acta physica slovaca vol. 45 No. 3, 279 – 288 June 1995

QUANTUM THEORY OF NONLINEAR COUPLERS[1]

J. Peřina[2]
Laboratory of Quantum Optics, Palacký University,
17.listopadu 50, 772 07 Olomouc, Czech Republic

Received 25 April 1995, accepted 10 May 1995

Quantum theory is presented of symmetric and asymmetric codirectional and contradirectional nonlinear couplers composed of linear waveguides and a nonlinear waveguide operating by the second harmonic generation. Two methods are adopted to solve the Heisenberg equations of motion: (i) a power solution is employed including the use of a computer symbolic method giving the operator solutions up to the twelfth order, (ii) linearization of nonlinear operator equations is performed assuming a strong classical pumping in the second harmonic beam. In this way solutions can be found for the field operators and quantum statistical characteristics, such as photocount distribution, its factorial moments, quadrature and integrated intensity variances, principal squeezing variances, correlations of fluctuations, etc. Incident vacuum, coherent and squeezed states and their superposition with external noise are considered. Regimes for generation and transmission of squeezed and/or sub-Poissonian light are found.

1. Introduction

In this paper we discuss the asymmetric and symmetric couplers composed of linear and nonlinear waveguides from the point of view of quantum statistical properties of optical beams adopting the Heisenberg equations. The nonlinear waveguide is assumed to operate by the second harmonic generation. Both the forward and backward arrangements for quantum propagation are considered. We use two approximations to be able to obtain complete quantum statistics of single and compound modes: (i) short-length approximation explicitly specified up to the second order in the interaction length, and using symbolic computations, we can obtain the quantum statistical quantities up to the twelfth order by iterations, (ii) parametric approximation in the second harmonic mode based on the assumption of stimulating strong coherent field in this mode, which makes it possible to linearize the problem. The particular attention is paid to a selfconsistent

[1]Presented at the **3rd central-european workshop on quantum optics**, Budmerice castle, Slovakia, 28 April - 1 May, 1995
[2]E-mail: perina@risc.upol.cz

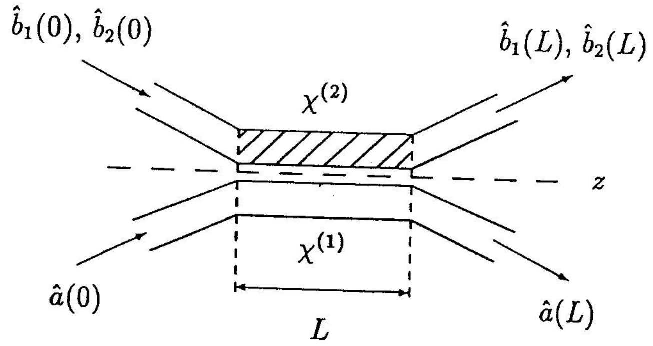

Fig. 1. Scheme of quantum nonlinear asymmetric coupler formed from linear and nonlinear waveguides described by susceptibilities $\chi^{(1)}$ and $\chi^{(2)}$, respectively. The beams are described by the photon annihilation operators as indicated; L is the interaction length.

quantum description of contrapropagating optical beams. In this way we are able to determine all quantum statistical characteristics, including photon number (photocount) distribution, its factorial moments, quadrature and principal squeezing variances, integrated intensity variances and correlations of fluctuations. Incident beams are assumed in various statistical states, including vacuum state, coherent and squeezed states and their superpositions with external noise.

2. Equations of motion and their solution

The asymmetric coupler is assumed to be composed of a linear waveguide and a nonlinear waveguide operating by second harmonic generation (Fig. 1). Thus the propagation of optical beams can be described in the interaction picture with the help of the momentum operator

$$\hat{G}_{\rm int} = -\hbar\kappa\hat{a}\hat{b}_1^\dagger - \hbar\Gamma\hat{b}_1^2\hat{b}_2^\dagger + {\rm h.c.}, \tag{1}$$

where $\hat{a}$, $\hat{b}_1$, $\hat{b}_2$ are annihilation operators of the linear, fundamental and second harmonic modes, κ is a linear coupling constant and Γ is a nonlinear coupling constant proportional to the second order susceptibility. Frequencies of the linear and fundamental modes are ω and the frequency of the second harmonic mode is 2ω. Phase matching is assumed, i.e. $\Delta k = |k_2 - 2k_1| = 0$ holds for the corresponding wavevectors k_1 and k_2 along the z-axis of the fundamental and second harmonic waves. The wavevector of the linear mode a is equal to k_1 or $-k_1$ with respect to forward or backward propagation. If the linear mode a is backward propagating, then we substitute $\hat{a} \leftrightarrow \hat{a}^\dagger$ in (1) and in the corresponding equations of motion to have a quantum consistent treatment; these operators are returned to their original positions at the final form of the solution [1]. This procedure is equivalent to the change of the sign at derivatives in the equations of motion of the contrapropagating beams, in agreement with the traditional classical approach. Further details can be found in [2].

