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THREE-WAVE MIXING WITH ENTANGLED AND DISENTANGLED STATES[1]

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We show that the three wave interaction can serve as a source of highly subpoissonian light. The possible regimes are compared and discussed. Special attention is paid to those arrangements enabling one to get highly subpoissonian light before the first energy exchange maximum.

The interaction of three waves [1,2,3] is one of the most important processes in quantum optics. It refers to a process, where the annihilation of a pump photon leads to the generation of a pair of highly correlated photons, one in the signal mode and the other in the idler mode. In the interaction picture the Hamiltonian (we assume exact resonance between the modes) of the studied system reads [3,4]

$$\hat{H}_{int} = \lambda(\hat{a}\hat{b}\hat{c}^{\dagger} + \hat{a}^{\dagger}\hat{b}^{\dagger}\hat{c}), \tag{1}$$

where $\hat{a}, \hat{a}^{\dagger}$ and $\hat{b}, \hat{b}^{\dagger}$ are the signal and the idler annihilation and creation operators respectively, and $\hat{c}, \hat{c}^{\dagger}$ are the pump annihilation and creation operators. Depending on the initial conditions the model can describe the parametric amplifier or a frequency converter. In the case of the parametric amplifier the initial state are usually chosen to be:

$$|\alpha\rangle_a|0\rangle_b|\gamma\rangle_c, \quad |\alpha\rangle_a|\beta\rangle_b|\gamma\rangle_c, \quad |0\rangle_a|0\rangle_b|\gamma\rangle_c, \quad |\gamma| \gg |\alpha|, |\beta| \tag{2}$$

where α, β and γ are the amplitudes of coherent states, however more general states are possible as well. An excited signal (a) and an empty idler (b) with a strongly excited coherent pump (c) describes a phase insensitive parametric amplifier that is exactly

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equivalent to a laser amplifier [5,6]. The process of amplification with the signal and idler in the vacuum initially gives the to the two-mode squeezed vacuum.

The other often used regime of the three wave interaction is the case of a difference-frequency and sum-frequency generation. Corresponding initial states read:

$$|\alpha\rangle_a|0\rangle_b|\gamma\rangle_c \quad (|\alpha| \gg |\gamma|), \qquad |\alpha\rangle_a|\beta\rangle_b|0\rangle_c, \quad (|\alpha| \gg |\beta|), \tag{3}$$

respectively. In the frequency conversion the idler mode at the difference frequency $\omega_b = \omega_c - \omega_a$ is coherently build up simultaneously with the depletion of the pump mode while during sum-frequency generation the idler is depleted and a pump is build up at the sum-frequency of the signal and idler.

Apart from the above mentioned initial states where at least one wave is very strong, the intermediate regimes with modes of comparable intensities exhibit also interesting effects. The results for the initial states of the form

$$|\alpha\rangle_a|0\rangle_b|\gamma\rangle_c \qquad |\alpha| \approx |\gamma|, \tag{4}$$

have been reported in [7]. To quantify the properties of the fields we utilize the following parameters:

- Mandel's q_x-parameter [8]

$$q_x = \frac{\langle \hat{n}_x^2\rangle - \langle \hat{n}_x\rangle^2}{\langle n_x\rangle} - 1, \tag{6}$$

where $\hat{n}_x = \hat{x}^\dagger\hat{x}$ is the photon number operator of the mode x.

- Squeezing [9]

$$s_{1x} = 4\langle(\Delta\hat{X}_x)^2\rangle - 1 \quad , \quad s_{2x} = 4\langle(\Delta\hat{Y}_x)^2\rangle - 1 \tag{7}$$

where

$$\langle(\Delta\hat{X}_x)^2\rangle = \langle\hat{X}_x^2\rangle - \langle\hat{X}_x\rangle^2 \quad , \quad \langle(\Delta\hat{Y}_x)^2\rangle = \langle\hat{Y}_x^2\rangle - \langle\hat{Y}_x\rangle^2$$

$$\hat{X}_x = \frac{\hat{x}e^{(i\phi_x)} + \hat{x}^\dagger e^{(-i\phi_x)}}{2} \quad , \quad \hat{Y}_x = \frac{\hat{x}e^{(i\phi_x)} - \hat{x}^\dagger e^{(-i\phi_x)}}{2i}.$$

- Entanglement [10,11]

$$S_x^{corr} = 1 - Tr_x\{\hat{\rho}_x^2\}, \tag{5}$$

where $\hat{\rho}_x = Tr_{y\neq x}\hat{\rho}$ is the reduced density matrix of the particular mode. For a pure state of the xth mode S_x^{corr} equals zero. The entanglement indicates, how strongly the modes are coupled to each other.

