acta physica slovaca vol. 45 No. 3, 253 – 260 June 1995

MUTUAL INFLUENCE OF TWO $\chi^{(2)}$-NONLINEARITIES AND THE GENERATION OF SUB-POISSONIAN TWIN BEAMS[1]

M.A.M. Marte[2]
Institut für Theoretische Physik, Universität Innsbruck
Technikerstr. 25, A-6020 Innsbruck, Austria

Received 29 April 1995, accepted 10 May 1995

The mutual influence of two $\chi^{(2)}$-nonlinearities in a cavity is investigated: the 'competition' of doubly–resonant second harmonic generation and nondegenerate optical parametric oscillation. This leads to novel squeezing effects such as twin beams with sub-Poissonian photon statistics in the difference *and* the sum of the two beams.

1. The Model

$\chi^{(2)}$-nonlinearities have proven a very effective tool for the generation of nonclassical 'squeezed' light [1] - [7]. Recently, also the generation of very large effective third order nonlinearities with phase mismatch from cascades of second order nonlinearities has attracted a lot of attention and has already been realized experimentally in KTP and organic crystals [8].

In the proposed system the interaction of two $\chi^{(2)}$ nonlinearities in one, or possibly two, crystal(s) inside a cavity – or a crystal representing the cavity itself, such as a monolithic optical internal reflection resonator [9] – is investigated: two photons of frequency ω_1 in the fundamental mode a_1, which is resonantly driven by a classical field $\mathcal{E}_1(t) = \mathcal{E}_1 e^{-i\omega_1 t}$, can be annihilated to form a photon of frequency $\omega_2 = 2\omega_1$ in mode a_2(second harmonic generation, SHG); this photon with frequency ω_2 can split into an pair of unequal photons $\omega_+ = \omega_1 + \Delta$ and $\omega_- = \omega_1 - \Delta$(nondegenerate optical parametric oscillation, NDOPO)– and vice versa. We adopt the model Hamiltonian

$$\begin{aligned} H = \quad & \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar\omega_+ a_+^\dagger a_+ + \hbar\omega_- a_-^\dagger a_- \\ & + i\hbar\frac{\kappa}{2}\left(a_1^{\dagger 2} a_2 - a_1^2 a_2^\dagger\right) + i\hbar\chi\left(a_2 a_+^\dagger a_-^\dagger - a_2^\dagger a_+ a_-\right) \\ & + i\hbar\left(\mathcal{E}_1(t)\, a_1^\dagger - \mathcal{E}_1^*(t)\, a_1\right) \end{aligned} \qquad (1)$$

[1] Presented at **3rd central-european workshop on quantum optics**, Budmerice castle, Slovakia, 28 April - 1 May, 1995

[2] E-mail address: marte@bozon.uibk.ac.at

to describe these processes. The frequency offset Δ between the OPO-modes and the fundamental ω_1 is not a free parameter: among all the cavity resonances usually only the pair of sideband modes satisfying the phase matching condition that has the lowest ratio of losses to coupling constant the will oscillate; all others and higher harmonics will not contribute significantly and thus do not have to be considered.

One motivation for studying the combination of these two individually very successful 'squeezers' is the idea that each nonlinearity can be understood as a nonclassical 'pump'–mechanism driving the other and thus the wealth of squeezing effects (including quadrature phase squeezing as well as sub-Poissonian photon statistics) that can be anticipated. It will be shown that the 'competing nonlinearities' system is a source for output beams with the following useful properties:

- *resonant above threshold oscillation* $\Longrightarrow$ *large brightness of the source*
- *no external regularization or feedback of any kind needed*
- *twin beams: two output beams of correlated photon pairs*
- *in each individual beam reduction below shot noise (even* $< 50\%$)
- *the sum of the twin beams can drop below shot noise*
- *up to* 100% *squeezing in very high frequency components.*