In the symmetric coupler we assume that the nonlinear waveguide is connecting two linear waveguides, so that the system is described by the momentum operator

$$\hat{G}_{\rm int} = -\hbar g\hat{a}\hat{c}_1^\dagger - \hbar\kappa\hat{b}\hat{c}_2^\dagger - \hbar\Gamma\hat{c}_1^2\hat{c}_2^\dagger + {\rm h.c.}, \tag{2}$$

where $\hat{a}$ and $\hat{b}$ are the annihilation operators of the linear waveguides, $\hat{c}_1$ and $\hat{c}_2$ are the annihilation operators of the fundamental and the second harmonic beams, g and κ are linear coupling constants and Γ is a nonlinear coupling constant. In the contrapropagating case we assume that the linear modes a and b are backward propagating.

The corresponding equations of motion are obtained for the asymmetric coupler in the form

$$\begin{aligned} \frac{d\hat{a}}{dz} &= -\epsilon i\kappa^*\hat{b}_1, \\ \frac{d\hat{b}_1}{dz} &= -i\kappa\hat{a} - 2i\Gamma^*\hat{b}_1^\dagger\hat{b}_2, \\ \frac{d\hat{b}_2}{dz} &= -i\Gamma\hat{b}_1^2, \end{aligned} \tag{3}$$

with the conservation laws $\epsilon\hat{a}^\dagger\hat{a}+\hat{b}_1^\dagger\hat{b}_1+2\hat{b}_2^\dagger\hat{b}_2 = constant$, and for the symmetric coupler in the form

$$\begin{aligned} \frac{d\hat{a}}{dz} &= -\epsilon ig^*\hat{c}_1, \\ \frac{d\hat{b}}{dz} &= -\epsilon i\kappa^*\hat{c}_2, \\ \frac{d\hat{c}_1}{dz} &= -ig\hat{a} - 2i\Gamma^*\hat{c}_1^\dagger\hat{c}_2, \\ \frac{d\hat{c}_2}{dz} &= -i\kappa\hat{b} - i\Gamma\hat{c}_1^2, \end{aligned} \tag{4}$$

with the conservation laws $\epsilon\hat{a}^\dagger\hat{a} + \epsilon 2\hat{b}^\dagger\hat{b} + \hat{c}_1^\dagger\hat{c}_1 + 2\hat{c}_2^\dagger\hat{c}_2 = constant$, where $\epsilon = 1$ for forward propagating beams and $\epsilon = -1$ for backward propagating beams a and b.

These systems of equations can be explicitly solved up to z^2 [3, 4]. Denoting the length of the coupler L, we can write, for example for the contradirectional asymmetric coupler, for the second order solutions

$$\begin{aligned} \hat{a}(0) &= \hat{a}(L)(1 - |\kappa|^2L^2/2) - i\kappa^*L\hat{b}_1(0) - \kappa^*\Gamma^*L^2\hat{b}_1^\dagger(0)\hat{b}_2(0), \\ \hat{b}_1(L) &= \hat{b}_1(0)(1 - |\kappa|^2L^2/2) - i\kappa L\hat{a}(L) - 2i\Gamma^*L\hat{b}_1^\dagger(0)\hat{b}_2(0) \\ &\quad - |\Gamma|^2L^2\hat{b}_1^\dagger(0)\hat{b}_1^2(0) + 2|\Gamma|^2L^2\hat{b}_1(0)\hat{b}_2^\dagger(0)\hat{b}_2(0) + \Gamma^*\kappa^*L^2\hat{a}^\dagger(L)\hat{b}_2(0), \\ \hat{b}_2(L) &= \hat{b}_2(0) - i\Gamma L\hat{b}_1^2(0) - |\Gamma|^2L^2(2\hat{b}_1^\dagger(0)\hat{b}_1(0)+1)\hat{b}_2(0) - \Gamma\kappa L^2\hat{a}(L)\hat{b}_1(0). \end{aligned} \tag{5}$$

