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# QUANTUM STATE ENGINEERING VIA SUPERPOSITION OF SCHRÖDINGER KITTENS[1]

**J. Janszky[2], P. Adam, S. Szabó, P. Domokos**
*Research Laboratory for Crystal Physics P.O. Box 132, H-1502 Budapest, Hungary*

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The possibility to construct quantum states via discrete superpositions of coherent states is discussed. It is shown that even a small number of coherent states can approximate the given quantum states at a high accuracy when the distance of coherent states is optimized. The representation of Fock states $|n\rangle$ by discrete superpositions can be constructed from $n+1$ coherent states lying in the vicinity of the vacuum state.

## 1. Introduction

Recently, much attention has been paid to the problem of generating quantum states of an electromagnetic field mode. In micromaser experiments various schemes have been proposed that allows us to create states with controllable number-state distribution [1, 2]. There are theoretical results presenting that certain quantum states can be arbitrarily well approximated by discrete superpositions of coherent states [3, 4]. The significance of applying a coherent-state expansion instead of the number-state one is to open new prospects in "quantum state engineering". Nonlinear interaction of the field, being initially in a coherent state, with a Kerr-like medium [5] or in degenerate parametric oscillator [6] leads to superpositions of finite number of coherent states. Back-action evading and quantum nondemolition measurements can also yield such superposition states [7, 8]. An atomic interference method has been developed, which can result in arbitrary superposition of coherent states on a circle in phase space [9]. Based on these promising schemes, implementation of experiments capable to produce required superpositions of coherent states can be anticipated.

In this paper we shall discuss the possibility to construct quantum states using coherent states superpositions. We find a simple set of superposition states which coincides with the Fock basis for any practical purpose.

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[2] E-mail address: H1084JAN@ELLA.HU

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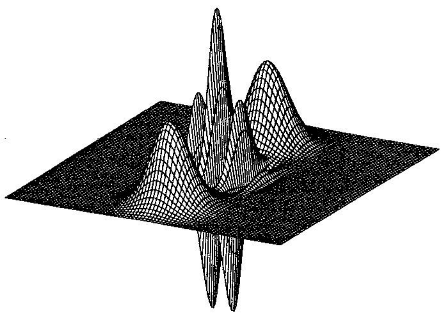

Fig. 1. Wigner function of a Schrödinger cat state consisting of two coherent states on the real axis of the phase space. Between the Gaussian bells of individual coherent states a fringe can be seen emerging from the quantum interference of the coherent states.

## 2. Schrödinger-cats and the Fock states

The superpositions of coherent states [10],

$$|\,\alpha,\phi\rangle = c_\phi(|\,\alpha\rangle + e^{i\phi}\,|-\alpha\rangle), \tag{1}$$

referred to as Schrödinger-cat states when the constituent coherent states are macroscopically distinguishable, have attracted much interest.

The two most typical cat states are the even ($\phi = 0$) and odd ($\phi = \pi$) superposition states. The case with small difference between the constituent states, by analogy, could be called Schrödinger-kitten states.

Although the coherent states are the most classical of all pure states of light, their simple superposition described by Eq. (1) shows remarkable nonclassical features as a consequence of the quantum interference [11, 12].

The Wigner function of the cat state leads us to better understanding of the interference pattern (Fig. 1.). The two Gaussian bells of the Wigner function correspond to the composite coherent states while an interference fringe pattern occurs between the bells. We note that although two coherent states with strongly different arguments are almost orthogonal to each other, the maximal amplitude of the interference fringe remains two times larger than the amplitudes of the composite coherent states, independently from the distance between them. The wavelength of the fringe decreases with the increase of the distance between the coherent states, the phase of the fringe depends on the relative phase $\phi$ in Eq. (1) between the composite part of the cat state. The picture becomes more complicated if we superpose more than 2 coherent states. In this case multiple fringes can constructively or destructively interfere with each other and also with the original coherent state bells to produce different nonclassical states.

It was shown that a single-mode electromagnetic field interacting with a two-level atom can evolve approximately to an odd coherent state in the framework of the Jaynes-Cummings model with certain initial conditions [14, 15]. Odd coherent state can also be generated in micromaser experiments [8]. The number state expansion of the odd coherent states contains only the odd-number states. They tend to the Fock state $|\,1\rangle$ in the $\alpha \to 0$ limit.