In a first approach one may neglect quantum effects and solve the 'semiclassical' equations of motion for the damped slowly varying amplitudes α_1 and α_2 of the fundamental and the second harmonic, and $\alpha_\pm$ for the OPO modes [10]. This, however, is not a good choice of variables because of the phase diffusion that is intrinsic to parametric oscillation: the phases $\phi_\pm$ of $\alpha_\pm$ are in neutral equilibrium (the corresponding eigenvalue is zero). Thus we transform the equations of motion to the OPO-intensities $I_+ = \alpha_+^*\alpha_+$ and $I_- = \alpha_-^*\alpha_-$ and the product $U = \alpha_+\alpha_- \propto e^{i(\phi_+ + \phi_-)}$:

$$\begin{aligned}
\dot\alpha_1 &= -\gamma_1 \alpha_1 + \kappa\, \alpha_1^*\, \alpha_2 + \mathcal{E}_1 \\
\dot\alpha_2 &= -\gamma_2\, \alpha_2 - \frac{\kappa}{2}\, \alpha_1^2 - \chi\, U \\
\dot U &= -(\gamma_+ + \gamma_-)\, U + \chi\, \alpha_2\, (I_+ + I_-) \\
\dot I_\pm &= -2\gamma_\pm\, I_\pm + \chi\, (\alpha_2 U^* + \alpha_2^* U)\,.
\end{aligned} \tag{2}$$

It is an easy task to verify that

$$\begin{aligned}
\alpha_1^0 &= \frac{\mathcal{E}_1}{\gamma_1\left(1 + r\,\frac{\bar\gamma}{\gamma_1}\right)}\,; \qquad \alpha_2^0 = -\,\frac{\bar\gamma}{\chi} \\
U^0 &= -\,\frac{1}{2}\, r\, (\alpha_1^0)^2 - \frac{\gamma_2}{\chi}\, \alpha_2^0\,; \qquad I_\pm^0 = \sqrt{\frac{\gamma_\mp}{\gamma_\pm}}\, (-\,U^0)
\end{aligned} \tag{3}$$

represents a stationary solution of Eq.(2). The constant – i.e. pump-field-free – value of the amplitude of the second harmonic in Eq.(3) can be interpreted as an equilibrium

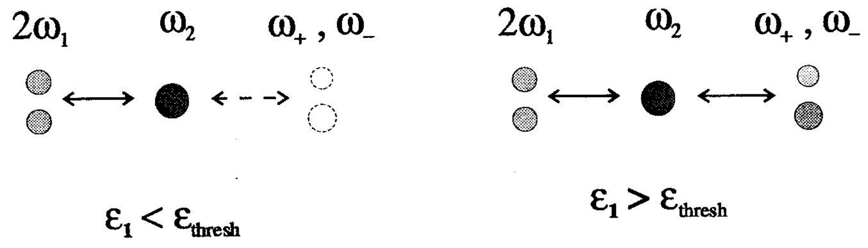


Fig. 1: Steady state equilibrium below and above the threshold for oscillation of all four modes

between the two 'channels' that the second harmonic can 'flow' into above threshold (cf. Fig. 1).

A *sufficient* (but not necessary) condition for the stability of the quadruply oscillating solution is given by

$$e_1 \equiv \mathcal{E}_1/\mathcal{E}_{thresh} > 1 \qquad \text{with} \qquad \mathcal{E}_{thresh} = \frac{\gamma_1}{\chi}\,(1 + r\,\frac{\bar\gamma}{\gamma_1})\,\sqrt{\frac{2\gamma_2\bar\gamma}{r}} \tag{4}$$

$$r \equiv \kappa/\chi < r_{max} \qquad \text{with} \qquad r_{max} = \frac{\gamma_1}{\bar\gamma}\left(1 + \frac{\gamma_2}{\gamma_1}\right), \tag{5}$$

The first condition is needed to make the solution Eq.(3) physically reasonable with $U^0 < 0$ and thus positive intensities $I_\pm^0 > 0$. The second condition ensures that one is in a regime without multistability [10]. For $r < r_{max}$ the threshold $\mathcal{E}_{thresh}$ lies below the threshold value $\mathcal{E}_{SP}^{(\chi=0)}$ of the selfpulsing instability in the SHG system [11] and for $r = r_{max}$ the two thresholds coincide.