These solutions are the same as for codirectional coupler, only $\hat{a}(0)$ is replaced by $\hat{a}(L)$, which means that the statistical properties for the codirectional and contradirectional couplers are the same in this approximation. This approximation can be improved up to the twelfth order for the codirectional coupler applying a symbolic recursive procedure to the Heisenberg equations [4]. A closed form solutions of systems (3) and (4) were found [2, 5] provided that the second harmonic mode is stimulated by a strong coherent (classical) field. In this case we put the classical amplitude ξ_2 instead of the operator $\hat{b}_2$ in (3) or instead of the operator $\hat{c}_2$ in (4). Now the solution for the annihilation operators can be expressed in terms of the annihilation and creation operators of the incident beams a and b_1 for the asymmetric coupler; the symmetric coupler can be reduced to the asymmetric one in this case, concentrating our attention to photon statistics. As an example we provide the solution for contradirectional coupler:

$$\begin{aligned} \hat{a}(0) &= U_1(L)\hat{a}(L) + V_1(L)\hat{a}^\dagger(L) + W_1(L)\hat{b}_1(0) + Y_1(L)\hat{b}_1^\dagger(0), \\ \hat{b}_1(L) &= U_2(L)\hat{a}(L) + V_2(L)\hat{a}^\dagger(L) + W_2(L)\hat{b}_1(0) + Y_2(L)\hat{b}_1^\dagger(0), \end{aligned} \tag{6}$$

where

$$\begin{aligned} U_1(L) &= \frac{1}{Det}u_1^*(L), \\ V_1(L) &= -\frac{1}{Det}v_1(L), \\ W_1(L) &= \frac{1}{Det}[v_1(L)y_1^*(L) - u_1^*(L)w_1(L)], \\ Y_1(L) &= \frac{1}{Det}[v_1(L)w_1^*(L) - u_1^*(L)y_1(L)], \\ U_2(L) &= \frac{1}{Det}[u_2(L)u_1^*(L) - v_2(L)v_1^*(L)], \\ V_2(L) &= \frac{1}{Det}[u_1(L)v_2(L) - u_2(L)v_1(L)], \\ W_2(L) &= \frac{1}{Det}\{u_2(L)[v_1(L)y_1^*(L) - u_1^*(L)w_1(L)] \\ &\quad + v_2(L)[v_1^*(L)w_1(L) - u_1(L)y_1^*(L)]\} + w_2(L), \\ Y_2(L) &= \frac{1}{Det}\{u_2(L)[v_1(L)w_1^*(L) - u_1^*(L)y_1(L)] \\ &\quad + v_2(L)[v_1^*(L)y_1(L) - u_1(L)w_1^*(L)]\} + y_2(L), \\ Det &= |u_1(L)|^2 - |v_1(L)|^2, \end{aligned} \tag{7}$$

and

$$\begin{aligned} u_1(z) &= \frac{1}{\lambda_1^2 - \lambda_2^2}[(\lambda_1^2 - |\kappa|^2 - 4|\Gamma|^2|\xi_2|^2)\cosh(\lambda_1 z) \\ &\quad - (\lambda_2^2 - |\kappa|^2 - 4|\Gamma|^2|\xi_2|^2)\cosh(\lambda_2 z)], \\ v_1(z) &= 2i\frac{\kappa^{*2}}{|\kappa|^2}\Gamma^*\xi_2\frac{\lambda_2\sinh(\lambda_1 z) - \lambda_1\sinh(\lambda_2 z)}{\lambda_1^2 - \lambda_2^2}, \\ w_1(z) &= u_2^*(z) = \frac{i\kappa^*}{\lambda_1 + \lambda_2}[\sinh(\lambda_1 z) + \sinh(\lambda_2 z)], \end{aligned}$$

$$\begin{aligned} y_1(z) &= v_2(z) = 2\kappa^*\Gamma^*\xi_2\frac{\cosh(\lambda_1 z) - \cosh(\lambda_2 z)}{\lambda_1^2 - \lambda_2^2}, \\ w_2(z) &= \frac{1}{\lambda_1^2 - \lambda_2^2}[(\lambda_1^2 - |\kappa|^2)\cosh(\lambda_1 z) - (\lambda_2^2 - |\kappa|^2)\cosh(\lambda_2 z)], \\ y_2(z) &= -2i\Gamma^*\xi_2\frac{\lambda_1\sinh(\lambda_1 z) - \lambda_2\sinh(\lambda_2 z)}{\lambda_1^2 - \lambda_2^2}, \end{aligned} \tag{8}$$

