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ENTANGLED FIELDS AND ENTANGLED ATOMS: TOOLS TO TEST FUNDAMENTALS OF QUANTUM MECHANICS[1]

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First, we study quantum coherence effects in two micromasers coupled in series by sharing a common pumping atomic beam. We consider the complete two-field system and focus on the quantum correlations arising between the two spatially separated fields as atomic coherence is transferred from the first to the second cavity. The scheme is analogous to Young's double-slit experiment as we have two indistinguishable atomic paths to reach the same final state. Quantum engineering of entangled states of the two nonlocal micromaser fields is investigated in the two cases when the final states of the atoms are measured conditionally or nonselectively. We found that arbitrary steady state entanglement of the two nonlocal fields can be produced in the form of entangled trapping states. These are experimentally feasible in the short-time transient regime when dissipation does not exceed a certain threshold. In the second part we show a way to translate entanglement of the nonlocal fields into entanglement of spatially separated atomic beams. The entangled atoms can have several applications in, e.g., studies of interatomic correlations in lasers and micromasers, teleportation, or tests of local realistic theories. Here, in particular, we discuss an application to test complementarity using a Ramsey type setup. The correlation between the atoms provides us with "Welcher Weg" information resulting in a destruction of the interference fringes without leading to Heisenberg's uncertainty principle. Manipulation of information results in an experimentally feasible version of the "quantum eraser".

1. Introduction

Micromasers [1] have been extensively used for direct studies of the quantum features in the interaction of two-level atoms with a single quantized mode of the electromagnetic field. This device can be used to generate different kinds of nonclassical states of the

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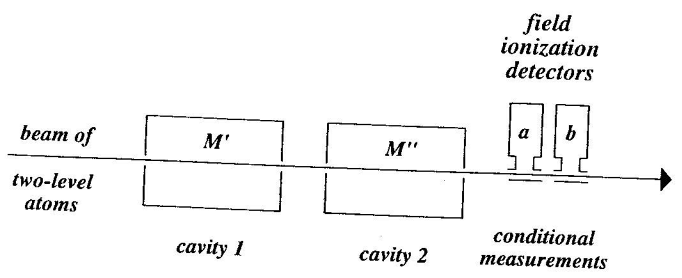

Fig. 1. Schematic arrangement of two micromasers coupled by a beam of two-level atoms the states of which are conditionally measured after the interaction.

radiation field [2-4] such as macroscopical quantum superpositions when injecting the atoms in a coherent superposition of the two levels into the cavity [5]. In the present paper we investigate two lossless micromasers coupled by the common pumping atomic beam [6-8] as depicted in Fig.1. Atoms enter the first cavity in one of their two definite states and then proceed to the second one without any time delay between the cavities. In this case the atomic coherence to be injected into the second micromaser is prepared by the first one. As we will see at optimum conditions, this can result in a quantum superposition of the two (macroscopically separated) micromaser fields. We assume that there is at most one atom in the cavities at a time to avoid collective effects. The final states of the atoms are measured after the interaction with 100% detector efficiency but we impose a condition on the outcome: only those sequences of atomic states are considered that follow our prescription. We do not know prior to the experiment which sequence will be produced, but we can redo the experiment until the desired one is obtained. This is called conditional measurement [9]. In particular, we consider the four simplest sequences of the final atomic states. In the case of the energy preserving (transferring) schemes we require each atom to be injected and detected in the same (different) states. We investigate the evolution of the fields under these schemes and show that a correlation and ultimately a steady state *entanglement* of the two *nonlocal* fields can arise due to the interference of the two atomic paths that an atom can follow to reach the same final state [7, 8, 10].

We want to note that the lecture given at the meeting also covered nonselective measurement schemes and the effect of dissipations on the entanglement of the fields. Here, we just mention that the two fields can also be entangled using nonselective measurement schemes. In the framework of that formalism it is shown that entanglement can be generated in the transient regime even when cavity losses and finite temperature thermal radiation are present. We find that entanglement of fields is experimentally accessible for tens of milliseconds using presently available state of the art facilities. We refer the interested reader to Ref.[8] for further details.

