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QUANTUM MEASUREMENT ON DRESSED ATOMS[1]

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The concept of an atom dressed by the electromagnetic field is first illustrated from a physical point of view, successively the state vector of the dressed atoms presented. The theory of quantum measurement of finite duration τ is also briefly introduced and it is used to describe a gedanken experiment, aimed at measuring the atomic population distribution of a two-level atom. It is shown that the result of the experiment depends on τ in an essential way. In particular, in the limit of short measurement the apparatus is capable of measuring the bare atomic population as expected, whereas in the opposite limit the same apparatus yields the population of the dressed atom. This result is interpreted in the context of the theory of dressed atoms.

1. The nature of dressed atoms

The expression *dressed atom* is of current use in QED and in quantum optics. It is referred however, to two quite different physical contexts.

In the first context a ground-state neutral atom is assumed to interact with the vacuum fluctuations of the electromagnetic field. In the ground state of the total (atom + field) system these vacuum fluctuations induce virtual absorption and re-emission processes of photons by the atom. These photons are virtual because the bare energy of the (atom + field) system, i.e. its energy disregarding the atom-field interaction, is not conserved. Thus the corresponding fluctuation δE of this bare energy must have a finite duration δt according to the uncertainty relation

$$\delta E \sim \frac{\hbar}{\delta t} \tag{1.1}$$

During one of these fluctuations photons are emitted and reabsorbed after time δt. Because of their limited lifetime, as we have mentioned, these photons are called *virtual*

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photons. Since the fluctuations take place continuously, a steady-state *cloud* of virtual photons is formed around the atom. The complex object (bare atom + cloud of virtual photons) is called a *dressed atom* [1]. In this sense the concept described by this expression belongs to the class of dressed sources introduced long ago in quantum field theory [2]. In contrast with a real photon, which can leave the region of the source in view of its infinite lifetime, a virtual photon can only attain a finite distance

$$r \sim c\delta\tau \sim \frac{\hbar c}{\delta E} \qquad (1.2)$$

from the atom. Consequently we should expect the linear dimensions of the *virtual cloud* which surrounds the atom to coincide roughly with r in (1.2), at least for virtual transitions characterized by a bare energy unbalance δE. For an atom, an obvious energy scale is provided by its energy eigenvalue distribution. For our purposes it is sufficient to model the atom as a two-level system with an energy separation $\hbar\omega_0$ corresponding to a natural frequency in the optical range ($\omega_0 \sim 10^{15}\,Hz$). For such an atomic model, for example, the obvious energy scale for energy fluctuations is $\delta E = \hbar\omega_0$, which can be substituted in (1.2) to yield typical linear dimensions of the virtual cloud $r \sim \frac{c}{\omega_0}$. This result, given by such an elementary approach, has been confirmed by rather sophisticated calculations [3] which we will mention in the next section. Here we remark that the detailed shape of the virtual photon cloud has been related to the form of the van der Waals forces between two neutral atoms [4].

A second context, where in quantum optics the expression *dressed atom* is also used, exist which is quite different from that outlined above, namely in the presence of real photons. In this second context the (atom + field) system is not in its ground state, a strong monochromatic external field of frequency ω being provided by an external source, such as a laser. This intense radiation field mixes and splits the levels of the system, and it is taken to overwhelm additional fields acting on the atom, including that of the vacuum fluctuations. The resulting energy eigenstates of the (atom + field) system display correlations between bare atomic occupation numbers and photon numbers, yielding an eigenvalue spectrum of the coupled (atom + field) system which for a two level atom reduces to the familiar Jaynes-Cummings series of doublets characterized by a splitting $\hbar\Delta$ [5]. Also these mixed and correlated levels $|u_n^{(\pm)}\rangle$ are called *dressed atomic states* [6], although their nature is different from that of the states dressed by vacuum fluctuations discussed above. The new energy scale $\hbar\Delta$, where Δ is called *Rabi frequency* [7], is determined by interaction between the atom and strong field and it plays an important role in a wealth of quantum-optical phenomena [8].

In order to distinguish between the two contexts described in what precedes, in the rest of this paper we will indicate the first by *Zero-Point* (ZP) *field dressing* and the second by *strong field dressing*.