and it holds that

$$\begin{aligned} \lambda_{1,2} &= \left[|\kappa|^2 + 2|\Gamma|^2|\xi_2|^2 \pm 2|\Gamma||\xi_2|(|\kappa|^2 + |\Gamma|^2|\xi_2|^2)^{1/2}\right]^{1/2}, \\ \lambda_1\lambda_2 &= |\kappa|^2, \qquad \lambda_1 - \lambda_2 = 2|\Gamma||\xi_2|, \\ \lambda_1 + \lambda_2 &= 2(|\kappa|^2 + |\Gamma|^2|\xi_2|^2)^{1/2}, \\ \lambda_1^2 - \lambda_2^2 &= 4|\Gamma||\xi_2|(|\kappa|^2 + |\Gamma|^2|\xi_2|^2)^{1/2}. \end{aligned} \tag{9}$$

Of course, this solution satisfies the commutation rules

$$\begin{aligned} [\hat{a}(0), \hat{a}^\dagger(0)] = [\hat{b}_1(L), \hat{b}_1^\dagger(L)] &= \hat{1}, \\ [\hat{a}(0), \hat{b}_1(L)] = [\hat{a}(0), \hat{b}_1^\dagger(L)] &= \hat{0}, \end{aligned} \tag{10}$$

and consequently the following identities are fulfilled:

$$\begin{aligned} |U_1(L)|^2 - |V_1(L)|^2 + |W_1(L)|^2 - |Y_1(L)|^2 &= 1, \\ |U_2(L)|^2 - |V_2(L)|^2 + |W_2(L)|^2 - |Y_2(L)|^2 &= 1, \\ U_1(L)V_2(L) - V_1(L)U_2(L) + W_1(L)Y_2(L) - Y_1(L)W_2(L) &= 0, \\ U_1(L)U_2^*(L) - V_1(L)V_2^*(L) + W_1(L)W_2^*(L) - Y_1(L)Y_2^*(L) &= 0. \end{aligned} \tag{11}$$

3. Quantum statistical properties of single and compound beams

To describe quadrature fluctuations of single beams we adopt the principal squeeze variance [6]

$$\lambda_a = 1 + 2[\langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle - |\langle(\Delta\hat{a})^2\rangle|], \tag{12}$$

and similarly for λ_{b_1}, λ_{b_2}, etc., and in the compound modes we have for example

$$\begin{aligned} \lambda_{ab_1} &= 1 + \langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle + \langle\Delta\hat{b}_1^\dagger\Delta\hat{b}_1\rangle + 2\mathrm{Re}\langle\Delta\hat{a}^\dagger\Delta\hat{b}_1\rangle \\ &\quad - |\langle(\Delta\hat{a})^2\rangle + \langle(\Delta\hat{b}_1)^2\rangle + 2\langle\Delta\hat{a}\Delta\hat{b}_1\rangle|. \end{aligned} \tag{13}$$

Then squeezing of vacuum fluctuations occurs if $\lambda < 1$ for the corresponding modes.

Denoting the number operators in single modes as $\hat{n}_a(z) = \hat{a}^\dagger(z)\hat{a}(z)$, etc., the integrated intensity fluctuations are described by

$$\langle(\Delta W_a(z))^2\rangle = \langle\hat{\mathcal{N}}\hat{n}_a^2(z)\rangle - \langle\hat{n}_a(z)\rangle^2, \tag{14}$$

etc., where $\hat{\mathcal{N}}$ is the normal ordering operator. For sub-Poissonian beams these variances are negative, for super-Poissonian beams they are positive and they are zero for

