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TESTS OF QUANTUM MECHANICS[1]

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The structure of quantum mechanics inposes constraints on the types of correlations it will allow. This makes very strong tests of quantum mechanics possible

1. Introduction

There are now three different proposed tests of local hidden-variables (or locally realistic) theories. ,The first, suggested by Bell, is an inequality for certain correlation functions [1]. The inequality is satisfied by locally realistic theories but can be violated by quantum mechanics. A much stronger result was derived by Greenberger, Horne, and Zeilinger (GHZ) [2]. They found a situation in which local hidden-variables theories require that a particular measurement to produce a specific value. It is possible, however, to find quantum mechanical states for which the measurement will produce a definite result which is different from that predicted by hidden variables. In an experimental test of the GHZ result it would be necessary to make only one measurement to decide between the two theories. The only problem is that the required states are hard to produce. This led Hardy to devise a scheme which is easier to implement than the GHZ one, but which is a stronger test of locally realistic theories than that proposed by Bell [3].

One can also ask whether it is possible to test quantum mechanics itself. Quantum mechanics derives probabilities from an underlying Hilbert space structure. If this structure imposes constraints on the types of correlations which are allowed a test becomes possible. The virtue of a test of this kind is that the assumptions it makes are minimal. No particular Hamiltonian is assumed, for example. This means that if a result which contradicts quantum mechanics were to be discovered, it could not be explained away by saying that some parameter wasn't quite right or that there were other interactions which were ignored but should not have been. What we would

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like, then, is a test of the way quantum mechanics produces probabilities which is independent of quantum dynamics.

The first constraint of this type was derived by Tsirelson [4]. He considered the same quantity as Bell, and found that quantum mechanics imposes constraints on the values it can achieve. These constraints are less severe than those imposed by local realism but, none the less, provide a means of testing quantum mechanics. A different constraint was found by Landau [5]. These two constraints are independent. This is shown by the fact that there are sets of correlations which violate Landau's constraint but do not violate Tsirelson's [6].

Here we shall review the work of Tsirelson and Landau as well as some new results. We shall begin with a discussion of the basic framework in which the results are set. This will allow us to define the correlation functions and probabilities which we need to explicitly state the constraints. Finally, we shall discuss some considerations which will play a role in the implementation of a test of quantum mechanics.

2. System and Basic Quantities

The gedanken experiment we shall consider is the same as the one which is used in the discussion of the Bell inequality. We have a source which emits two signals which we can think of as particles. These then proceed to two detectors, detectors A and B, one particle to each detector. Each detector has a switch which can assume one of two positions, i.e. at each detector we can measure two possible quantities. At detector A we can measure a_1 or a_2 and at detector B, b_1 or b_2. For simplicity we shall assume that each quantity has only two possible values, 1 or -1. After running the experiment many times what we have determined from our measurements are the 16 probabilities $P(a_j = l, b_k = m)$ where j and k can be 1 or 2 and l and m can be either 1 or -1. From these we define the correlation function

$$\langle a_j b_k \rangle = \sum_{l=\pm 1} \sum_{m=\pm 1} l\, m\, P(a_j = l; b_k = m) \quad . \tag{2.1}$$

The probabilities $P(a_j = l, b_k = m)$ should obey the laws of probability and they should satisfy one further constraint. If we only look at detector A the result of a measurement should not depend on the setting of the switch on detector B. The reverse should also be true. In terms of the basic probabilities this conditions which we shall call the causal communication constraint, is

$$\sum_{m=\pm 1} P(a_j = l; b_1 = m) = \sum_{m=\pm 1} P(a_j = l; b_2 = m);$$

$$\sum_{l=\pm 1} P(a_1 = l; b_k = m) = \sum_{l=\pm 1} P(a_2 = l; b_k = m) \quad . \tag{2.2}$$

We shall only consider sets of probabilities which satisfy this condition.

3. Results

Tsirelson considered the combination of correlation functions

$$K = |\langle a_1 b_1 \rangle + \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle - \langle a_2 b_2 \rangle| \quad . \tag{3.1}$$

The laws of probability demand that K be at most 4. If the correlation functions come from a local realistic theory we have that K must be less than or equal to 2. The natural question, then, is whether quantum mechanics can fill the entire gap between 2 and 4. Tsirelson found that it cannot. In particular he found that if the correlation functions are calculated from a quantum mechanical density matrix then $K \leq 2\sqrt{2}$. It is possible to find sets of probabilities which obey the causal communication constraint for which $K = 4$. A specific example was given by Popescu and Rohrlich [7]. Therefore, quantum mechanics cannot generate all reasonable sets of probabilities.

Landau found a different constraint. For quantum mechanical correlation functions it must be the case that

$$|\langle a_1 b_1 \rangle\langle a_1 b_2 \rangle - \langle a_2 b_1 \rangle\langle a_2 b_2 \rangle| \leq \left(1 - \langle a_1 b_1 \rangle^2\right)^{1/2} \left(1 - \langle a_1 b_2 \rangle^2\right)^{1/2} + \left(1 - \langle a_2 b_1 \rangle^2\right)^{1/2} \left(1 - \langle a_2 b_2 \rangle^2\right)^{1/2} \quad . \tag{3.2}$$

This inequality is not implied by Tsirelson's because one can find sets of probabilities which satisfy the causal communication constraint and which violate Landau's inequality but do not violate Tsirelson's.

We can derive a new constraint by starting with a special case of Landau's inequality. Consider a state for which

$$\langle a_1 b_2 \rangle = \langle a_2 b_1 \rangle = 1 \quad . \tag{3.3}$$

Eq.(3.2) then implies that

$$\langle a_1 b_1 \rangle = \langle a_2 b_2 \rangle \quad . \tag{3.4}$$

This result can also be proved in an elementary fashion [6]. We can, however, go a bit further. We find that Eq.(3.3) implies that

$$P(a_1 = l; b_1 = m) = P(a_2 = m; b_2 = l), \quad . \tag{3.5}$$

a result which is stronger than that in Eq.(3.4). Eq.(3.5) implies Eq.(3.4) but the reverse is not the case. Thus, Eqs.(3.3) and (3.5) represent yet another constraint which quantum mechanical probabilities must obey.

4. Conclusions

How can we use the result in the proceeding section to test quantum mechanics? With Tsirelson's and Landau's inequalities one could prepare a quantum state which is on the "edge" of the allowed region, i.e. one which satisfies either inequality as an equality. One could then perform the necessary measurements to determine both sides of the inequality and see whether a violation can be found. In the case of Eq.(3.3) one can prepare a quantum state which satisfies it. For example, if our source emits two spin 1/2 particles and we make the identification

$$a_1 \to \sigma_{z\,a}; \qquad b_1 \to \sigma_{z\,b};$$
$$a_2 \to \sigma_{x\,a}; \qquad b_2 \to \sigma_{x\,b}; \quad . \tag{4.1}$$

the state

$$|\Psi\rangle = \frac{1}{2}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)\,, \quad . \tag{4.2}$$

will satisfy Eq.(3.3). We can then perform measurements to see if either Eq.(3.4) or Eq.(3.5) is violated.

Quantum mechanics has provided us with spectacularly successful description of nature for the last 70 years. We really do not expect this situation to change. It is, however, good to put beliefs such as this to the test. The recent work on the constraints imposed by quantum mechanics make stringent tests possible.

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