2. The dressed atom

The Hamiltonian for a two-level atom fixed at the origin of the reference frame and taking account of the interaction with the electromagnetic field in the electric dipole

approximation is [8]

$$\begin{aligned} H &= H_0 + V_1 + V_2; \\ H_0 &= \hbar\omega_0 S_z + \sum_{kj} \hbar\omega_k (a^\dagger_{kj} a_{kj} + \frac{1}{2}) \;; \\ V_1 &= \sum_{kj} (\epsilon_{kj} a_{kj} S_+ + \epsilon^*_{kj} a^\dagger_{kj} S_-) \;; \\ V_2 &= \lambda \sum_{kj} (\epsilon_{kj} a^\dagger_{kj} S_+ + \epsilon^*_{kj} a_{kj} S_-) \end{aligned} \qquad (2.1)$$

where S_i ($i = \pm, z$) are the usual pseudospin $S = \frac{1}{2}$ operators $a^\dagger_{kj}$ and a_{kj} being Bose creation and annihilation operators for photons in plane-wave field modes of momentum $\mathbf{k}$ polarization j and frequency $\omega_k = ck$. Moreover the multipolar and minimal coupling versions of H have respectively [8]

$$\lambda = 1 \quad, \epsilon_{kj} = -i\sqrt{\frac{2\pi\hbar\omega_0^2}{V\omega_k}}\mu_{21}.\mathbf{e}_{kj} \qquad (minimal);$$

$$\lambda = -1 \quad, \epsilon_{kj} = -i\sqrt{\frac{2\pi\hbar\omega_k}{V}}\mu_{21}.\mathbf{e}_{kj} \qquad (multipolar) \qquad (2.2)$$

where in the atom-photon coupling constant ϵ_{kj} the vector μ_{21} is the matrix element of the electric dipole operator between the two atomic levels and $\mathbf{e}_{kj}$ is the polarization vector of the $\mathbf{k}_j$ mode of the field. Here for simplicity we take both μ_{21} and $\mathbf{e}_{kj}$ as real. A complete account of the minimal coupling and of the multipolar theories of the atom-photon interaction can be found in reference [9]. Finally, we note that the *Rotating Wave Approximation* (RWA) is obtained by putting $\lambda = 0$ in (2.1), and that the λ-term in V_2 gives rise to energy nonconserving processes which create photons and simultaneously excite the atom. These terms should obviously be expected to contribute to the virtual cloud in the ZP field dressing of the atom.

The state corresponding to the atom dressed by the ZP field is evidently the ground state of H, which we obtain by perturbation theory. We start from the ground state $|\{0_{kj}\},\downarrow\rangle$ of H_0 with no photons and the atom in its bare ground state, and we obtain the dressed ground state, normalized up to terms of order ϵ^2, as

$$|\{0_{kj}\},\downarrow\rangle' = [1 + \frac{1}{E_0 - H_0}(1-P_0)V_2 + \frac{1}{E_0 - H_0}(1-P_0)V_1\frac{1}{E_0 - H_0}(1-P_0)V_2 - \frac{1}{2}\langle\{0_{kj}\},\downarrow|V_2\frac{1}{(E_0 - H_0)^2}(1-P_0)V_2|\{0_{kj}\},\downarrow\rangle]|\{0_{kj}\},\downarrow\rangle \qquad (2.3)$$

where $E_0 = \frac{-\hbar\omega_0}{2}$ and P_0 is the projector on the ground state $|\{0_{kj}\},\downarrow\rangle$ of H_0. Thus the virtual cloud is contributed by fluctuations which excite the atom and create one photon (terms in V_2, one-photon fluctuations) as well as by other fluctuations whose net effect is to leave the atom in the ground state and to create a photon pair (terms in V_1V_2, two-photon fluctuations). The second kind of fluctuations had not been anticipated in the