In the second part of the paper we show that nonlocal entanglement of fields can be translated into an entanglement of macroscopically separated atomic beams. These atoms can have several applications from testing fundamentals of quantum mechanics to studies of effects of atomic correlations in lasers and micromasers. Here, in particular,

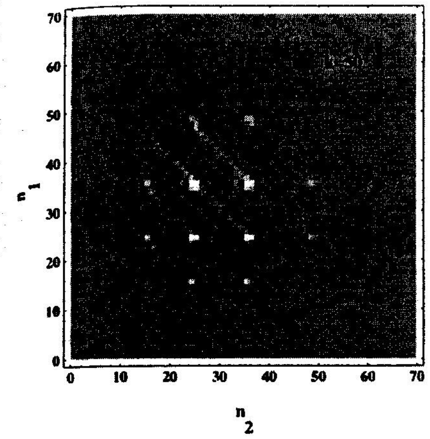

Fig.2 (a) Density plot of the amplitude distribution of the fields in scheme $a-M'M''-a$ at atom number $k = 50$ for $g\tau = \pi$ starting from coherent fields of parameter $\alpha^2 = 30$. The generated Fock states are located at photon numbers that are square integers minus one under the envelope of the amplitude distribution of the initial fields.

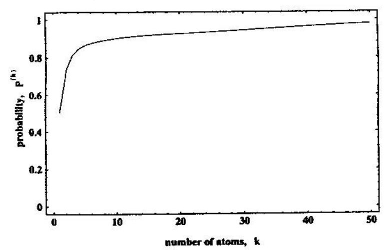

Fig.2 (b) The probability of detecting the upper state of atom number k in the scheme in Fig.2(a).

we apply them to test the principle of complementarity using Ramsey's two-field atomic interferometer setup. It will be shown that the correlation between the atoms itself makes "Welcher Weg" information available and destroys interference. There is no need to include Heisenberg's uncertainty principle in the analyses. We also show that information can be manipulated leading to a "quantum eraser" setup.

The paper is organized as follows. The two-cavity problem is formulated in Sect.II. Then the production of entangled states of nonlocal fields for the above mentioned conditional measurement schemes is studied in Sect.III. In Sect.IV we use entangled fields to generate entangled atomic beams and apply the latter in tests of complementarity. Section V is devoted to discussions and summary.

2. Coupled micromasers with conditional measurements of atoms

Let us consider two micromaser fields coupled by the common monoenergetic pumping beam of two-state atoms (upper, $|a\rangle$, and lower, $|b\rangle$) in such a way that atoms first interact with the field in cavity 1 and then proceed to cavity 2 (see Fig.1). We consider single quantized modes of the microwave radiation in the two lossless cavities of zero temperature. The interaction is resonant and we assume that there is at most one atom in the cavities at a time. The final states of the atoms are measured with 100% detection efficiency. We investigate the two conditional measurement schemes denoted by $a-M'M''-a$ and $a-M'M''-b$ indicating the state of each atom, $|a\rangle$ or $|b\rangle$, before and after the two maser cavities, M' and M''. We want to note here that schemes, $b-M'M''-b$ and $b-M'M''-a$, provide very similar results (see Ref.[7]) and, therefore, are not discussed in this paper. The state of the fields after the $k-1^{st}$ atom left but before the k^{th} atom entered the cavity is given by

$$|\Psi^{(k-1)}\rangle = \sum_{n_1,n_2} \Psi^{(k-1)}_{n_1,n_2}|n_1,n_2\rangle \quad . \tag{2.1}$$

After the interaction of the k^{th} atom with the fields the state of the atom-fields system reads as

$$|\Psi^{(k)}\rangle = \sum_{n_1,n_2} \Psi^{(k-1)}_{n_1,n_2}[C'_{n_1+1}(C''_{n_2+1}|a,n_1,n_2\rangle - iS''_{n_2+1}|b,n_1,n_2+1\rangle)-$$
$$-iS'_{n_1+1}(C''_{n_2}|b,n_1+1,n_2\rangle - iS''_{n_2}|a,n_1+1,n_2-1\rangle)] \quad , \tag{2.2}$$

where $S'_{n_1} \equiv sin(g'\tau'\sqrt{n_1})$ and $C'_{n_1} \equiv cos(g'\tau'\sqrt{n_1})$ with atom-field coupling constant, g', and interaction time, τ', corresponding to the first and the double-primed ones, S''_{n_2} and C''_{n_2}, with g'' and τ'' to the second micromaser. Then, the state of the k^{th} atom is measured resulting in a reduction of the state vector to a pure state of the fields given by

$$|\Psi^{(k)}\rangle = N^{(k)} \sum_{n_1,n_2} \Psi^{(k)}_{n_1,n_2}|n_1,n_2\rangle \quad . \tag{2.3}$$

Each measurement is followed by a renormalization of the state vector by $N^{(k)}$. The new amplitudes, $\Psi^{(k)}_{n_1,n_2}$, are functions of the old ones, $\Psi^{(k-1)}_{n_1,n_2}$, and for our two schemes they read as follows.

