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FIELD STATE MANIPULATION USING A MULTIATOMIC SYSTEM[1]

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We point out that a conditional measurement performed on an ensemble of two level atoms can transform the field into a highly nonclassical state. In contrast to the scheme employing a sequence of atoms the probability of such a process is relatively high.

In recent years great attention was given to the possibility to manipulate a given field mode state. There are several possibilities how to perform such a task. The relatively most straightforward way is the unitary transformation. One has to find a physical process that induces the proper transformation leading so to the desired state. However, the number of "available" processes is limited especially due to the fact, that for more complicated states we need to rely on processes of higher order with very small interaction constants or the needed process can be accompanied with equally probable competing processes. The second quite often used method is the so called quantum state engineering [1]. The desired target state is generated via the interaction of an initial cavity field mode with a sequence of atoms prepared initially for instance in their excited state. Each atom after the passage through the cavity is detected, i.e. its state is determined. Moreover, the determination of the atomic state is accompanied with a state reduction of the cavity mode. Using such a configuration we force the cavity field into a pure state with certain properties depending on which measurement outcomes we select for our purpose. On the other hand, a preparation of nonclassical states utilizing the repeated conditional measurement procedure is accompanied by decreasing probability of a favorable sequence.

Most of the field state engineering schemes have a basic characteristic in common. A controllable system (in terms of preparation and measurement) is brought into interaction with the field. The presented idea can have different realizations. Instead of using a sequence of atoms (and selecting the favorable outcomes of measurements) we can use at one "shot" a cluster of atoms and after the interaction perform only one measurement. In the present paper we illustrate this type of realization of the quantum state engineering idea. In particular we emphasize, that the use of an collection

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of atoms can lead to strongly nonclassical state, what is one of the main goals of the method. In addition the performed conditional measurement has a greater probability than the process with a sequence of atoms.

The interaction of an ensemble of two-level atom with one cavity mode is described within the Tavis-Cummings model [2]. The interaction Hamiltonian reads (we suppose exact resonance between the atoms and the field)

$$\hat{H}_{int} = \lambda \left(\hat{a} \sum_j \hat{\sigma}_j^+ + \hat{a}^\dagger \sum_j \hat{\sigma}_j^- \right) , \qquad (1)$$

where $\hat{a}, \hat{a}^\dagger$ are the cavity mode annihilation and creation operators and $\hat{\sigma}_j^\pm$ are the spin-flip operators of the atoms. We choose the initial state of the system in the disentangled form. The field will be considered in the coherent state $|\alpha\rangle_f$ while the atomic subsystem in the state $|N\rangle_a$ of all atoms excited, i.e.

$$|\psi(t=0)\rangle = |\alpha\rangle_f \otimes |N\rangle_a . \qquad (2)$$

What can be expected from such an initial configuration? Let us illustrate this for the choice of one excited atom, i.e. for the Jaynes-Cummings model [2]. To quantify the nonclassical effects to be expected we calculate squeezing and Mandel's q parameter to characterize the sub-Poissonian character of the cavity mode. In the case of squeezing we evaluate the maximum quadrature squeezing to be seen, i.e. we always look for the quadrature (depending on the phase ϕ_f) giving the higher degree of squeezing at the particular time moment t

$$s_{1f} = 4\langle(\Delta\hat{X}_f)^2\rangle - 1 \quad , \quad s_{2f} = 4\langle(\Delta\hat{Y}_f)^2\rangle - 1$$
$$\hat{X}_f = \frac{\hat{a}e^{i\phi_f} + \hat{a}^\dagger e^{-i\phi_f}}{2} \quad , \quad \hat{Y}_f = \frac{\hat{a}e^{i\phi_f} - \hat{a}^\dagger e^{-i\phi_f}}{2i}$$
$$\langle(\Delta\hat{X}_f)^2\rangle = \langle\hat{X}_f^2\rangle - \langle\hat{X}_f\rangle^2 \quad , \quad \langle(\Delta\hat{Y}_f)^2\rangle = \langle\hat{Y}_f^2\rangle - \langle\hat{Y}_f\rangle^2 . \qquad (3)$$