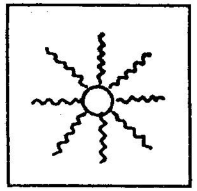

Fig.1. Virtual photons

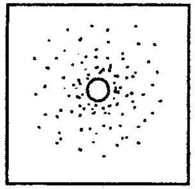

Fig.2. Virtual cloud

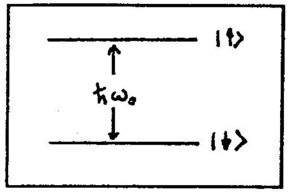

Fig.3. 2-level atom

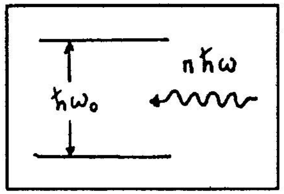

Fig.4. Strong field

qualitative model presented in the previous section, and it has effect of extending the boundaries of the photon cloud beyond the limit $r \sim \frac{c}{\omega_0}$ of the one-photon fluctuations because the energy unbalance $\delta E = \hbar(\omega_{k1} + \omega_{k2})$ can be arbitrarily small. It turns out that the virtual cloud at point $\mathbf{x}$ in space, as expressed by the quantum average of the electric energy density of the state (2.3), can be approximated as

$$\langle H_{el}(\mathbf{x})\rangle = \begin{cases} \frac{1}{8\pi}\mu_{21}^2(1+3cos^2\theta)\frac{1}{x^6} & (x \ll \frac{c}{\omega_0}) \\ \frac{1}{16\pi^2}\frac{c}{\omega_0}\mu_{21}^2(13+7cos^2\theta)\frac{1}{x^7} & (x \gg \frac{c}{\omega_0}) \end{cases} \tag{2.4}$$

where θ is the angle between $\mathbf{x}$ and μ_{21} [8].

As for the atom dressed by a strong field, the Hamiltonian can be obtained from (2.1) by neglecting all the field modes except that populated by laser photons and by performing the RWA. This leads to the Jaynes-Cummings Hamiltonian [5]

$$H = H_0 + V \;\; ; H_0 = \hbar\omega_0 S_z + \hbar\omega a^\dagger a \;\; ; V = \epsilon a S_+ + \epsilon^* a^\dagger S_- \tag{2.5}$$

The eigenstates of (2.5) for each doublet are [6]

$$|u_n^{(+)}\rangle = -cos\frac{\theta}{2}|n,\downarrow\rangle + \frac{\epsilon}{|\epsilon|}sin\frac{\theta}{2}|n-1,\uparrow\rangle \;\; ;$$

$$|u_n^{(-)}\rangle = \frac{\epsilon}{|\epsilon|}cos\frac{\theta}{2}|n-1,\uparrow\rangle - sin\frac{\theta}{2}|n,\downarrow\rangle \;\; ;$$

$$sin\frac{\theta}{2} = \frac{1}{\sqrt{2}}[1+\frac{\delta}{\Delta}]^{\frac{1}{2}} \;\; ; \tag{2.6}$$

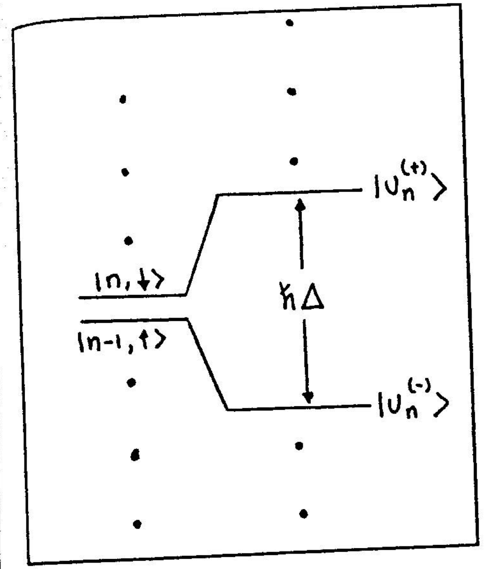

Fig.5. JC doublets

$$cos\frac{\theta}{2} = -\frac{1}{\sqrt{2}}[1-\frac{\delta}{\Delta}]^{\frac{1}{2}} \;\; ;$$

$$\delta = \hbar(\omega_0 - \omega) \;\; ; \;\; \Delta = (\delta^2 + 4|\epsilon|^2 n)^{\frac{1}{2}}$$

corresponding to the eigenvalues

$$E_n^{(\pm)} = (n - \frac{1}{2})\hbar\omega \pm \frac{1}{2}\Delta \tag{2.7}$$

The dressed states (2.6) clearly display correlation between bare atomic and field states.

