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FINE STRUCTURE IN THE ^{20}O-CLUSTER DECAY OF THE ^{229}Th NUCLEUS WITHIN THE ENLARGED SUPERFLUID MODEL

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Within one level R-matrix approach several hindrance factors for the ^{20}O radioactive decay are calculated. The interior wave functions are supposed to be given by the shell model with effective residual interactions, e.g. the enlarged superfluid model. The exterior wave functions are calculated from a cluster-nucleus double-folding model potential obtained with the M3Y interaction. We analyzed the fine structure of cluster decay in the case of the ^{20}O-decay of ^{229}Th as an example. Good agreement with the experimental data is obtained.

1. Introduction

Recently Hourani and his co-workers [1] experimentally discovered the fine structure in the ^{14}C radioactivity [2]. The theoretical studies of α- [3,4] and heavy-cluster decay [4,7] have very much in common. We study the decay process as composed of two main steps: First the mother nucleus makes a kind of phase transition from the initial state, which could be of any structure (Fermi liquid, superfluid, spherical or deformed, one or many α- cluster state, one or many combined heavy cluster state etc), to the final state composed of at least one cluster, which is to be emitted, and the residual nucleus, which may have also any structure as above. One mechanism of such a transition could be the cluster condensation, or what usually is assumed to be a formation of the cluster from already formed condensates of smaller cluster [5]. Another mechanism could be the slow shape deformation [6] from an initial shape configuration of the studied many-particle system through the shapes that are energetically very unfavored to the shape corresponding to two daughter nuclei in contact.

Secondly, the two daughter nuclei tunnel through the potential barrier in their relative motion, without further change in shape.

Most of the theoretical models of heavy cluster decay [7] is based assentially on Gamov's theory [8], which was the first success of quantum mechanics applied to the α-decay phenomenon, i.e. a detailed description of the second step - the tunneling through the potential barrier. The differences in approaches are related to the way of

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calculating the potential barrier defined by the interaction potential acting between the emitted cluster and the residual nucleus. All these theoretical treatments fit a special dependence of the favored cluster transitions, analogous to the Geiger-Nuttal law [9] for favored alpha-decay, which emerges directly from the simplest Jeffreys-Wentzel-Kramers-Brillouin (JWKB) expression of the penetrability determined by the square well plus Coulomb interaction potential. The unfavored transitions do not follow the Geiger - Nuttal law, because of the large variations of the reduced widths [3,4,10–12], which have a key role in the understanding of the decay process and require a precise knowledge of the structures of the initial and final quantum states. We can learn much about the structure of the atomic nuclei and the mechanism of the decay phenomenon from such transition. For instance, when treating the favored cluster decays, one assumes that the nucleons used to build the cluster are more or less strongly correlated in the initial state. This fact leads to small hindrance factors [4,10,11]. On the contrar, unfavored transitions (with large hindrance factors) are characterized by the fact that the nucleons used to build the cluster are collected from different strongly correlated groups of nucleons entering the structure of the initial state. In this last case it is necessary to breakup the correlated groups of nucleons first and then to build the cluster, which is going to be emitted.

In paper [4], the formal expressions for the theoretical hindrance factors are derived.

In the present paper we continue this work and calculate several hindrace factors for the ^{20}O radioactivity. The calculations will be performed by using the enlarged superfluid model (ESM) [4].

2. The ^{20}O - cluster decay of the ^{229}Th

The ^{229}Th nucleus belongs [13] to the well known region of soft nuclei with $Z \approx 88$ and $N \approx 134$, with strong octupole correlations in the ground and low-lying excited states, where the $lj_{15/2}$ intruder orbital interacts strongly with the $2g_{9/2}$ natural parity orbital. The hindrance factors for both the alpha and ^{20}O - decays of the ground state of the ^{229}Th are very difficult to be calculated at the moment, due to the lack of accurate knowledge of structure of the mother and daughter nuclei. Studying the experimental hindrance factors for alpha-decays to ^{225}Ac ground and low lying excited states [14] we learn that about fifteen transitions have small (≤ 100) hindrance factors and five haved hindrance factors of these transitions less or equal to 10.

The corresponding excited states have very different structure and this tells us that the structure of the ground state of ^{229}Th is not as simple as e.g. the ^{225}Fm case [15], and it may contain many more-or-less equal components of single quasi-particle or quasi-particle-phonon structure.

Unfortunately, not all the spins and parities of the ^{225}Ac excited states, populated by alpha-decay, are known. Thus it is a really difficult problem to describe the quantum states involved in the alpha and ^{20}O-decay of the the ^{229}Th. It is not sufficient to describe these states within an independent particle model only [16], [17]. Residual interactions could play an important role [10]. The restrictions concerning the number of quasiparticles and phonons lead to inaccurate structure of the ^{229}Th-nucleus.