$$\Psi^{(k)}_{n_1,n_2} = \Psi^{(k-1)}_{n_1,n_2}C'_{n_1+1}C''_{n_2+1} - \Psi^{(k-1)}_{n_1-1,n_2+1}S'_{n_1}S''_{n_2+1} \quad , \tag{2.4a}$$

$$\Psi^{(k)}_{n_1,n_2} = \Psi^{(k-1)}_{n_1,n_2-1}C'_{n_1+1}S''_{n_2} + \Psi^{(k-1)}_{n_1-1,n_2}S'_{n_1}C''_{n_2} \quad , \tag{2.4b}$$

for $a-M'M''-a$ and $a-M'M''-b$, respectively, providing us with the iteration rules determining the evolution of the state of the fields from atom to atom. The probability of finding the k^{th} atom in the desired state is

$$P^{(k)} = \sum_{n_1,n_2} |\Psi^{(k)}_{n_1,n_2}|^2 = \frac{1}{N^{(k)2}} \quad . \tag{2.5}$$

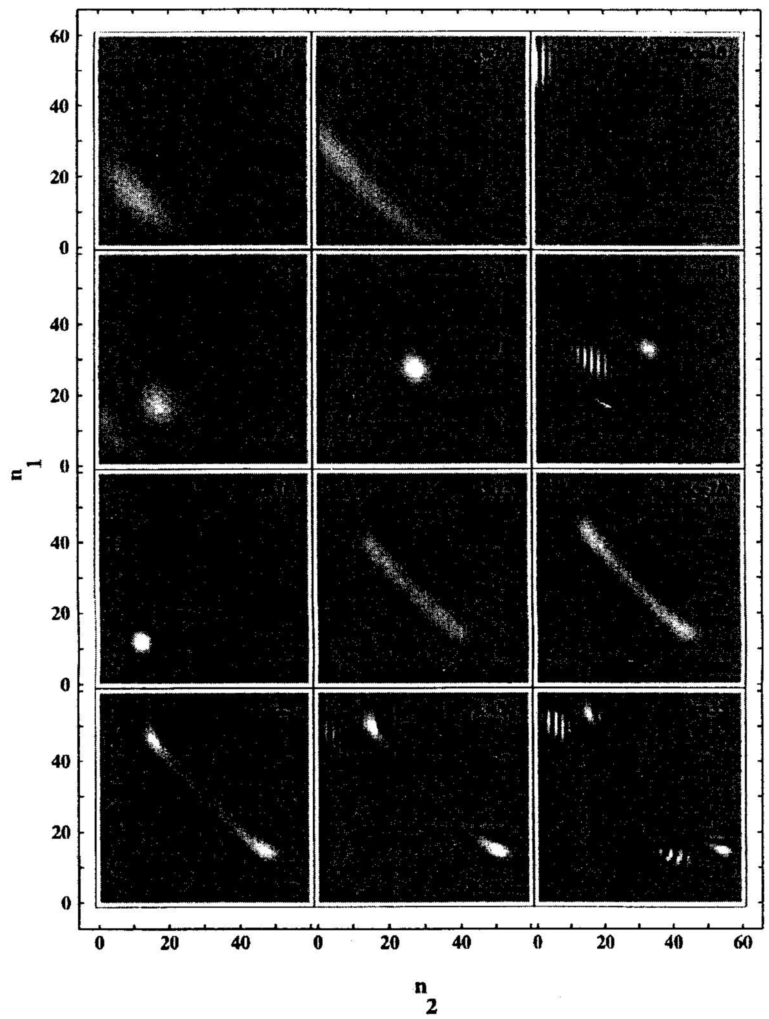

Fig. 3 (a) Density plots showing the evolution of the amplitude distribution of the fields in scheme $a-M'M''-b$ starting from coherent fields for $k=0$, where $g\tau = 0.3$ for the first, 0.5 for the second and 0.8 for the third and fourth rows. The first and second rows represent the correlated and uncorrelated regimes, respectively, while a transition between these two regimes as well as a double-peaked distribution at $k=30$ can be seen in the third and fourth ones.