Mandels q parameter is defined as

$$q = \frac{\langle(\hat{a}^\dagger\hat{a})^2\rangle - \langle\hat{a}^\dagger\hat{a}\rangle^2}{\langle\hat{a}^\dagger\hat{a}\rangle} - 1. \qquad (4)$$

The solution for the initial state (2) is given as

$$|\psi(t)\rangle = |\psi_1(t)\rangle_f |1\rangle_a + |\psi_0(t)\rangle_f |0\rangle_a . \qquad (5)$$

The field states $|\psi_1(t)\rangle_f$ and $|\psi_0(t)\rangle_f$ can be each written in the form of a superposition of two states

$$|\psi_1(t)\rangle_f = \frac{1}{2}(|\alpha(t)\rangle_f + |\alpha(-t)\rangle_f), \qquad (6)$$

with

$$|\alpha(t)\rangle_f = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n \exp(i\lambda t\sqrt{n+1})}{\sqrt{n!}} |n\rangle_f , \qquad (7)$$

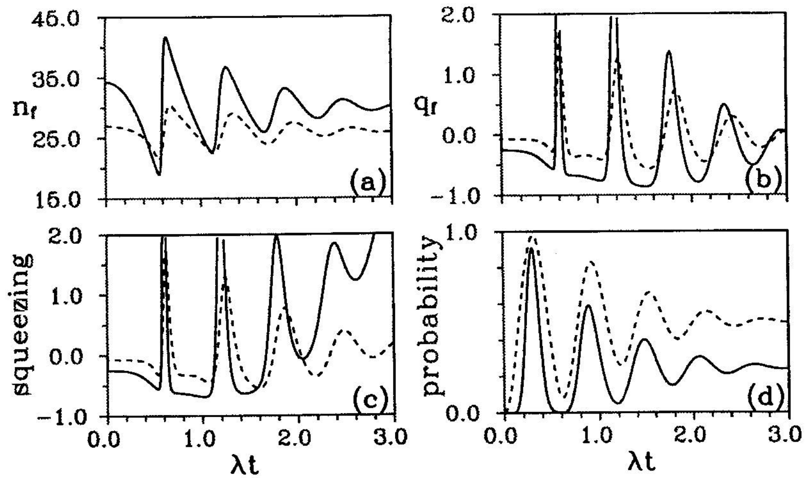

Fig.1: Properties of the cavity field in the case when the performed measurement at the time moment λt finds the atomic subsystem in the deexcited state – (a) cavity field mean photon number, (b) Mandel's q parameter, (c) squeezing, (d) conditional probability of a successful measurement. Solid line corresponds to the case $N = 5$, the dashed line correspond to the case $N = 1$. The initial coherent state amplitude was set $\alpha = 5$.

and

$$|\psi_0(t)\rangle_f = -\frac{1}{2}(|\alpha'(t)\rangle_f - |\alpha'(-t)\rangle_f), \qquad (8)$$

with

$$|\alpha'(t)\rangle_f = \frac{1}{\sqrt{\hat{a}^\dagger\hat{a}}} \hat{a}^\dagger |\alpha(t)\rangle_f . \qquad (9)$$

The field state after the interaction with the atom is given generally by a mixture of the states (6) and (8). Each of these states is a superposition and it was shown in great detail that especially such superpositions are to be accounted for the nonclassical properties found [3]. However, when we analyse only the mixture the resulting cavity mode can have this nonclassical properties suppressed. The easiest way out is to perform on the atom a measurement and so force the cavity field (depending on the result) into the state

$$|\Phi(t)\rangle_f = \frac{1}{\sqrt{_f\langle\psi_i(t)|\psi_i(t)\rangle_f}} |\psi_i(t)\rangle_f . \qquad (10)$$

In our case we looked into the case when the atom is found in the lower state. The time dependencies of squeezing and Mandels's q parameter are shown in Fig.1 (dashed line). In addition we plot the probability to find the atom in the lower state and the behaviour of the mean photon number. The surprising effect is, that the probability to find the atom in the lower state is higher then 50%. Even though nonclassical effects for the Jaynes-Cummings model are seen, they are not extremely pronounced. It was pointed out, that one way how to improve the nonclassical effects in the superposition of coherent–like components is to increase their number [3].