In what follows we will investigate the physical nature of both kinds of dressed states (2.3) and (2.6) using the tools provided by one of the modern version of the quantum theory of measurement [10]. The main questions we would like to answer are:

-Can we measure *in principle* physical quantities pertaining to dressed atoms?
-Which properties should the measurement possess in order to achieve this aim?
-Can we learn something about the structure of the dressed atom trough such a measurement?

3. The measurement of atomic variables

The textbook recipe for measuring a physical quantity U is based on the assumption that a corresponding operator $\hat{U}$ exists, endowed with a complete set of eigenstates $|u_k\rangle$

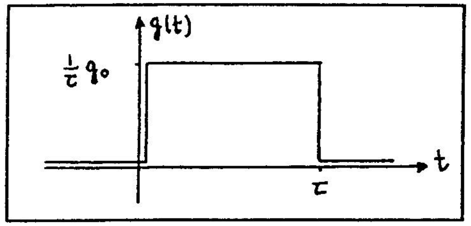

Fig.6. Atom pointer coupling

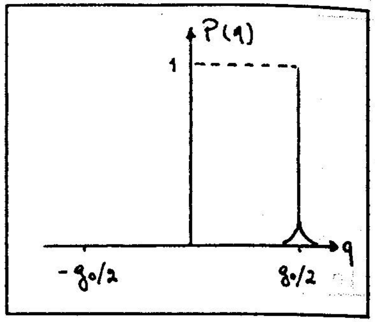

Fig.7. $|\phi\rangle = |\downarrow\rangle$

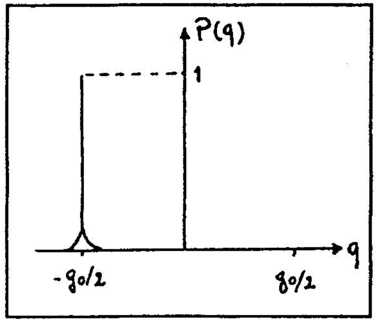

Fig.8. $|\phi\rangle = |\uparrow\rangle$

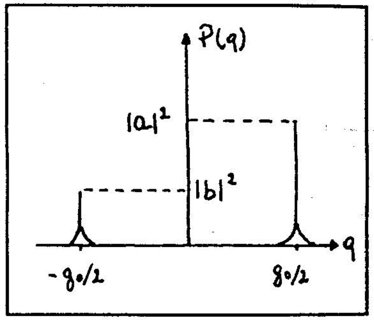

Fig.9. $|\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$

such that

$$\hat{U}|u_k\rangle = u_k|u_k\rangle \tag{3.1}$$

If the state of the atom at the observation time is $|\phi\rangle$, the observation of U yields one of the eigenvalues u_k with probability $|\langle u_k|\phi\rangle|^2$. Note that here the observation and measurement are synonymous and that the process corresponding to these terms is instantaneous. After the observation the system is in state $|u_k\rangle$, and the evolution leading from $|\phi\rangle$ to $|u_k\rangle$, usually called *wavefunction collapse*, cannot be described by the Shrödinger equation. It should also be noted that this procedure pays little or no attention to the measuring apparatus.

A more sophisticated procedure, which takes into account explicitly the role of the measuring apparatus, has been proposed by Peres and co-workers [10]. According to this procedure also the apparatus (equivalently indicated as the *pointer*) is treated quantum-mechanically and its macroscopic nature is exploited at a later stage. Before the measurement begins, the atom is taken to be in the state $|\phi\rangle$ and the pointer in the state $|\psi\rangle$. A pointer Hamiltonian is assumed to exist with a complete set of pointer

eigenstates $|\psi_k\rangle$. The measurement process is taken to start at time $t = 0$, when the state of the atom (atom + pointer) system is decorrelated and of the form

$$|\Psi(0)\rangle = |\psi_k\rangle \otimes |\phi_k\rangle \tag{3.2}$$

The atom-pointer interaction is described by a Hamiltonian containing both atomic and pointer variables, in such a way that the (atom + pointer) system evolves for a finite period of time τ according to the Schrödinger equation. The set of processes taking place during this period is defined as *measurement*, and

$$|\Psi(t)\rangle = T(t)|\Psi(0)\rangle = \sum_k c_k|u_k\rangle \otimes |\psi_k\rangle \tag{3.3}$$