So what happens in the case (4) of equally excited signal and pump? Let us comment on this in the context of quasiperiod energy exchange between the modes. Due to the empty idler the process starts with difference-frequency generation. In the initial moments the idler mode is build up and the modes became significantly entangled. Both signal and idler become superpoissonian (the photon number distribution broader than that the initial coherent state). During the depletion the pump becomes noticeable

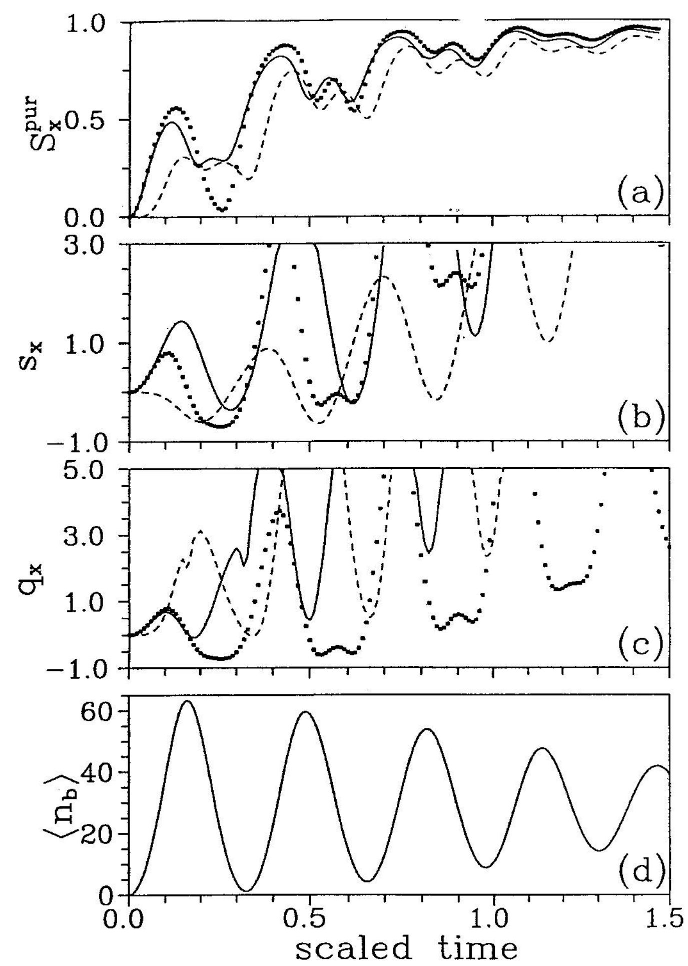

Fig.1. The time evolution of the entanglement (a), the squeezing parameter (b), Mandel's q_x parameter and the idler mean photon number for the initial state $|9\rangle_a|5\rangle|_b|0\gamma_c$. The pump mode is shown with a full line, the idler with a dashed and the signal with the line with squares.

squeezed while the other two modes are antisqueezed. The interesting effects appears after the pump is almost fully depleted. Due to the zero passage of the pump amplitude the process is reversed and *sum-frequency generation starts*. The pump mode is build up and the considerable non-classical properties in the signal mode appear. The signal becomes strongly squeezed and highly subpoissonian [7]. In addition, it becomes gradually decoupled from the other two modes. So we conclude that the intermediate regime can lead to the generation of a pure, highly subpoissonian state of light. Let us

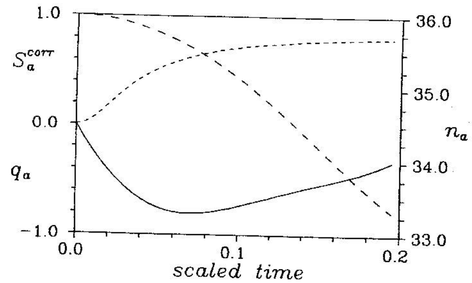

Fig.2. The time evolution of the signal mode entanglement (short dashed line), q_a parameter (full line) and mean photon number (long dashed line) for the initial state $|6_\epsilon\rangle_a|3\rangle_b|4\rangle_c$ (with $\epsilon = -0.052$). The subpoissonian character of the signal is build up right from the beginning.

also note that the supplementary condition $\alpha \approx \gamma$ indicates the region with the best (optimalized) nonclassical signal for the initial state (4).