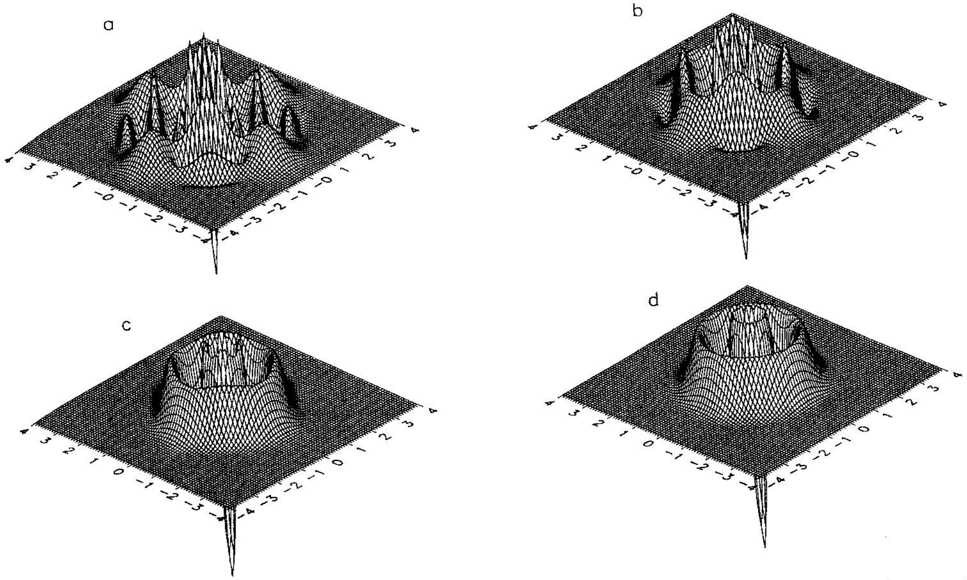

Fig. 2. The evolution of the Wigner function of the state $|\,n,r\rangle$ as the parameter $r$ decreases ($n = 3$). At large enough radius [(a) $r = 2.5$], the Gaussian bells of the coherent states and the fringe, characteristic of the quantum interference, are well announced. Decreasing the radius, the Wigner function resembles more and more that of a Fock state $|\,3\rangle$ [(b) $r = 1.5$, (c) $r = 1$] from which it cannot be practically distinguished in the case of (d) $r = 0.5$.

The idea of producing the Fock state $|\,1\rangle$ via an odd coherent state can be generalized for Fock states with more photon numbers [16].

Let us consider the following discrete superposition of $n+1$ coherent states situated symmetrically on a circle with radius $r$ in phase space

$$|\,n,r\rangle = c(r)\frac{\sqrt{n!}e^{\frac{r^2}{2}}}{(n+1)r^n}\sum_{k=0}^{n} e^{\frac{2\pi i}{n+1}k}\,|\,re^{\frac{2\pi i}{n+1}k}\rangle\,. \tag{2}$$

For $n = 1$ the superposition in Eq. (2) gives back the odd coherent state with the parameter $\alpha = r$. The Fock state expansion reads

$$\begin{aligned}|\,n,r\rangle &= c(r)\sqrt{n!}\sum_{l=0}^{\infty}\frac{r^{l(n+1)}}{\sqrt{(n+l(n+1))!}}\,|\,n+l(n+1)\rangle \\ &= c(r)\left(|\,n\rangle + \sqrt{\frac{n!}{(2n+1)!}}r^{n+1}\,|\,2n+1\rangle + \ldots\right).\end{aligned} \tag{3}$$

Only the first term in the sum survives the $r \to 0$ limit

$$|\,n,r\to 0\rangle = |\,n\rangle\,. \tag{4}$$

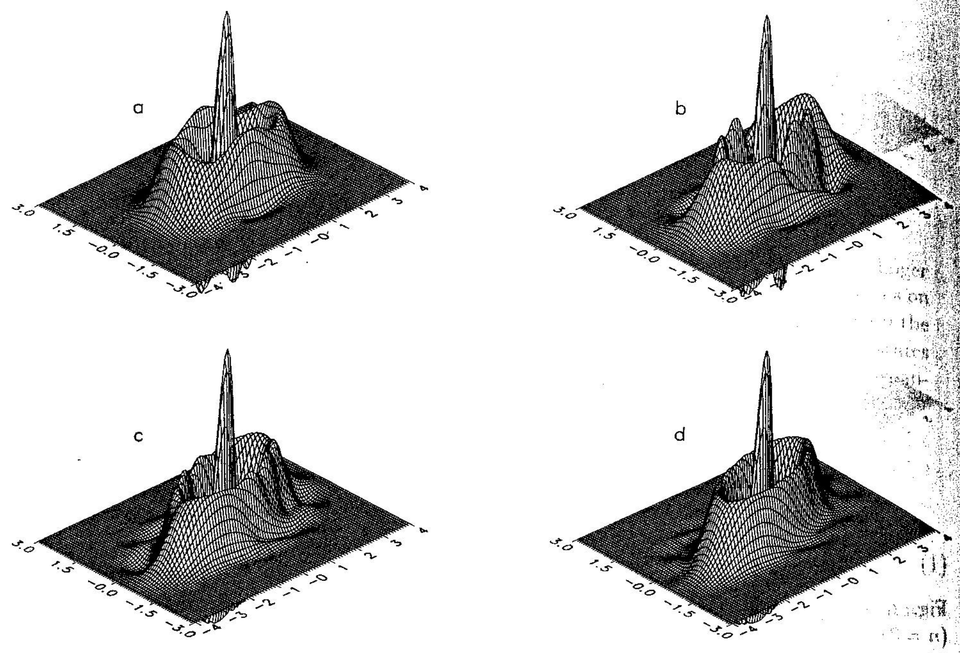


Fig. 3. Wigner functions of the squeezed and rotated 2-photon Fock state approximated by 4 (a), 6 (b), 8 (c) and 10 (d) coherent states, that are situated along the real axis of the phase space. Adding a small number of coherent state the resulting state would be indistinguishable from the desired state.