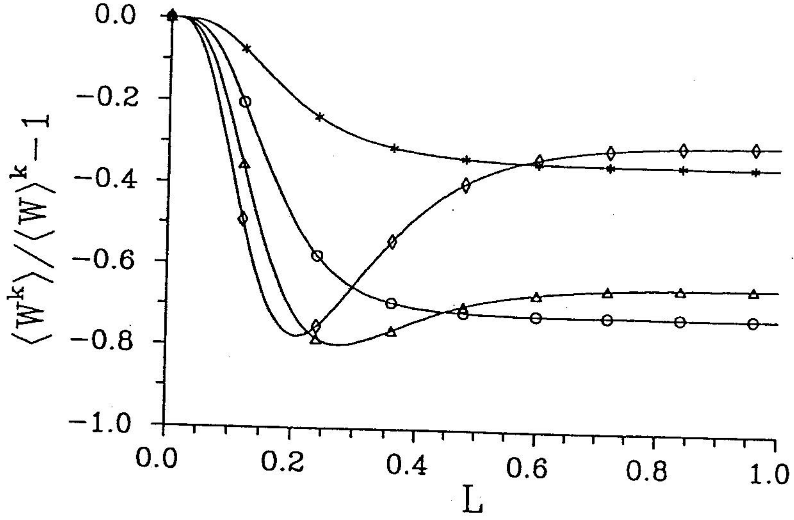

Fig. 2. Reduced factorial moments $\langle W^k\rangle\langle W\rangle^k - 1$ for $k = 2(*)$, $3(\circ)$, $4(\triangle)$ and $5(\diamond)$ for mode a; $\Gamma = 1$, $\kappa = 10$, $\xi_2 = 2$, $\xi = 1$, $\xi_1 = \exp(i\pi/4)$, $r_1 = r_2 = 0$, $\langle n_{\rm ch1}\rangle = \langle n_{\rm ch2}\rangle = 0$.

coherent beams. The correlations of fluctuations are specified by the integrated intensity correlations

$$\langle \Delta W_a(z)\Delta W_{b_1}(z)\rangle = \langle \hat n_a(z)\hat n_{b_1}(z)\rangle - \langle \hat n_a(z)\rangle\langle \hat n_{b_1}(z)\rangle, \qquad (15)$$

etc. In the linearized form the quantum statistical properties of the processes under discussion are derived in terms of the generalized superposition of coherent fields and quantum noise [7] (Sec. 8.5) and the photocount distribution and its factorial moments are expressed in terms of the Laguerre polynomials:

$$\begin{aligned} p(n_j,L) &= (E_jF_j)^{-1/2}\left(1-\frac{1}{F_j}\right)^{n_j}\exp\left(-\frac{A_{1j}}{E_j}-\frac{A_{2j}}{F_j}\right) \\ &\times \sum_{k=0}^{n_j}\frac{1}{\Gamma(k+1/2)\Gamma(n_j-k+1/2)}\left(\frac{1-1/E_j}{1-1/F_j}\right)^k \\ &\times L_k^{-1/2}\left(-\frac{A_{1j}}{E_j(E_j-1)}\right)L_{n_j-k}^{-1/2}\left(-\frac{A_{2j}}{F_j(F_j-1)}\right), \qquad j=1,2, \qquad (16) \end{aligned}$$

$$\begin{aligned} \left\langle \frac{n_j!}{(n_j-k)!}\right\rangle &= \langle W_j^k\rangle = k!(F_j-1)^k \\ &\times \sum_{l=0}^{k}\frac{1}{\Gamma(l+1/2)\Gamma(k-l+1/2)}\left(\frac{E_j-1}{F_j-1}\right)^l \\ &\times L_l^{-1/2}\left(-\frac{A_{1j}}{E_j-1}\right)L_{k-l}^{-1/2}\left(-\frac{A_{2j}}{F_j-1}\right), \qquad j=1,2, \qquad (17) \end{aligned}$$