Where c_k is a c-number amplitude and $T(t)$ is an appropriate unitary operator. The observation, which here is kept distinct from the measurement, takes place at time τ and induces a nonSchrödinger collapse of $|\Psi(\tau)\rangle$ into one of the states $|u_k\rangle \otimes |\psi_k\rangle$. Note that here *observation* is separated from *measurement* and that a new time scale is introduced which is defined as a finite measurement time preceding the observation.

In preparation for the application of these ideas to the case of a dressed atom, we now discuss the measurement of the atomic population of a two-level atom in the absence of atom radiation coupling. Consider the atom-pointer Hamiltonian [10]

$$H_M = \frac{1}{2M}p^2 + \hbar\omega_0 S_z + g(t)pS_z \quad ; g(t) = \frac{1}{\tau}g_0[\theta(t) - \theta(t-\tau)] \tag{3.4}$$

where p is the momentum of a particle of large mass M, which we take to be pointer, and $g(t)$ is the atom-pointer coupling. Thus the atom pointer interaction starts at $t = 0$ and lasts for a time τ. During this measurement the pointer gets correlated with the atom, and one can evaluate the time evolution of the position operator $q(t)$ of the pointer. We are interested in the quantities

$$\langle\Psi|q(\tau)|\Psi\rangle \equiv \langle q(\tau)\rangle \tag{3.5}$$

$$\langle\Psi|q^2(\tau)|\Psi\rangle - (\langle\Psi|q(\tau)|\Psi\rangle)^2 \equiv \langle q^2(\tau)\rangle - \langle q(\tau)\rangle^2$$

where $|\Psi\rangle$ is given by (3.2). We also assume that $|\Psi\rangle$ is such that

$$\langle\Psi|q(0)|\Psi\rangle = \langle\Psi|p(0)|\Psi\rangle = 0 \tag{3.6}$$

and also, in view of the large mass of the pointer, that it is an approximate eigenstate of both $q(0)$ and $p(0)$. A simple calculation yields

$$\langle q(\tau)\rangle = g_0\langle S_z(0)\rangle \quad ; \langle q^2(\tau)\rangle - \langle q(\tau)\rangle^2 = g_0^2(\frac{1}{4} - \langle S_z(0)\rangle^2) \tag{3.7}$$

Thus if the atom is initially in the $|\uparrow\rangle$ state, the pointer will always be found at position $q = \frac{g_0}{2}$ with zero variance. Similarly if the atom is initially in the state $|\downarrow\rangle$, the pointer will always be found at position $q = -\frac{g_0}{2}$ and the variance of $P(q)$ is zero. If the