The evolution of the two fields will be studied in the next section by iterating the amplitudes according to one of the rules above starting from coherent states of the two fields. We are going to investigate the correlations building up between the two micromasers as a result of the interference between the two paths that each atom can

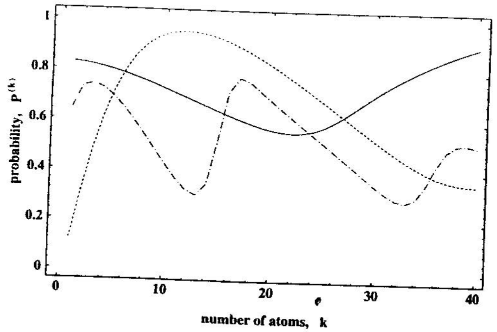

Fig.3 (b) The probabilities of detecting the lower state, $|b\rangle$, of atom number k in scheme $a - M'M'' - b$ during the evolution of the fields shown in Fig.3(a) for $g\tau = 0.3$, 0.5 and 0.8 depicted by the solid, dashed and dot-dashed lines, respectively.

follow when traversing the cavities. In order to do so we define m^{th} order correlation by the nonseparability condition given by

$$\langle(\hat{a}_1\hat{a}_2^\dagger)^m\rangle \neq \langle\hat{a}_1^m\rangle\langle\hat{a}_2^{\dagger m}\rangle \quad , \tag{2.6}$$

where $\hat{a}_1$ and $\hat{a}_2$ are the field operators of micromasers 1 and 2, respectively. We would like to draw attention to the fact that this is a correlation between fields of two different micromasers, i.e. an entanglement of two *nonlocal* subsystems. Thus, carrying out a measurement that reduces the state of one of the fields results in a reduction of the state of the other field located at a different point in space. One example of such state vector of m^{th} order correlation is given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|n, n+m\rangle \pm |n+m, n\rangle) \quad , \tag{2.7}$$

a possible production of which will be shown in the next section.

3. Transient and steady state entanglement from initial coherent states

Let us assume that both fields are initially in coherent states of the same amplitude, α, given by

$$|\Psi^{(0)}\rangle \equiv |\alpha, \alpha\rangle = e^{-\alpha^2} \sum_{n_1,n_2} \frac{\alpha^{n_1+n_2}}{\sqrt{n_1!\, n_2!}} |n_1, n_2\rangle \quad . \tag{3.1}$$

According to Eq.(2.6) the two fields are uncorrelated because their state vector is separable into a tensor product of two coherent states in the two cavities as $|\alpha\rangle_{cavity1} \otimes |\alpha\rangle_{cavity2}$. We are going to consider typical examples for the evolution of the fields for this initial condition using the energy-preserving and transferring conditional measurement schemes, applying the corresponding iteration rules of Eq.(2.4), and assuming equal interaction parameters, $g\tau \equiv g'\tau' = g''\tau''$, in both cavities.