2. Squeezing variances

Starting from a Fokker-Planck equation in the same variables as in equation Eq.(3), in the linearized theory the matrix of squeezing spectra is given by [1], [12]

$$S(\omega) = \left[i\omega + A^0\right]^{-1} D^0 \left[-i\omega + (A^0)^T\right]^{-1}, \tag{6}$$

where A^0 and D^0 denote the Jacobian matrix of the drift vector and the diffusion matrix of the Fokker–Planck equation, evaluated at the stable stationary operating point. It is convenient to define squeezing variances in the following way

$$\begin{aligned}
V_{ij}^{coh}(\omega) &= 1 + \sqrt{2\gamma_i\, 2\gamma_j}\; S_{ij}(\omega) \geq 0 \qquad \text{for } i,j \in \{a_1, b_1, a_2, b_2\} \\
V_{i,j}^{incoh}(\omega) &= 1 + \sqrt{\frac{2\gamma_i\, 2\gamma_j}{I_i I_j}}\; S_{ij}(\omega) \geq 0 \qquad \text{for } i,j \in \{I_+, I_-\}
\end{aligned} \tag{7}$$

for the 'coherent' and 'incoherent' case of quadrature phase squeezing and sub-Poissonian photon statistics in the output fields; with these definitions nonclassical effects manifest themselves as a drop below the shot noise level which corresponds to unity in both cases.

Only symbolic–algebra-program-packages make it possible to tackle this problem of dimension 8×8; it was finally possible to bring the general analytical solutions (without adiabatic elimination of any kind) for the squeezing spectra into a form of managable length (with the indices a_i and b_i referring to the amplitude and phase quadratures, respectively):

for the fundamental:

$$V_{\substack{a_1a_1\\b_1b_1}}(\omega) = 1 + 2\gamma_1 S_{\substack{a_1a_1\\b_1b_1}}(\omega) \tag{8}$$

$$= 1 \mp 4\gamma_1|\epsilon_2| \frac{\Omega^2_\pm(\omega)(\omega^2+\gamma_2^2) - 2|\epsilon_0|^2(\omega^2 \pm \gamma_2\bar\gamma)}{\Omega^2_\pm(\omega)\left[\omega^2(\gamma_1+\gamma_2\pm|\epsilon_2|)^2 + N^2_\pm(\omega)\right] + |\epsilon_0|^2 M_\pm(\omega)}$$

for the second harmonic:

$$V_{\substack{a_2a_2\\b_2b_2}}(\omega) = 1 + 2\gamma_2 S_{\substack{a_2a_2\\b_2b_2}}(\omega) \tag{9}$$

$$= 1 \mp 4\gamma_2|\epsilon_2| \frac{\Omega^2_\pm(\omega)|\epsilon_1|^2 - |\epsilon_0|^2\Gamma^2_\pm(\omega)\frac{1}{r}}{\Omega^2_\pm(\omega)\left[\omega^2(\gamma_1+\gamma_2\pm|\epsilon_2|)^2 + N^2_\pm(\omega)\right] + |\epsilon_0|^2 M_\pm(\omega)}$$

with the stationary fields

$$\epsilon_1 \equiv \kappa\alpha_1^0, \quad \epsilon_2 \equiv \kappa\alpha_2^0, \quad \epsilon_0 \equiv \chi\sqrt{I^0} \tag{10}$$

and the definitions

$$\begin{aligned}
\Gamma^2_\pm(\omega) &= \omega^2 + (\gamma_1 \pm |\epsilon_2|)^2\\
\Omega^2_\pm(\omega) &= \omega^2 + 2(1\mp 1)\bar\gamma^2\\
N_\pm(\omega) &= \left[\omega^2 - \gamma_2(\gamma_1 \pm |\epsilon_2|) - |\epsilon_1|^2\right]\\
M_\pm(\omega) &= -2\omega^2\left[\Gamma^2_\pm(\omega) - |\epsilon_1|^2\right] + |\epsilon_0|^2\,\Gamma^2_\pm(\omega)\\
&\quad +2(1\mp 1)\bar\gamma\left[\gamma_2\Gamma^2_-(\omega) + |\epsilon_1|^2\gamma_1 - |\epsilon_2|\right].
\end{aligned}$$

For the intensity fluctuation spectra in the OPO-beams one finds

for the single beams:

$$V_{single}(\omega) = 1 + \frac{2\bar\gamma}{I^0_\pm} S_{I_\pm I_\pm}(\omega) \tag{11}$$

$$= 1 + \frac{2\bar\gamma^2}{\omega^2+4\bar\gamma^2}\,\frac{4\bar\gamma^2 R(\omega) + |\epsilon_0|^2 T_-(\omega) - |\epsilon_0|^4\Gamma^2_\mp(\omega)}{\omega^2 R(\omega) - 2\omega^2|\epsilon_0|^2(\Gamma^2_+(\omega) - |\epsilon_1|^2) + |\epsilon_0|^4\Gamma^2_+(\omega)}$$