hence the superposition state $| n, r\rangle$ realizes the Fock state $| n\rangle$ at small enough radius of the circle. It can be noticed that the addition of each coherent state to the superposition decimates a series of Fock states from the Poisson distribution. The only remaining series ($| n\rangle$, $| 2n+1\rangle$, $| 3n+2\rangle$,...) is reduced to a single value due to the limit $r \to 0$. It is straightforward to infer from this derivation that the superposition state given in Eq.(2) contains the minimum number of coherent states to express a Fock state. The bigger the $n$ number of quanta in the Fock state, the state $| n, r\rangle$ can provide an appropriate representation of $| n\rangle$ at a bigger radius $r$.

Fig. 2. shows in Wigner function picture how the interference fringe characteristic of a Schrödinger-cat state [Fig. 2(a)] forms the annuli of the Wigner function of the Fock state $| 3\rangle$. In the case of a circle with radius $r = 1.5$ [Fig. 2(b)] the traces of the coherent components and the interference fringes can be noticed, while at radius $r = 1$ the emerging Wigner function [Fig. 2(c)] resembles very much to that of a Fock state. Decreasing further the radius [Fig. 2(d), $r = 0.5$], the phase space distribution of the superposition state $| n = 3, r = 0.5\rangle$ and that of a Fock state $| 3\rangle$ practically cannot be distinguished.

There are several experimental schemes which are appropriate to generate superposition states composed of coherent states lying on a circle in phase space. Making an initial coherent field interact with a sequence of two-level atoms detuned from the cavity resonance leads to such superpositions [8]. In the special case of their scheme, when the "phase-shift per photon" accumulated by the atomic dipoles crossing the cavity is a rational multiple of $\pi$, a symmetrical superposition of finite number of coherent states on a circle emerges. Required discrete superpositions on a circle, including the elements of the basis set given in Eq.(2), can be prepared in a single-atom interference method in a designed apparatus [9]. Superposition on a circle with small radius, that is essential in our case, can be generated in both of the above mentioned experimental schemes by starting with a field initially in coherent state with a small amplitude. The progress in quantum optics seems to enable us in the near future to create experimentally these superposition states.



## 3. Discretization of the 1-D representation

Let us consider a pure state written as a superposition of coherent states along the real axis in phase space [11, 3, 17]

$$| \psi\rangle = \int F(x) \,| x\rangle dx \,. \tag{5}$$

Let us consider the following discrete superposition of coherent states along the real axis of the phase space

$$| \psi_N\rangle = \sum_{k=1}^{N} F_k \,| x_k\rangle \,. \tag{6}$$

Here the coherent states $| x_k\rangle$ are chosen to be equally distributed at distances $d$ along the real axis around the coherent state $| x_0\rangle$ that belongs to the center of the corresponding one-dimensional distribution function $F(x)$ (Eq. 5), i.e.

$$x_k = x_0 + \left(k - \frac{N+1}{2}\right) d \,, \quad k = 1, \ldots N \,. \tag{7}$$

The coefficients $F_k$ are derived from the one-dimensional continuous distribution

$$F_k = cF(x_k) \,, \tag{8}$$

where $c$ is a normalisation constant.

As an example we consider displaced squeezed number states $| n, \zeta, Z\rangle$. Their interesting nonclassical properties were widely discussed in the literature [18]. The one-dimensional coherent-state representation of squeezed displaced number state along the real axis of phase space has the form

$$F(x) = \tilde{c}_n H_n \left(\frac{x - Z}{\sqrt{2uv}}\right) \exp\left(-\frac{u - v}{2v} x^2 + \frac{(uZ - vZ^*)}{v} x\right) , \mathrm{Re}\left(\frac{u - v}{2v}\right) > 0 \,. \tag{9}$$

The parameters $u$ and $v$ are connected to the complex squeezing parameter $\zeta$ in the usual way

$$u = \cosh r\,, \quad v = e^{i\theta}\sinh r\,. \tag{10}$$

We note that states $|\,0,\zeta,Z\rangle$ are the well-known squeezed coherent states.

In Fig. 3. we show how a squeezed Fock state builds up as we use more and more coherent states in the superposition. Here $n = 2$, the complex squeezing parameter $\zeta = 0.5\exp(0,5i)$ (the direction of squeezing closes 14 degrees with the imaginary axis). The sampling distance $d$ for each $N$ was optimized, minimizing the mismatch between the desired and the approximating states. In Fig. 3a at $N = 4$ coherent states the desired state has not emerged yet (the resulting state has a squeezing direction rather different from the planned one). Fig. 3b shows state 6 coherent states. The emerging desired state can be clearly seen. As we added more coherent states ($N = 8$ and $N = 10$ for Figs. 3c and 3d respectively) the approximation became more and more perfect.

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