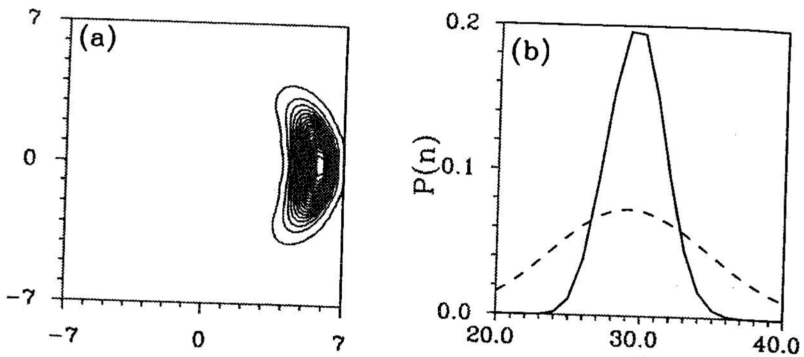

Fig.2: The Q function and the photon number distribution (compared to the initial coherent state - dashed line) in the moment of strongest sub-Poissonian character of the cavity mode ($\lambda t = 1.5$).

It was shown for the present situation, that within the interaction (1) the initial coherent Q function splits into $N+1$ components [4], where N is the number of atoms. This would imply in our case the use of more then one atom. For the input state (2) the system can be written in the form

$$|\psi(t)\rangle = \sum_{j=0}^{N} |\psi_j(t)\rangle_f |j\rangle_a . \tag{11}$$

where each field component $|\psi_j(t)\rangle_f$ is coupled to the fully symmetric atomic state (the so called Dicke state) with j atoms excited. Each of the component out of the initial coherent state (in the case $j = N$ and relatively short times) or a state related to the coherent state (for other j). Therefore it can be expected that each of the states $|\psi_j(t)\rangle_f$ itself can exhibit strong nonclassical properties due to quantum interference between its coherent–like components. By projecting out (performing measurement on the atoms) the field state $|\psi_j(t)\rangle_f$ we force the cavity field to exhibit fully the nonclassical properties of this superposition state. The improvement of the nonclassical properties for $N = 5$ and $j = 0$ when in the measurement all the atoms are found to be deexcited is shown in Fig.1 (solid line). The squeezing properties and in particular the sub-Poissonian character of the field is strongly improved. Mandel's q parameter goes almost to -0.9. In addition the probability to find the field in such a state after measuring the atomic subsystem is about 0.4. So the process to generate a nonclassical state in such a highly idealized way is fairly high, especially when we compare it with the scheme using a sequence of atoms. To illustrate the field mode even in more detail we plotted in Fig.2 the Q function of the cavity mode. The Q function has a crescent shape. To underline the strong sub-Poissonian character we plotted also the initial and the actual photon number distribution. From this is again nicely seen the strong sub-Poissonian character of the cavity mode. Let us note, that similar strong nonclassical effects after performing a conditional measurement can be found also for other initial

states of the atomic system. For instance when we prepare the atomic subsystem in a coherent atomic [SU(2)] state we can find the field in similar nonclassical states.

We pointed out that our scheme offers the possibility to generate highly nonclassical states with considerable probability on a time scale considerable shorter than the revival time of the system. However, we have to keep in mind that it is not a trivial task to prepare an ansemble of atoms in the needed state as well as to detect the atomic states at the output of the cavity.

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