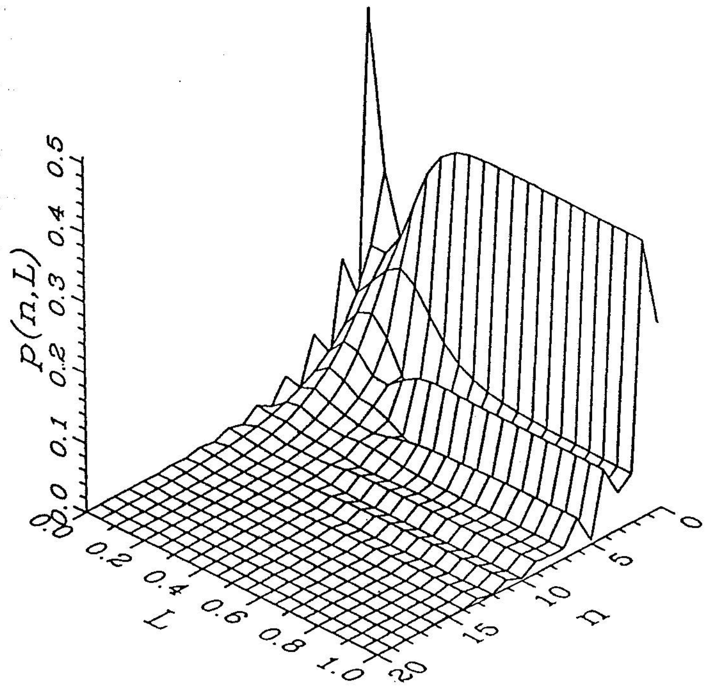

Fig. 3. Photon number distribution $p(n,L)$ for mode a; $\Gamma = 1$, $\kappa = 10$, $\xi_2 = 2$, $\xi = 1$, $\xi_1 = \exp(i\pi/4)$, $r_1 = r_2 = 1$, $\langle n_{\rm ch1}\rangle = \langle n_{\rm ch2}\rangle = 0$.

where Γ is the gamma function, $L_k^{-1/2}$ is the Laguerre polynomial, and

$$\begin{aligned} E_j &= \mathcal{B}_j - |\mathcal{C}_j| + 1, \qquad F_j = \mathcal{B}_j + |\mathcal{C}_j| + 1, \\ A_{1,2j} &= \frac{1}{2}\left[|\zeta_j|^2 \mp \frac{1}{2|\mathcal{C}_j|}(\zeta_j^2\mathcal{C}_j^* + \text{c.c.})\right], \qquad j=1,2 \qquad (18) \end{aligned}$$

are the chaotic and coherent components, respectively.

The spatial development of the beams is described by the normal characteristic function

$$\begin{aligned} C_{\mathcal{N}}(\beta_1,\beta_2) &= \mathrm{Tr}\{\hat\rho\exp[\beta_1\hat a^\dagger(0)+\beta_2\hat b_1^\dagger(L)]\exp[-\beta_1^*\hat a(0)-\beta_2^*\hat b_1(L)]\} \\ &= \exp\left\{\sum_{j=1}^{2}\left[-\mathcal{B}_j|\beta_j|^2 + \left(\frac{1}{2}\mathcal{C}_j\beta_j^{*2} + \text{c.c.}\right)\right]\right. \end{aligned}$$

$$+(\mathcal{D}\beta_1^*\beta_2^* + \bar{\mathcal{D}}\beta_1\beta_2^* + \text{c.c.}) + \sum_{j=1}^{2}(\beta_j\zeta_j^* - c.c.)\Big\}, \quad (19)$$

where c.c. denotes the complex conjugate terms, $\hat{\rho}$ is the statistical operator of the incident beams, and the complex amplitudes of the developing fields are

$$\begin{aligned}\zeta_1 &\equiv \xi(0) = U_1(L)\xi(L) + V_1(L)\xi^*(L) + W_1(L)\xi_1(0) + Y_1(L)\xi_1^*(0),\\ \zeta_2 &\equiv \xi_1(L) = U_2(L)\xi(L) + V_2(L)\xi^*(L) + W_2(L)\xi_1(0) + Y_2(L)\xi_1^*(0);\end{aligned} \quad (20)$$

here $\xi(L)$ and $\xi_1(0)$ are complex amplitudes of the incident beams corresponding to the operators $\hat{a}(L)$ and $\hat{b}_1(0)$, and the quantum noise functions $\mathcal{B}_j$, $\mathcal{C}_j$, $\mathcal{D}$ and $\bar{\mathcal{D}}$ are obtained with the help of the following expressions