for the difference of the intensities in the twin beams:

$$V_{diff}(\omega) = 1 + \frac{2\bar\gamma}{I^0_\pm}\left(S_{I_\pm I_\pm}(\omega) - S_{I_\pm I_\mp}(\omega)\right) = \frac{\omega^2}{\omega^2+4\bar\gamma^2} \tag{12}$$

for the sum of the intensities in the twin beams:

$$V_{sum}(\omega) = 1 + \frac{2\bar\gamma}{I^0_\pm}\left(S_{I_\pm I_\pm}(\omega) + S_{I_\pm I_\mp}(\omega)\right) \tag{13}$$

$$= 1 + 4\bar\gamma^2\,\frac{R(\omega) - r|\epsilon_0|^2\,|\epsilon_1|^2}{\omega^2 R(\omega) - 2\omega^2|\epsilon_0|^2(\Gamma^2_+(\omega) - |\epsilon_1|^2) + |\epsilon_0|^4\Gamma^2_+(\omega)}$$

with the additional definitions

$$\begin{aligned}
R(\omega) &= (\omega^2+\gamma_2^2)\,\Gamma^2_+(\omega) - 2|\epsilon_1|^2 N_+(\omega) - |\epsilon_1|^4\\
T_\pm(\omega) &= 2\omega^2(\Gamma^2_+(\omega) - |\epsilon_1|^2) \pm r|\epsilon_1|^2(\omega^2+4\bar\gamma).
\end{aligned}$$

The specific form of these results has the great advantage that the full analytical solutions of the two subsystems SHG/NDOPO can be immediately found by setting one parameter to zero and then reinterpreting the stationary amplitudes $|\epsilon_i|$:

$$V_{SHG}(|\epsilon_1|,|\epsilon_2|) \overset{|\epsilon_0|\to 0}{\longleftarrow} V_{\substack{SHG\\NDOPO}}(|\epsilon_0|,|\epsilon_1|,|\epsilon_2|) \overset{|\epsilon_1|,|\epsilon_2|\to 0}{\longrightarrow} V_{NDOPO}(|\epsilon_0|)$$

The most interesting features do not occur in the frequency component $\omega = 0$, which makes it difficult to find the optimal parameters: the exact result e.g. for the frequency components with maximum squeezing in the second harmonic is a polynomial of high order. Thus in practice the following approach has proven more useful: in SHG the best squeezing is found near the threshold of the selfpulsing instability. At the instability point $\mathcal{E}_1 = \mathcal{E}_{SP}^{\chi=0}$ one complex conjugate pair of eigenvalues

$$\lambda^{SHG} = \frac{1}{2}\left[-(\gamma_1+\gamma_2) - \epsilon_2 \pm \sqrt{(\epsilon_2+\gamma_1+\gamma_2)^2 + 4(-\gamma_1\gamma_2 - \gamma_2\epsilon_2 - |\epsilon_1|^2)}\right] \tag{14}$$

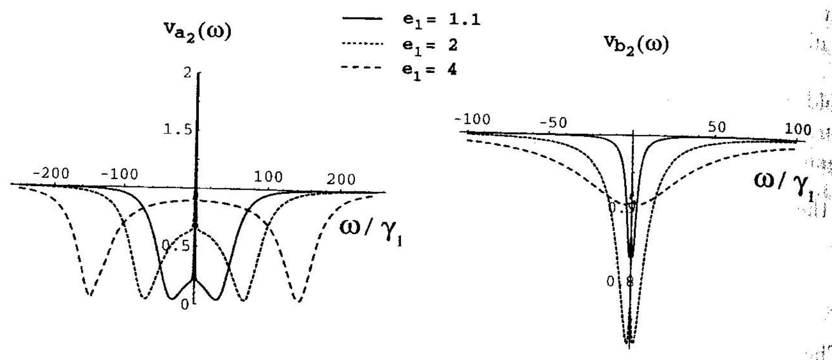
becomes purely imaginary, because at this point $\epsilon_2 = -(\gamma_1+\gamma_2)$, and for $\mathcal{E}_1 \gtrsim \mathcal{E}_{SP}^{\chi=0}$ the imaginary part determines the frequency of the pulsing. Inserting the quadruply oscillating solution for the amplitudes ϵ_1 und ϵ_2 in the above equation, the real part $Re(\lambda^{SHG}) = r\bar\gamma - (\gamma_1+\gamma_2)$ stays *nonzero* for the combined system for $r < r_{max} = (1+\gamma_2/\gamma_1)/s$ and the imaginary part takes on the form

$$\omega_0 = \sqrt{\gamma_1\gamma_2\,(1 - rs + 2e_1rs) - (\gamma_1+\gamma_2-rs\gamma_1)^2/4}\,. \tag{15}$$