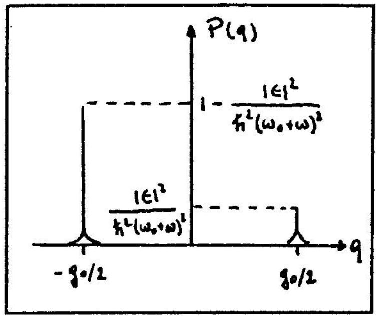

Fig.10. $|\phi\rangle = |0,\downarrow\rangle$; $\tau \ll (\omega_0+\omega)^{-1}$

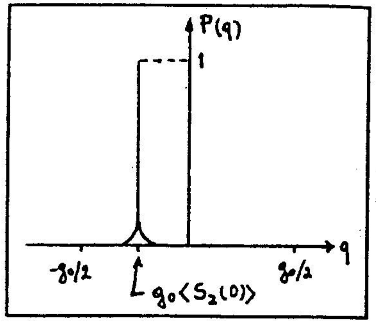

Fig.11. $|\phi\rangle = |0,\downarrow\rangle$; $\tau \gg (\omega_0+\omega)^{-1}$

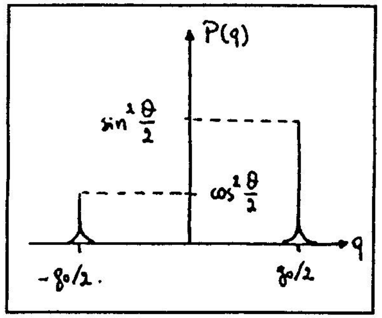

Fig.12. $|\phi\rangle = |u_n^{(+)}\rangle$; $\tau \ll \frac{\hbar}{\Delta}$

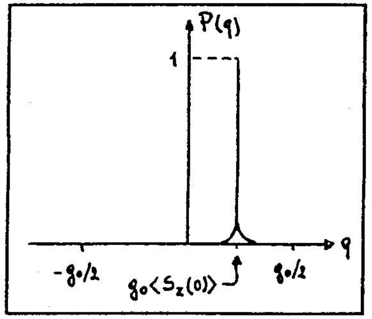

Fig.13. $|\phi\rangle = |u_n^{(+)}\rangle$; $\tau \gg \frac{\hbar}{\Delta}$

atom is in a superposition state, however, the variance does not vanish and the pointer distribution $P(q)$ will exhibit two peaks. It is possible to show that in all cases the position of the pointer is perfectly correlated with the atomic state.

We conclude that in the case of two-level atom decoupled from the radiation field, i.e. in the case of a *bare atom*, the pointer is perfectly capable of measuring the atomic population.

4.Measurement of atomic variables for an atom dressed by ZP fluctuations

We will now apply the same procedure to an atom coupled to the electromagnetic field. In order to simplify the discussion, we consider first the following single mode version of Hamiltonian (2.1)

$$H = H_0 + V_1 + V_2 \quad ; H_0 = \hbar\omega_0 S_z + \hbar\omega a^\dagger a \quad ;$$
$$V_1 = \epsilon a S_+ + \epsilon^* a^\dagger S_- \quad ; V_2 = \lambda(\epsilon a^\dagger S_+ + \epsilon^* a S_-) \tag{4.1}$$

The dressed ground state of H, accurate at order ϵ^2, can be explicitly obtained from (2.3) with $\lambda = -1$ as

$$|\phi\rangle \equiv |0,\downarrow\rangle' = [1 - \frac{1}{2}\frac{|\epsilon|^2}{\hbar^2(\omega_0+\omega)^2}]|0,\downarrow\rangle + \frac{\epsilon}{\hbar(\omega_0+\omega)}|1,\uparrow\rangle -$$
$$\frac{1}{\sqrt{2}}\frac{|\epsilon|^2}{\hbar^2\omega(\omega_0+\omega)}|2,\downarrow\rangle \tag{4.2}$$

The Hamiltonian for the measurement of S_z is

$$H_M = \frac{1}{2M}p^2 + \hbar\omega_0 S_z + g(t)S_z p + \hbar\omega a^\dagger a + \epsilon a S_+ + \epsilon^* a^\dagger S_- - \epsilon a^\dagger S_+ - \epsilon^* a S_- \tag{4.3}$$

where $g(t)$ is the same as in (3.4). The relevant expressions in terms of the coordinate of the pointer can be obtained by direct integration of the Heisenberg equation

$$q(t) = \int_0^t S_z(t')g(t')dt' + \frac{1}{M}p(0)t + q(0) \quad ; \tag{4.4}$$
$$q^2(t) = \int_0^t \int_0^t S_z(t')S_z(t'')g(t')g(t'')dt'dt'' +$$
$$[\int_0^t S_z(t')g(t')dt', \frac{1}{M}p(0)t + q(0)]_+ + (\frac{1}{M}p(0)t + q(0))^2$$