$$\begin{aligned}\mathcal{B}_1 &= \langle\Delta\hat{a}^\dagger(0)\Delta\hat{a}(0)\rangle = [|U_1(L)|^2 + |V_1(L)|^2]B_1\\ &\quad + [|W_1(L)|^2 + |Y_1(L)|^2]B_2 + [U_1^*(L)V_1(L)C_1^* + W_1^*(L)Y_1(L)C_2^* + \text{c.c.}]\\ &\quad - |U_1(L)|^2 - |W_1(L)|^2,\\ \mathcal{B}_2 &= \langle\Delta\hat{b}_1^\dagger(L)\Delta\hat{b}_1(L)\rangle = [|U_2(L)|^2 + |V_2(L)|^2]B_1\\ &\quad + [|W_2(L)|^2 + |Y_2(L)|^2]B_2 + [U_2^*(L)V_2(L)C_1^* + W_2^*(L)Y_2(L)C_2^* + \text{c.c.}]\\ &\quad - |U_2(L)|^2 - |W_2(L)|^2,\\ \mathcal{C}_1 &= \langle(\Delta\hat{a}(0))^2\rangle = U_1^2(L)C_1 + V_1^2(L)C_1^* + W_1^2(L)C_2 + Y_1^2(L)C_2^*\\ &\quad + 2U_1(L)V_1(L)B_1 + 2W_1(L)Y_1(L)B_2 - U_1(L)V_1(L) - W_1(L)Y_1(L),\\ \mathcal{C}_2 &= \langle(\Delta\hat{b}_1(L))^2\rangle = U_2^2(L)C_1 + V_2^2(L)C_1^* + W_2^2(L)C_2 + Y_2^2(L)C_2^*\\ &\quad + 2U_2(L)V_2(L)B_1 + 2W_2(L)Y_2(L)B_2 - U_2(L)V_2(L) - W_2(L)Y_2(L),\\ \mathcal{D} &= \langle\Delta\hat{a}(0)\Delta\hat{b}_1(L)\rangle = [U_1(L)V_2(L) + V_1(L)U_2(L)]B_1\\ &\quad + [W_1(L)Y_2(L) + Y_1(L)W_2(L)]B_2 + U_1(L)U_2(L)C_1 + V_1(L)V_2(L)C_1^*\\ &\quad + W_1(L)W_2(L)C_2 + Y_1(L)Y_2(L)C_2^* - U_2(L)V_1(L) - Y_1(L)W_2(L),\\ \bar{\mathcal{D}} &= -\langle\Delta\hat{a}^\dagger(0)\Delta\hat{b}_1(L)\rangle = -[U_1^*(L)U_2(L) + V_1^*(L)V_2(L)]B_1\\ &\quad - [W_1^*(L)W_2(L) + Y_1^*(L)Y_2(L)]B_2 - V_1^*(L)U_2(L)C_1 - U_1^*(L)V_2(L)C_1^*\\ &\quad - Y_1^*(L)W_2(L)C_2 - W_1^*(L)Y_2(L)C_2^* + U_1^*(L)U_2(L) + W_1^*(L)W_2(L),\end{aligned} \quad (21)$$

where $B_1 \equiv B_1(L)$, $B_2 \equiv B_2(0)$ are the corresponding quantities of the incident beams related to the antinormal ordering, which enables us to describe nonclassical states of the incident beams, and $C_1 \equiv C_1(L)$, $C_2 \equiv C_2(0)$; these quantities are expressed under the condition of independence of the incident beams as

$$\begin{aligned}\langle\Delta\hat{a}(L)\Delta\hat{a}^\dagger(L)\rangle &= \cosh^2 r_a + \langle n_{cha}\rangle,\\ \langle(\Delta\hat{a}(L))^2\rangle &= \frac{1}{2}\exp(i\theta_a)\sinh(2r_a),\end{aligned} \quad (22)$$

where r_a is squeeze parameter, θ_a is squeeze phase and $\langle n_{cha}\rangle$ is mean photon number of external noise of the incident field in mode a; we define these quantities similarly for

the other beams. In this way incident coherent beams (all $r = \langle n_{ch}\rangle = 0$), squeezed beams (all $\langle n_{ch}\rangle = 0$) and their superpositions with external noise can be described. More details can be found in [5].

4. Discussion

Based on quadrature fluctuations we have obtained for the asymmetric nonlinear coupler that the signal and second harmonic beams remain coherent up to the second iteration, whereas the subharmonic beam changes its statistical properties during propagation as a consequence of self-interaction. Particularly, it can exhibit squeezing of vacuum fluctuations in stimulated or partially spontaneous regime. Similar results can be obtained for the compound modes. The signal–second harmonic mode is again coherent up to the second iteration, whereas the signal–fundamental and fundamental–second harmonic modes can be squeezed in this approximation in the stimulated or partially spontaneous regime. In the spontaneous process coherence is conserved in this approximation. Based on the fourth-order moments similar conclusions can be obtained. The signal and the second harmonic beams remain coherent up to the second iteration, whereas the fundamental mode can exhibit phase dependent photon antibunching in the stimulated process and phase independent antibunching in partially spontaneous process. The signal and second harmonic beams are uncorrelated, whereas the fundamental and second harmonic beams are always anticorrelated in this approximation. The signal and fundamental beams can be phase correlated or anticorrelated as a consequence of the coupling of linear and nonlinear waveguides. In the spontaneous process coherence is conserved in all single and compound modes.