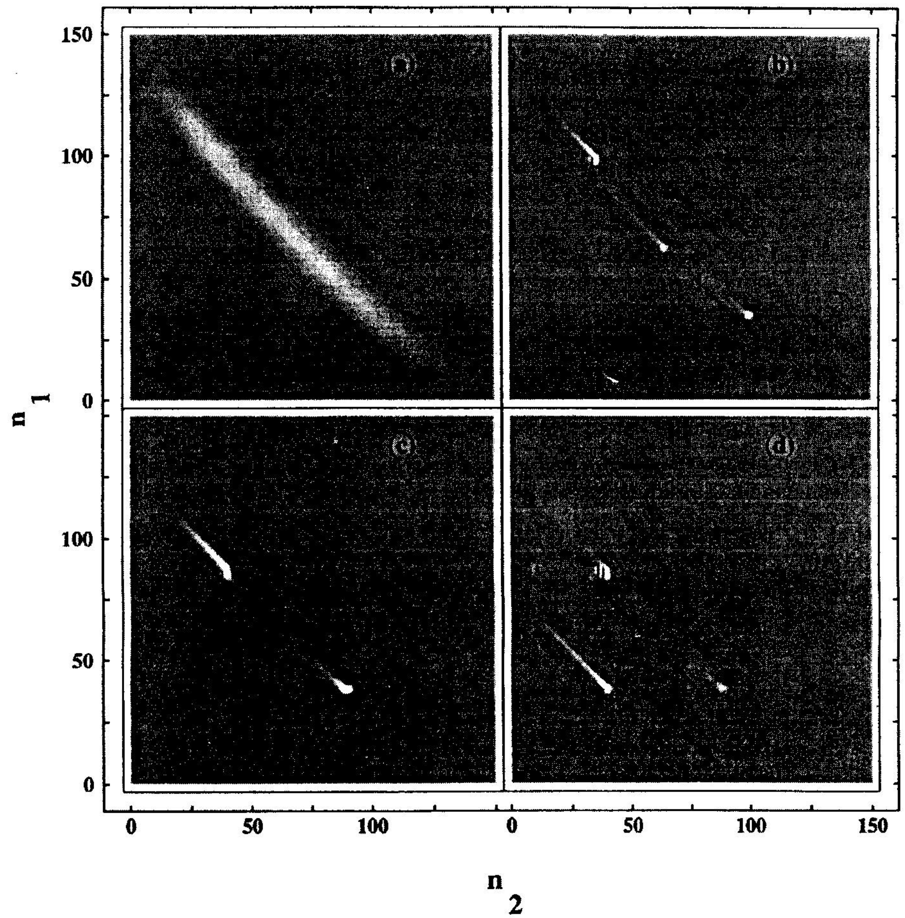

Fig. 4 Density plots of the amplitude distributions of the fields. (a): at the 100^{th} atom starting from coherent fields of $\alpha^2 = 30$ in scheme $a - M'M'' - b$ for $g\tau = 0.142$; (b): at the 300^{th} atom after switching from the field generated in (a) at the 100^{th} atom to $a - M'M'' - a$ for $g\tau = \pi/2$; (c): same as (b) but switching to $g\tau = 1.0$; (d): same as (c) but switching at the 50^{th} atom.

In the case of the energy-preserving scheme, $a - M'M'' - a$, the fields in both micromasers settle to a superposition of Fock states where the corresponding Rabi angles are multiples of π at steady state. As a result of this the atoms are in their upper states before, between and after the cavities. In the example depicted in Fig.2(a), where $\alpha^2 = 30$ and the interaction parameters are $g\tau = \pi$, $(g\tau \equiv g'\tau' = g''\tau'')$ the Fock states are located at integer squares minus one, predominantly at 24 and 35. It is easy

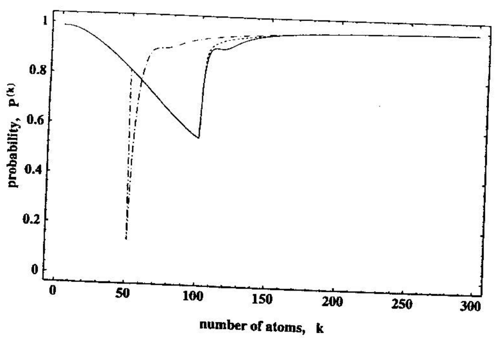

Fig. 5 Probabilities of detecting the required state of atom number k corresponding to the evolution of the fields in the schemes given in Fig. 4 (b), (c) and (d) depicted by solid, dashed and dot-dashed lines, respectively. The probability decreases before the switching, then after a transient at the switching it jumps to a high value and then increases to unity at steady state.

to see that the two fields are uncorrelated at steady state, because the state vector is approximately separable as $(|24\rangle+|35\rangle)_{cavity1}\otimes(|24\rangle+|35\rangle)_{cavity2}$ (unnormalized). The probabilities of detecting the atoms in their upper states as required by the conditional measurement scheme are depicted in Fig.2(b).

Let us now consider the energy-transferring schemes, $a-M'M''-b$. Fig.3(a) shows typical evolutions of the fields for interaction parameters, $g\tau = 0.3, 0.5$, and 0.8, starting from coherent fields of $\alpha^2 = 10$. It can be seen in the first row of the figure for $g\tau = 0.3$ that the distribution is stretched along a straight line as it is shown for $k = 20$. The fields are correlated in this regime. The distribution then separates into two regions and finally ends its regular evolution in a rapidly oscillating structure around $k = 40$ that corresponds to the so-called "single-cavity" trapping effect. In the second row for $g\tau = 0.5$ the photon number increases in both cavities in such a way that the distribution localizes around a single point where $n_1 \cong n_2$ showing a balance between the fields. The two micromasers are uncorrelated in this regime. At $n_1 \cong n_2 \cong 40$ the oscillatory structure shows that the system reached a single- cavity trapping point.

For larger interaction parameters, $g\tau = 0.8$, shown in the third and fourth rows of Fig.3(a) the system undergoes a transition between the above two uncorrelated and correlated regimes. On the other hand, the stretched distribution becomes double peaked around $k = 30$ showing fields with state vector approximately of the form, $|15,50\rangle+|50,15\rangle$ (unnormalized). This is the result of a new trapping effect called "two-cavity" trapping. Finally, the double-peaked distribution built up by $k = 30$ begins to be destroyed at $k = 35$ by the co-existing single- and two-cavity trapping mechanisms. The probabilities that the atoms are detected in their lower state according to the measurement scheme are depicted in Fig.3(b) for the three examples given in Fig.3(a). It can be seen that the probability drops at the transitions between the uncorrelated and correlated regimes as well as at the trapping. According to Eqs.(2.6) and (2.7), the transient state above, $|15,50\rangle + |50,15\rangle$, exhibits 35^{th} order correlation.