The quantum averages of these operators are easily taken on the state $|\psi\rangle \otimes |\phi\rangle$ of the total system, consisting of pointer, atom and radiation field, and the following expressions for the average position of the pointer and for its variance are obtained [11]

$$\langle q(\tau)\rangle = g_0\langle S_z(0)\rangle \quad ; \tag{4.5}$$
$$\langle q^2(\tau)\rangle - \langle q(\tau)\rangle^2 = g_0^2(\frac{1}{4} - \langle S_z(0)\rangle^2)\frac{2[1-\cos(\omega_0+\omega)\tau]}{(\omega_0+\omega)^2\tau^2}$$

This result should be compared with (3.7) for a bare atom. In the limit of a *short measurement* $\tau \ll (\omega_0+\omega)^{-1}$ the variance (4.5) coincides with bare-atom result (3.7). Thus the pointer perceives the dressed atom as bare. This means that in these conditions the measurement is not influenced by the coupling with the zero-point fluctuations of the field, except of the trivial way. On the contrary for $\tau \gg (\omega_0+\omega)^{-1}$ the variance of the dressed atom vanishes, but $\langle q(\tau)\rangle \neq \pm\frac{g_0}{2}$. Thus we have a single peak in $P(q)$ at $|\phi\rangle = |0,\downarrow\rangle; \tau \ll (\omega_0+\omega)^{-1}$

$$q = g(0)\ '\langle 0,\downarrow|S_z(0)|0,\downarrow\rangle' = g_0(-\frac{1}{2} + \frac{|\epsilon|^2}{\hbar^2(\omega_0+\omega)^2}) \tag{4.6}$$

We conclude that *long measurements* are strongly influenced by the coupling with vacuum fluctuations, and that in this kind of measurement the pointer perceives a new object which originates because of the atom-photon coupling: the dressed atom.

These results can also be related to the energy which is conveyed to the (atom + field) system during the measurement. This energy, because of the uncertainty principle, is given by $\delta E \sim \frac{\hbar}{\tau}$. Thus for short measurements $\delta E > \hbar(\omega_0 + \omega)$, which is sufficient to create a real simultaneous excitation of the atom and of one photon. This breaks the virtual cloud apart from the atom,with the result that the latter is perceived as bare. It is thus clear that a dressed atom cannot be detected by any "textbook measurement", which as we have seen is instantaneous. On the other hand in a long measurement the energy conveyed is $\delta E < \hbar(\omega_0 + \omega)$, which is not sufficient to transform a virtual fluctuation into a real one, and the atom is perceived as dressed [11].

The single mode model can be generalized to the many-mode case with the result that [12]

$$\langle q(\tau)\rangle = g_0\langle S_z(0)\rangle \quad ; \tag{4.7}$$

$$\langle q^2(\tau)\rangle - \langle q(\tau)\rangle^2 = g_0^2 \sum_{kj} \frac{|\epsilon_{kj}|^2}{\hbar^2(\omega_0+\omega_k)^2} \frac{2[1-\cos(\omega_0+\omega_k)\tau]}{(\omega_0+\omega_k)^2\tau^2}$$

where

$$\langle S_z(0)\rangle = -\frac{1}{2} + \sum_{kj} \frac{|\epsilon|^2}{\hbar^2(\omega_0+\omega_k)^2} \tag{4.8}$$

We see if τ is short, such that $\tau \ll \hbar(\omega_0+\omega_k)^{-1}$ for any of the field modes, the variance takes the form appropriate for a bare atom. If on the contrary $\tau \gg \hbar(\omega_0+\omega_k)^{-1}$ for any of the field modes, the variance of the pointer distribution vanishes and we have only one peak as in the dressed-atom case. In an intermediate situation the atom is perceived by the pointer as dressed only by the high-frequency modes, or *partially dressed* [13].

We emphasize that one is likely to obtain quite different results on a dressed atom depending on duration of the measurement.