For the special case $r = r_{max}$, and thus $\mathcal{E}_{SP}^{\chi=0} = \mathcal{E}_{thresh}$, we get the expression for the selfpulsing frequency at the instability point of SHG, derived in [11]

$$\omega_0 \longrightarrow \omega_{crit}^{SHG} = \sqrt{\gamma_2(2\gamma_1+\gamma_2)} \qquad \text{for} \qquad e_1 \longrightarrow 1\,. \tag{16}$$

ω_0 is a good estimate for the frequency components with the best squeezing – at least for relatively large r and moderate $\mathcal{E}_1$. This means that the same nonlinear effects that



(a) (b)

Fig.2: Amplitude (a) and (b) phase quadrature squeezing in the second harmonic; parameters: (a) $\gamma_2/\gamma_1 = 25$, $\bar{\gamma}/\gamma_1 = 1$, $r = r_{max}$, (b) $\gamma_2/\gamma_1 = 10^3$, $\bar{\gamma}/\gamma_1 = 1$, $r = 0.1$

lead to selfpulsing in SHG lead to phase sensitive amplification or damping of noise, i.e. squeezing; the important difference, however, is that – even for $\mathcal{E}_1 > \mathcal{E}_{SP}^{X=0}$ – the real part is still negative, provided that $r < r_{max}$, i.e. the system does not undergo a real *hard mode instability*!

Some of the most interesting results are summarized below and at the same time contrasted to the results for the independent subsystems (see e.g. [13] for SHG and [14] for NDOPO):

$V_{a_2a_2}$: always super-Poissonian at $\omega = 0$, but $V_{a_2a_2} = \frac{\gamma_2}{\gamma_1+\gamma_2} \to 0$ for $\gamma_2 \gg \gamma_1$ at ω_0 and $r = r_{max}$
Comparison with SHG: also $V_{a_2a_2} \to 0$ for $\gamma_2 \gg \gamma_1$, but at $\omega_{crit}^{SHG} \ll \omega_0$, since $e_1 > 1$ is required

$V_{b_2b_2}$: squeezing possible for $\omega = 0$ for $r < 1/3$
Comparison with SHG: no squeezing in the phase quadrature

V_{diff} : Lorentz-curve of width $\bar{\gamma}$ with 100% squeezing at $\omega = 0$
Comparison with NDOPO: identical result

V_{single} : 50% squeezing at $\omega = 0$ and better than 50% at $\omega \neq 0$
Comparison with NDOPO: also 50% at $\omega = 0$, but large excess noise in side peaks

V_{sum} : squeezing possible at $\omega \neq 0$
Comparison with NDOPO: always super-Poissonian.

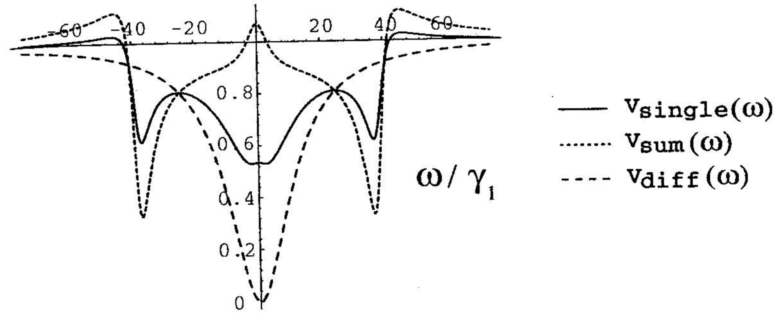


Fig.3: Squeezing spectra for the OPO-beams: single intensity, sum intensity and intensity difference; parameters: $\gamma_2/\gamma_1 = 0.5$, $\bar{\gamma}/\gamma_1 = 6$, $r = 1.525 = 6.1\, r_{max}$

The fact that the best squeezing is found in symmetrical satellites which occur at relatively large frequencies, the separation of which scales with pump strength (and is thus tunable), might be particularly useful for applications requiring fast amplitude modulation at low noise, since the large separation of the two dips is tantamount to having an extremely large *effective* squeezing bandwidth. An example for this effect is shown in Fig. 2a, whereas in Fig. 2b a case with phase quardature squeezing in the second harmonic is depicted.