Now, we want to combine these two kinds of schemes starting with the energy-transferring one and then at an optimum number of atoms switching to the energy-preserving one. This way the Fock states generated by the latter scheme will be located under the envelope of the fields generated by the former scheme at steady state. Let us start for example the system in the energy-preserving scheme at $g\tau = 0.142$, from uncorrelated coherent fields of $\alpha^2 = 30$. After 100 atoms the generated fields will exhibit a long stretched distribution as depicted in Fig. 4(a) showing strong correlation between the fields. This correlation would be destroyed by the atoms to come due to the appearance of the oscillatory structures if the energy-transferring scheme was followed any further. We switch our system to the other scheme instead. From the 101^{st} atom we continue in the energy-preserving scheme using different parameters, $g\tau = \pi/2$. (The interaction time can be changed in an experiment by changing the velocity of the atoms.) After the next 200 atoms we get a superposition of three Fock states at squares of even integers minus one depicted in Fig.4(b) that could be approximated by the state, $|99,35\rangle + |63,63\rangle + |35,99\rangle$ (unnormalized). This is approximately equal to the steady state of the fields. A double-peaked superposition of the fields can be produced if instead of $\pi/2$ we chose $g\tau = 1.0$ as new parameters. The generated fields are depicted in Fig.4(c) and can be approximated with the state, $|88,38\rangle + |38,88\rangle$ (unnormalized) showing a 50^{th} order correlation at steady state. Although the method is not particularly sensitive to the atom number where the switch is to be made, we show what happens if we switch between the schemes too early in Fig.4(d): instead of $k = 100$ now at $k = 50$. Since the distribution at the moment of the switch is much broader this time than it was in Fig.4(a) a peak at $|38,38\rangle$ can arise. The probabilities of detecting the atoms in the desired states ($|b\rangle$ before and $|a\rangle$ after the switch) are depicted in Fig.5. In principle, arbitrary superpositions can be produced by choosing the appropriate initial coherent states and interaction parameters. In the energy- transferring scheme $g\tau|\alpha| = \pi/4$ needs to be satisfied in order to produce a stretched distribution. When it is prepared we need to switch to new interaction parameters for the energy-preserving scheme that will generate the desired Fock states assuring Rabi angles of multiples of π under the envelope of the stretched distribution.

4. Entanglement of atomic beams: a tool to test complementarity

After having the entangled states of macroscopically separated fields prepared we now want to utilize them to produce entangled states of distinct atomic beams. Employing two atomic beams, one traversing the first and the other traversing the second cavity (see Fig.6) the correlation between the two nonlocal fields can be translated into a correlation between the two distinct atomic beams. Let us start from the atom-field state given by

$$(|n+m,n\rangle \pm |n,n+m\rangle)\cdot|b_1',b_1''\rangle \quad , \qquad (4.1)$$

where $|b_i'\rangle$ and $b_i''\rangle$ are the lower states of the i^{th} atoms in beams 1 and 2, respectively. (For the sake of simplicity we are going to omit normalization constants throughout the rest of the paper.) Apparently, the two atoms are initially uncorrelated, while the state of the two fields exhibits m^{th} order correlation. Using the same interaction parameter,

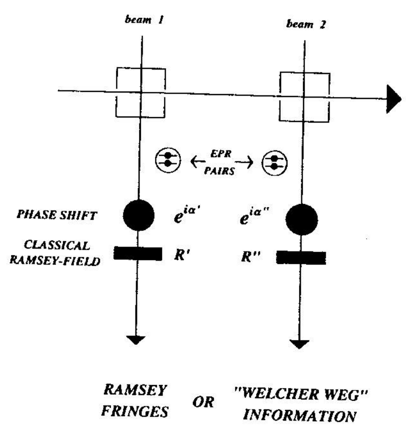

Fig. 6 Schematic arrangement of two entangled atomic beams to test complementarity by detecting Ramsey-fringes (interference) in one and atomic state ("Welcher Weg") in the other beam. First we apply a Ramsey-apparatus including a phase shift, $e^{i\alpha'}$, and a classical field, R', in beam 1 alone, and then together with the second one of $e^{i\alpha''}$ and R'' in beam 2.