One of the basic difficulties with the described effect is, that it appears *after* the first maximum in the energy exchange, i.e., on a long time scale at least from the experimental point of view. So we ask, whether it is possible to make such an arrangement of the initial conditions that the strong subpoissonian character of the signal appears already on a short time scale or in other words before the first maximum of the energy exchange is reached.

The simplest and most straightforward candidate for this purpose is to start with sum-frequency generation from the beginning. So let us look on the initial state of the form:

$$|\alpha\rangle_a|\beta\rangle_b|0\rangle_c \qquad |\alpha| \approx 2|\beta|. \tag{8}$$

The results are summarized in Fig.1. The process starts with sum-frequency generation, however none of thé modes exhibits nonclassical behaviour. Only after the first quasioscillation when *the sum-frequency starts again* we observe a similar effect like in the case of difference-frequency generation. The signal becomes highly subpoissonian and squeezed. However, the mode does not reach a pure state even when there is a clear reverse in the degree of entanglement. The comparison with the case (4) indicates that for the appearance of the spontaneous disentanglement effects the initial phases of evolution play an important role and we cannot neglect the initially formed entanglement between the modes.

In our particular case when we would like to reproduce the strong subpoissonian effect on a much shorter time scale we should feed the system initially with a state exhibiting definite correlations among the modes. It is clear, that such an initial state is quite difficult to realize. In addition we have to treat the case with all the modes initially excited, as the entanglement of the modes (apart the disentanglement moment) does not drop down to zero. Because of the difficulties to feed the system with a specially entangled initial state we looked at the possibility to obtain the observed subpoissonian character using three initially excited modes. The appearance of enlarged

phase fluctuations in the intermediate regime indicates also, that a pure coherent state ansatz will not lead to the desired effect. Because of this we have chosen as a good candidate the initial state of the form:

$$|\alpha_\epsilon\rangle_a|\beta\rangle_b|\gamma\rangle_c, \tag{9}$$

where

$$|\alpha_\epsilon\rangle_a = \exp(-i\epsilon\hat{a}^{\dagger 2}\hat{a}^2)|\alpha\rangle_a \tag{10}$$

is a Kerr state that has similarly enhanced phase fluctuations [12,13].

To see already in the initial moments a strong tendency towards subpoissonian signal state it turned out to be necessary to fix the phases of the modes in such a way, that the (coherent) energy exchange between the modes vanishes in the linear approximation. Indeed, for the phase choice:

$$\varphi_a + \varphi_b - \varphi_c - |\alpha|^2\sin(\epsilon) = 0, \tag{11}$$

the mean photon number of the signal mode reads:

$$\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = |\alpha|^2 + \kappa^2t^2[|\alpha|^2(|\gamma|^2 - |\beta|^2) + |\gamma|^2(|\beta|^2+1)] + O(\kappa^3t^3). \tag{12}$$

The coherent (phase dependent) interaction between the modes is switched off and the time dependent change of the photon number starts with the quadratic term. The evolution of the photon number variance behaves differently. The variance for the phase choice (11) reads:

$$\langle[\hat{a}^\dagger(t)\hat{a}(t)]^2\rangle - \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle^2 = |\alpha|^2 + \kappa t|\alpha|^3|\beta||\gamma|\sin(\epsilon)$$
$$2\kappa^2t^2|\gamma|^2|\alpha|^2|\beta|^2(1-\exp(-|\alpha|^2(1-\cos(2\epsilon)))\cos(\epsilon))+$$
$$\kappa^2t^2[|\alpha|^2(|\gamma|^2-|\beta|^2) + |\gamma|^2(|\beta|^2+1) + 2|\alpha|^2|\gamma|^2] + O(\kappa^3t^3). \tag{13}$$

For the possible choice of $\epsilon < 0$ the variance becomes smaller than the mean photon number and the short time expansion indicates a strong tendency toward subpoissonian statistics. To confirm this prediction we calculates the three wave interaction with the initial state (9) numerically. The results are summarized in Fig.2. We see that during the first moment of the interaction the signal mode becomes strongly subpoissonian. In fact the effect is even stronger than that in the intermediate regime (4). However, there is also a price to pay for the shortening of the time scale. The purity of the mode indicates a mixture, the signal rapidly departs from a pure state. The mode also does not exhibit quadrature squeezing.

We have shown, that for a special choice of initial states the three wave interaction offers a possibility to generate highly subpoissonian light. For this we used a combined system where one the modes (signal) was initially prepared in a Kerr state. Even though the Kerr state can be by itself considered as a nonclassical state we do not feed into the system any subpoissonian light or quadrature squeezing. The observed effect has to be attributed to a great extend to the internal dynamics of three wave mixing.

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