5. Measurement of atomic variables for an atom dressed by a strong field

As we have seen, the RWA Hamiltonian (2.5) can describe a two-level atom dressed by an external monochromatic field of frequency ω. We wish to apply the theory of measurement of finite duration to such an object. We start from the measurement Hamiltonian

$$H_M = \frac{1}{2M}p^2 + \hbar\omega_0 S_z + g(t)S_z p + \hbar\omega a^\dagger a + \epsilon a S_+ + \epsilon^* a^\dagger S_- \tag{5.1}$$

which includes the pointer as in the previous cases. The state of the total system, including the pointer, is taken as $|\psi\rangle \otimes |\phi\rangle$ also in this case, where $|\phi\rangle$ is the state of the (atom + field) system. Here we assume that $|\phi\rangle$ coincides with the dressed state $|u_n^{(+)}\rangle$. A procedure analogous to that previously adopted gives

$$\langle q(\tau)\rangle = g_0\langle S_z(0)\rangle \quad ;\langle S_z(0)\rangle = \frac{\delta}{2\Delta} \tag{5.2}$$

$$\langle q^2(\tau)\rangle - \langle q(\tau)\rangle^2 = g_0^2(\frac{1}{4} - \langle S_z(0)\rangle^2)\frac{1-\cos(\frac{\Delta\tau}{\hbar})}{(\frac{\Delta\tau}{\hbar})^2}$$

Clearly for $\tau \ll \frac{\hbar}{\Delta}$ the variance of $P(q)$ behaves as that of the bare atom, and we find two peaks at $q = \pm\frac{g_0}{2}$ as expected. For long measurements, however, $\tau \gg \frac{\hbar}{\Delta}$ and the variance of $P(q)$ vanishes. Consequently in the latter case we have only one peak at

$$q = g_0\langle u_n^{(+)}|S_z(0)|u_n^{(+)}\rangle = g_0\frac{\delta}{2\Delta} \tag{5.3}$$

In these conditions one observes a dressed atom rather then a bare one. It should be noted that here the time scale which separates the two kinds of measurement is the inverse Rabi frequency $\frac{\hbar}{\Delta}$. This is orders of magnitude larger than the time scale for dressing by zero-point fluctuations ($\omega_0^{-1} \sim 10^{-15}\ s$), since in a typical experiment at optical frequencies it is $\Delta \sim 10^{-9}\ s$ [15].

We remark that observing $q(\tau)$ amounts to observing the dressed atomic population if the measurement is long, and the bare atomic population if the measurement is short.

6. Conclusions

We have taken into account the finite time τ necessary to perform a measurement of the atomic population of an atom dressed by the electromagnetic field. During this time, which is followed by a sudden observation causing wavefunction collapse, dynamical correlations between the pointer and the dressed atom are established in a Schrödinger-like fashion. This amounts to introducing a new time-scale which depends on the experimental conditions. Such a new time-scale interferes with the time-scale typical of the interactions which leads to the atomic dressing. This is true for atoms dressed by zero-point fluctuations as well as for atoms dressed by a real field , although the time-scale of two dressing processes may be very different. As a result of this interference an experimental apparatus, devised to measure bare atomic quantities, may in fact turn out to measure dressed or partially dressed ones. In other words, whether an atom should be regarded as bare, as dressed or as partially dressed depends on experimental conditions under which the measurement is performed. In particular our analysis shows that if in a gedanken experiment an atom is subjected to any conventional instantaneous measurement it will always be detected as bare and not as dressed, although the notation of a perfectly bare atom must be interpreted as purely ideal and as a limit of the more realistic notion of a partially dressed atom.

In addition, the following considerations apply to an atom dressed by the vacuum fluctuations. The qualitative description of the virtual cloud, in terms of zero-point quantum fluctuations of duration $\delta\tau$, presented in section 1, is dynamic and essentially time dependent. On the other hand the quantitative theory of the dressed atom sketched in section 2 is a time-independent description of the atom interacting with the vacuum field, which is based on the form of the eigenstates of the total Hamiltonian of the (atom + field) system in which time plays no significant role. Given this apparent conflict one may well doubt the soundness of the dynamical picture, which refers to bare (hence normally unobservable) quantities which fluctuate as functions of time. The gedanken experiments described in section 4 resolve this apparent discrepancy by bringing out the time-dependent aspect of vacuum fluctuations even if the system is in an eigenstate of the total Hamiltonian. This is achieved by the introduction of the duration τ of the measurement and by setting this time scale against the (inverse) energy scale δE of the vacuum fluctuations, which therefore provides the conceptual foundation of the aesthetically appealing description of the virtual cloud in section 1.

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