$g\tau$, for both cavities satisfying the two equations, $sin(g\tau\sqrt{n}) = 0$ and $cos(g\tau\sqrt{n+m}) = 0$, simultaneously (or at least approximately) the final state after the first pair of atoms reads as

$$|n+m-1,n\rangle \cdot |a_1', b_1''\rangle \pm |n, n+m-1\rangle \cdot |b_1', a_1''\rangle \quad . \tag{4.2}$$

In the case of $m = 1$ the atoms and the fields disentangle into the EPR-state [11],

$$|a_i', b_i''\rangle \pm |b_i', a_i''\rangle \quad , \tag{4.3}$$

of the first ($i = 1$) pair of atoms and the number state, $|n,n\rangle$, of the fields. By periodically reconstructing the $m = 1$ field-state from $|n,n\rangle$ [8], and sending subsequent pairs of lower-state atoms through the cavities two beams can be produced where the atoms are correlated in pairs as shown in Eq.(4.3) for the i^{th} pair. For higher initial field-correlations, $m > 1$, the atoms and the fields do not disentangle. The subsequent atom-pairs will also become correlated among themselves forming larger clusters of entangled atoms. Various kinds of entangled beams of EPR-pairs or of arbitrarily large clusters of atoms can be engineered by using the proper initial states and interaction parameters. The correlation between the atomic beams can be measured by detecting (anti-) coincidences of atomic states using field ionization detectors.

Several possible applications of these entangled atomic beams can be envisioned such as to study the effect of interatomic correlations on the radiation fields of lasers and micromasers driven by the two beams simultaneously. The singlet states of the atom-pairs [taking the minus sign in Eq.(4.3)], in particular, play an essential role in the recently suggested teleportation of quantum states [12]. The correlated beams could also be used in fundamental tests of quantum mechanics, as the photon-coincidence experiments [13], experiments challenging local realistic theories [14], and others where the entangled atoms would be employed to substitute for the entangled photons.

In this paper we propose another experimentally feasible application of these correlated atomic beams to test the complementarity principle of quantum mechanics. To

this end we are going to use Ramsey's two-field method [15] that, when applying it to a single uncorrelated atomic beam, can be summarized as follows. Starting from a definite atomic state we apply two consecutive $\pi/2$-pulses on the atoms as Ramsey's first and second fields of $\Omega\tau = \pi/2$ in each pulse where Ω, Rabi frequency and τ, interaction time. Between the pulses the relative phase of the atomic levels is shifted by $e^{i\alpha}$ by, e.g., differential Stark shift using a static field. The probabilities of finding the atoms in definite final states display interference fringes. In the case of initial upper state, $|a\rangle$, they are given by

$$P_{\pm} = \frac{1}{2}(1 \pm \cos\alpha) \quad , \tag{4.4}$$

where $+(-)$ corresponds to the lower (upper) atomic state. The interference is the consequence of the two possible atomic "paths" to reach the same final state, viz., after the first pulse the atom is in a coherent superposition of the two states. The relative phase of the superposition can be varied via $e^{i\alpha}$ in order to display the fringes.

Let us now apply Ramsey's apparatus on one of the two entangled atomic beams discussed above. As a result of the correlation between the atoms "Welcher Weg" information about one of the beams can, in principle, be extracted from its correlated pair and according to complementarity interference fringes are bound to disappear. This can be seen by starting from the symmetric version of the correlated state of Eq.(4.3) that, after the phase shift of $e^{i\alpha'}$ and the subsequent classical Ramsey-field, R', in beam 1 (Fig.6), ends up in the final state given by

$$i|a'\rangle \cdot (S'|a''\rangle + ie^{i\alpha'}C'|b''\rangle) - |b'\rangle \cdot (C'|a''\rangle - ie^{i\alpha'}S'|b''\rangle) \quad . \tag{4.5}$$

(Here, the first of the two coupled micromasers serves as Ramsey's first field for beam 1.) The coefficients, $S' \equiv \sin(\Omega'\tau'/2)$ and $C' \equiv \cos(\Omega'\tau'/2)$, are equal and factor out in the case of a $\pi/2$ - pulse (for $\Omega'\tau' = \pi/2$). The detection probabilities of the individual atomic states and those of the (anti-) coincidences are constants showing no oscillations as functions of α' at all. As a consequence of the strong atom-atom correlation in Eq.(4.3) the two spatially separated atoms serve as "Welcher Weg" detectors of one another. The atomic state in one beam keeps track of the evolution of that in the other resulting in no indistinguishable paths that could interfere. We want to emphasize that, as in the interferometer schemes of Refs.[16], it is the correlation between the "interference" atom and its "Welcher Weg" detector pair in the other beam that, via making information available, enforces complementarity. There is no added noise in the system of any kind that would lead us to Heisenberg's uncertainty relation.