In the symmetric coupler squeezing of vacuum fluctuations can be exhibited by the fundamental mode of the nonlinear waveguide, including the spontaneous process in which the complex amplitudes of the incident fundamental and second harmonic beams are zero, as a consequence of the coupling of linear and nonlinear waveguides. The other modes are coherent. Also some compound modes can exhibit squeezing of vacuum fluctuations, particularly this holds for the modes of linear waveguides combined with the fundamental mode of the nonlinear waveguide and for the combined mode in the nonlinear waveguide (fundamental and second harmonic beams). This is also valid if the fields incident on the nonlinear waveguide are zero. All the other compound modes remain coherent. Sub-Poissonian statistics can be observed in the fundamental mode of the nonlinear waveguide, including phase dependent effects in the stimulated process and a phase-independent effect in partially spontaneous process. All the other modes are Poissonian. The signal mode of the linear waveguide coupled to the fundamental mode of the nonlinear waveguide and this fundamental mode can be anticorrelated depending on phases of the incident beams. The modes of the nonlinear waveguide are always anticorrelated. All the other modes are uncorrelated in this approximation.

The symbolic computations performed up to z^{10} for the symmetric codirectional coupler and up to z^{12} for the asymmetric codirectional coupler confirmed the analytical results and provided additional information about quantum spatial behaviour of light beams, including regimes of squeezing of vacuum fluctuations, anticorrelation of integrated intensity fluctuations and sub-Poissonian photon statistics [4].

In the linearized case based on strong stimulating coherent wave in the second harmonic mode, the discussion is to be divided to several cases because for codirectional coupler and for $|\Gamma\xi_2| > |\kappa|$ the roots of the characteristic polynomial are real, whereas for $|\Gamma\xi_2| < |\kappa|$ they are complex. For the contradirectional coupler they are always real. We have examined effects produced by the nonlinear dynamics of the process as well as by the nonclassical behaviour of the incident beams. We have demonstrated that the nonclassical properties, such as squeezing of vacuum fluctuations, sub-Poissonian photon statistics and oscillations in photocount distribution created in the nonlinear waveguide by the nonlinear dynamics can be transferred to the signal mode of the linear waveguide (Fig. 2 illustrates the spatial development of the reduced factorial moments for contradirectional asymmetric coupler exhibiting sub-Poissonian behaviour). Under suitable initial phase conditions this transfer is stronger when the linear coupling constant is increasing and it is more effective for contrapropagating beams. In codirectional coupler oscillations in the photocount distribution having quantum origin can interfere with oscillations arising if linear coupling prevails so that the roots of the characteristic polynomial are complex. The quantum statistical features are also exhibited in the compound signal and fundamental mode. Although the quantum uncertainty product is generally increasing along the way of propagation, in some cases the uncertainty product in single modes can periodically return to reduced values and the corresponding mode can return to a state very close to a pure state. This is especially actual for the contradirectional coupler. In general, additional external noise rules out any nonclassical behaviour of light beams. However, in some cases the quantum properties of the incident beams are fastly smoothed out, whereas the quantum properties arising from the nonlinear dynamics can servive in a reduced form (Fig. 3 demonstrates a development of quantum oscillations in the photon distribution for contradirectional asymmetric coupler, arising from the initial squeezed state and from nonlinear dynamics). Phase mismatch effectively reduces the power of interaction and supports conservation of the initial photon statistics.

Acknowledgements We acknowledge the support by the grant No 202/94/0789 of the Czech Grant Agency.

References

[1] M. Toren, Y. Ben-Aryeh: *Quantum Opt.* **6** (1994) 425;

[2] J. Peřina, J. Peřina, Jr.: *Quant. Semiclass. Opt.* (1995) submitted;

[3] J. Peřina: *J. Mod. Opt.* **42** (1995) in print;

[4] J. Peřina, J. Bajer: *J. Mod. Opt.* (1995) submitted;

[5] J. Peřina, J. Peřina, Jr.: *Quant. Semiclass. Opt.* **7** (1995) in print;

[6] A. Lukš, V. Peřinová, J. Peřina: *Opt. Commun.* **67** (1988) 149;

[7] J. Peřina: *Quantum Statistics of Linear and Nonlinear Optical Phenomena*, 2^{th} edition (Kluwer, Dordrecht–Boston, 1991);