In the combined system the second harmonic acts as a nonclassical 'mediator' converting amplitude noise reduction in the fundamental into suppression of fluctuations of the sum intensity in the OPO beams below shot noise, at least as far as the peak squeezing at nonzero frequencies is concerned – at the same time being strongly influenced by the OPO beams (cf. Fig. 1). In the ordinary NDOPO the maximum noise reduction in the single beam is limited to 50% at zero frequency and it can be shown analytically that the *sum* intensity of the twin beams is at best at shot noise level. In the combined system the interaction of the twin beams with the two other modes, which are amplitude squeezed, makes it possible to reduce the sum intensity below shot noise: again the two symmetrical dips that are characteristic of the system appear in the spectrum, at about the same location where the single beam intensity fluctuation are also suppressed below shot noise (cf. Fig. 3).

3. Conclusions and outlook

The competing nonlinearities system can be expected to exhibit strong *antibunching* [15] in the sum intensity of the twin beams in the limit of low intensities, and particularly just below threshold; this could naively be interpreted as a sub-Poissonian rate of *pair emissions* of correlated photons, 'smeared out' in time by the finite cavity

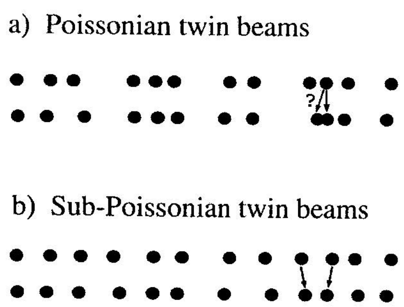


Fig.4: Photodetection of Poissonian and sub-Poissonian twin beams

lifetime (the spectral width of the correlations is limited by $\bar{\gamma}$). Regularization in the emission of pairs helps to avoid overlaps between photons from subsequent pairs and makes the identification of the partner of an individual photon that was registered more reliable, compared to Poissonian twin beams of the same intensity, as indicated in Fig. 4. Besides the investigation of the below threshold behaviour, the inclusion of cavity detunings as well as pump field fluctuations would seem worthwhile pursuing.

Acknowledgements We acknowledge the support by the Österreichischer Fonds zur Förderung der wissenschaftlichen Forschung, Projekt H012-PHY.

References

[1] G.J.Milburn, D.F.Walls: *Quantum Optics*, first edition (Springer, Berlin, 1994);
[2] J.Peřina: *Quantum statistics of linear and nonlinear phenomena*, second edition (Kluwer Academic, Dordrecht, 1991);
[3] R.Paschotta et al.: *Phys. Rev. Lett.* **72** (1994) 3807;
[4] L.A.Wu, H.J.Kimble, J.L.Hall, H.Wu: *Phys. Rev. A* **57** (1986) 2520;
[5] A.Heidmann et al.: *Phys. Rev. Lett.* **59** (1987) 2555;
[6] J.Martinez-Linares, P.Garcia-Fernandes: *J. Mod. Opt.* **41** (1994) 67;
[7] I.Jex, G.Drobný, V.Bužek: *acta physica slovaca* **44** (1994) 155;
[8] G.I.Stegeman et al.: *Opt. Lett* **17** (1992) 28;
[9] S.Schiller, R.L.Byer: *J. Opt. Soc. Am. B* **10** (1993) 1696;
[10] M.A.M.Marte: *Phys. Rev. A* **49** (1993) R3166; and *J. Opt. Soc. Am. B*, submitted;
[11] P.D.Drummond, K.J. McNeil, D.F.Walls: *Optica Acta* **27** (1980) 321;
[12] C.W.Gardiner: *Quantum Noise*, first edition (Springer, Berlin, 1992);
[13] M.J.Collett, D.F.Walls: *Phys. Rev. A* **32** (1985) 2887;
[14] A.S.Lane, M.D.Reid, D.F.Walls: *Phys. Rev. A* **38** (1988) 788;
[15] H.J.Carmichael, D.F.Walls: *J. Phys. B* **9** (1976) L43;