The information stored in the "Welcher Weg" atoms can, however, be manipulated by applying a phase shift, $e^{i\alpha''}$, and a classical field, R'', on the detector atoms in beam 2 (see Fig. 6). The new state vector starting from Eq.(4.5) reads as

$$i(e^{i\alpha'}C'S'' + e^{i\alpha''}S'C'')|a',a''\rangle + i(e^{i\alpha'}S'C'' + e^{i\alpha''}C'S'')|b',b''\rangle -$$
$$-(e^{i\alpha'}C'C'' - e^{i\alpha''}S'S'')|a',b''\rangle + (e^{i\alpha'}S'S'' - e^{i\alpha''}C'C'')|b',a''\rangle \quad , \tag{4.6}$$

where the coefficients, S', C' and S'', C'' corresponding to R' and R'', respectively, are equal and factor out in the case of $\pi/2$ -pulses ($\Omega'\tau' = \Omega''\tau'' = \pi/2$). The detection probabilities of coincident and anticoincident atomic states in the two beams (first and second two terms in Eq.(4.6), respectively) can be calculated as

$$P_{\pm} = \frac{1}{4}[1 \pm \cos(\alpha' - \alpha'')] \quad , \tag{4.7}$$

where the (lower) upper sign corresponds to the (anti-) coincidences. Apparently, these probabilities show interference (anti-) fringes oscillating as functions of the difference phase, $\alpha' - \alpha''$, similarly to Eq.(4.4). At the same time the detection of any individual atomic state in any beam disregarding the atomic state in the other beam exhibits constant probabilities ($P_+ + P_- = 1/2$) that are the same as the initial ones in Eq.(4.5). Consequently, in order to display Ramsey-fringes in, for example, beam 1 one needs to detect the final atomic state on its counterpart in beam 2 thereby undoing the correlation between the atoms and, similarly to the "quantum eraser" in Ref.[16], erasing the stored "Welcher Weg" information. This way, the detections in beam 1 can be separated into two groups: one when the state coincides with the state in beam 2 and another when there is anticoincidence. The coincidence and anticoincidence detection probabilities both exhibit interference fringes which, however, cancel from their sum and add up to the constant detection probability of a given final state in beam 1. The detections in beam 2 can take place at any point following the field, R'', even long after the detection event of the atomic state in beam 1, allowing for a delayed choice as we decide whether to emphasize wave-like (interference) or particle-like ("Welcher Weg") behavior. Together with other similar setups realizing the "quantum eraser" [16] these schemes provide experimentally feasible tests of quantum complementarity.

5. Summary

Two lossless micromasers are coupled by the common pumping beam of two-level atoms the fields of which are studied performing conditional measurements on the final state of the atoms. There are two paths that the atoms can follow when traversing the cavities to reach the same final state reminiscent of Young's double-slit experiment. The probabilities of these paths can be manipulated by the interaction times in the cavities correlating or decorrelating the two macroscopically separated micromaser fields. Four conditional measurement schemes have been considered: in the case of the energy preserving (transferring) schemes we require each atom to be injected and detected in the same (different) states. Starting from coherent states of the fields it has been shown that energy-transferring conditional measurement schemes can produce transient entanglement of the fields. In the case of energy-preserving schemes a set of Fock states is generated at steady state under the envelope of the initial fields. If the initial fields were uncorrelated (correlated) the generated ones would be uncorrelated (correlated) too. This suggests that, by switching from an energy-transferring to an energy-preserving scheme, steady state correlation between the fields can be produced. The Fock states generated by the latter scheme will in this case be located under the envelope of the correlated fields generated by the former scheme. This results in steady-state entanglement of the nonlocal fields such as described by the state, $|n, n+M\rangle \pm |n+M, n\rangle$. In the absence of dissipations steady-state quantum superpositions of arbitrary number states of macroscopically separated fields – sometimes referred to as nonlocal "Schrodinger-cats" – can be generated.

As an application of entangled fields we have proposed experimentally feasible tests of complementarity based upon spatially separated (EPR-) correlated atomic beams. Ramsey interference fringes are detected (or not) in one of the beams, while "Welcher Weg" information (or not) in its correlated pair. There is no added noise in the system that would lead to Heisenberg's uncertainty relation, complementarity is enforced by the correlation between the "interference" atom and its "Welcher Weg" detector pair. The available information and the contrast of the interference fringes can be manipulated using classical Ramsey fields realizing the "quantum eraser". We conclude by noting that further applications of entangled atoms can easily be envisioned. For example, quantum optical experiments that, thus far, have been realized with entangled photons only can be redone with entangled atoms. Teleportation of quantum states, EPR-experiments, or the study of collective atomic effects on the radiation field are a few of the many possibilities.

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