time $t \cdots t + \mathrm{d}t$ is therefore given by $rp\,\mathrm{d}t\,\mathrm{tr}\{\mathcal{A}\,\rho_A(t)\}$, and the probability for having no click of either kind in the mean time is $\exp(-rpt)$. Putting things together we have the result

$$\begin{aligned} p_0 &= rp\int_0^{\infty} \mathrm{d}t\,e^{-rpt}\,\mathrm{tr}\{\mathcal{A}\exp(\mathcal{L}^{(p)}t)\,\mathcal{A}\,\rho^{(SS)}\} + [\mathcal{A} \leftrightarrow \mathcal{B}] \\ &= 1 - 2rp\,\mathrm{tr}\{\mathcal{A}\,[rp - \mathcal{L}^{(p)}]^{-1}\mathcal{B}\,\rho^{(SS)}\}. \end{aligned} \tag{10}$$

According to eq. (5), the mean number of successive OAM detector clicks of the same kind is therefore given by

$$\bar{n} = \left[2rp\,\mathrm{tr}\{\mathcal{A}\,[rp - \mathcal{L}^{(p)}]^{-1}\mathcal{B}\,\rho^{(SS)}\}\right]^{-1} \tag{11}$$

which is the central result of the paper.

In the situation of very low detector efficiencies, that is $0 < p \ll 1$, the formula (11) reduces to

$$\bar{n}_{\mathrm{uncor}} = \left[2\,\mathrm{tr}\{\mathcal{A}\,\rho^{(SS)}\}\,\mathrm{tr}\{\mathcal{B}\,\rho^{(SS)}\}\right]^{-1}. \tag{12}$$

To be more specific, we consider the phase sensitive setup of the OAM experiments proposed in Ref. [2], which has been extensively used to study phase properties of the cavity field. In this setup the atom crosses a classical microwave field after exiting from the cavity and before reaching the detectors. The classical microwave field is resonant with the maser transition and effects $\pi/2$ pulse. As a result, the detectors respond to coherent superpositions of the atomic states rather than to the states themselves as in the standard OAM experiments. The linear operators $\mathcal{A}$ and $\mathcal{B}$ in eq. (11) are in this case given by

$$\left.\begin{matrix}\mathcal{A}\rho \\ \mathcal{B}\rho\end{matrix}\right\} = \frac{1}{2}\left[\cos(\varphi\sqrt{aa^\dagger}) \mp a^\dagger\frac{\sin(\varphi\sqrt{aa^\dagger})}{\sqrt{aa^\dagger}}\right]\rho\left[\cos(\varphi\sqrt{aa^\dagger}) \mp a^\dagger\frac{\sin(\varphi\sqrt{aa^\dagger})}{\sqrt{aa^\dagger}}\right]^\dagger, \tag{13}$$

where φ is accumulated Rabi angle, a and $a^\dagger$ are the photon annihilation and creation operators. The diagonality of $\rho^{(SS)}(a^\dagger a)$ supplies $\mathrm{tr}\,\{\mathcal{A}\,\rho^{(SS)}\} = \mathrm{tr}\,\{\mathcal{B}\,\rho^{(SS)}\} = 1/2$ and, therefore, the value $\bar{n}_{\mathrm{uncor}} = 2$.

For the phase sensitive setup discussed above, $\bar{n}$ has been computed recently in Ref. [2] by means of a Monte Carlo simulation that produced estimates for the probabilities P_n. These results are reproduced in Fig. 1(a), which shows $\bar{n}$ as a function of φ for $p = 100\%$ and 10%. The uncorrelated value $\bar{n}_{\mathrm{uncor}} = 2$ is also indicated in this figure. We observe that the detector clicks are bunched for almost the entire φ range of the plot; antibunching is seen only around $\varphi = \sqrt{2}\pi = 4.44$.

In Fig. 1(b) we plot $\bar{n}$ of eq. (11) for the same parameters that were used in the Monte Carlo simulation of Fig. 1(a). We observe perfect agreement between the numerical results and the analytical answer.

In conclusion we presented an analytical method for calculating the mean number of successive clicks of the same kind in one-atom-maser experiments. The expression we find is simple and can be evaluated for arbitrary detector efficiencies. We have applied the result to the phase sensitive setup of one-atom-maser experiments [1,2].

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