First, the valence single-particle space should be extended, and secondly, the number of quasiparticles and phonons should be increased at the next step when incorporating the quasiparticle-phonon interaction. Such a task is as hard as to perform the calculations within the shell model code with realistic residual interaction [4]. To understand this situation we construct a very simple model, which proves to deserve attention by itself and to suggest the highly nontrivial behaviour of any realistic model. Assume, for a moment, that the structure of the ground state of the ^{229}Th-nucleus consists of spherical core described by an independent-particle model. Above the core, there exists a deformed single particle orbital only. The wave function for this orbital can be expanded in terms of spherical orbitals. In this case the spectroscopic factor in the expression of the hindrance factor HF [4].

$$HF^{(I_iK_i\pi_i \to I_fK_f\pi_f)}_{lK_i-K_f} = \frac{|\sum_N \Theta^{OO+(g.s.)\to OO+(g.s.)}_{NOO} R_{NO}|^2}{|\sum_l F_l \sum_N \Theta^{(I_iK_i\pi_i \to I_fK_f\pi_f)}_{NlK_i-K_f} R_{Nl}|^2} \quad (1)$$

may be factorized [15] according to:

$$\Theta^{(I_iK_i\pi_i \to I_fK_f\pi_f)}_{NlK_i-K_f} = C_{\Omega_i} C_{\Omega_f} a^{\Omega_i=K_i}_{N_il_ij_i} a^{\Omega_F=K_f}_{N_fl_fj_f} \sqrt{2I_f+l} \begin{pmatrix} I_i l I_f \\ K_i K K_f \end{pmatrix} \Theta^{(j_i\pi_i \to j_f\pi_f)}_{core} \quad (2)$$

Here, $C_{\Omega_{i(f)}}$ are the weights of the single quasiparticle state in the structure of the i(f) - state (see eq. (5) of Ref. [23]) of the initial and final states, a^{Ω}_{Nlj} are the corresponding Nilsson-like amplitudes ($\chi_\Omega = \sum_{Nlj} a^{\Omega}_{Nlj} |Nlj\Omega>$), $\begin{pmatrix} I_i l I_f \\ K_i K K_f \end{pmatrix}$ stands for the 3-j symbol and $\Theta^{(j_i\pi_i \to j_f\pi_f)}_{core}$ acts as a spectroscopic amplitude between many-body sherical $|j_{i(f)}\pi_{i(f)}>$ states, including both the cluster overlaps [18], [4] and the intrinsic overlap integrals [4]. These spectroscopic factors may be calculated within the restricted Kuo-Herling model space [18], [24] including four neutron and proton orbitals above the shell closure at $Z = 82$, $N = 126$.

The expression of the hindrance factor (1) bacomes [15]:

$$\text{HF[mother nucleus}(I_i^{\pi_i} K_i) \to {}^{20}\text{O+ daughter nucleus}(I_f^{\pi_f} K_f)] \approx$$
$$\approx \{\sum_l F_l \,|\, C^{I_ilI_f}_{K_iKK_f} C_{\rho_i} C_{\rho_f} a^{\Omega_i=K_i}_{N_il_ij_i} a^{\Omega_f=K_f}_{N_fl_fj_f} (\text{RSA})_l l^{(i\to f)} |^2\}^{-1} \quad (3)$$

The ratio of the intrinsic spectroscopic amplitudes (RSA) from the eq. (3) is given by the ratio of $\Theta^{(j_i\pi_i \to j_f\pi_f)}_{core}$ calculated for the $^{229}Th \to {}^{20}O + {}^{209}Pb$ - transition and $\Theta^{(j_i\pi_i \to j_f\pi_f)}_{core}$ calculated for the $^{228}Th \to {}^{20}O + {}^{208}Pb$.

The quantities (RSA's) replace essentially the ratio of the favored intrinsic spectroscopic amplitudes [4] corresponding to the transitions between odd-mass and doubly even nuclei, respectively. The intrinsic spectroscopic amplitude (Θ_{int}) is defined as

$$\Theta_{int} = \sum_{\nu_1\ldots\nu_8} \sum_{\omega_1\ldots\omega_{12}} A^{LM}(\nu_1\ldots\nu_8|\omega_1\ldots\omega_{12}) \xi^{fav}(\nu_1\ldots\nu_8|\omega_1\ldots\omega_{12}) \quad (4)$$

E_f(keV)	0.	779.
RSA	0.52	
$I_f^{\pi_f}$(DSPC)	$\frac{9}{2}^+(2g_{9/2})$	$\frac{11}{2}^+(li_{11/2})$
$[Nn_z\Lambda]_i$	[633]	[622]
$[Nn_z\Lambda]_f$	[615]	[606]
$a_{nlj}^{\Omega_i}$	0.72	0.70
$a_{nlj}^{\Omega_f}$	1.0	1.0
C_{Ω_i}	1%	87%
C_{Ω_f}	98%	97%
HF_{exp}	1000.	17.
HF_{ESM}	≈ 1070	≈ 20

Table 1: The hindrance factors calculated within enlarged superfluid model [5] for the ^{20}O-transition ^{229}Th (g.s.) → ^{20}O+^{209}Pb. The abbreviations DSPC and RSA mean the dominant single particle configuration and the spectroscopic amplitude ratios.

Table 2: The structure of the ground and one excited state entering the cluster transition ^{229}Th (g.s.) →^{20} O+^{209}Pb, as calculated within enlarged superfluid model [5]

Nucleus	I^π K	E_{exp}(MeV)	E_{theo}(MeV)	Structure
^{229}Th	$\frac{5}{2}^+$ $\frac{5}{2}$	0.	0.	$87.91\%[633]\frac{5}{2}^+ + 1.1\%[622]\frac{5}{2}^+ + 2.1\%[743]\frac{7}{2}^- Q_{31} + 2.5\%[631]\frac{1}{2}^+ Q_{22}$
^{209}Pb	$\frac{11}{2}^+$ $\frac{11}{2}$	0.779	1.116	$97.23\%[606]\frac{11}{2}^+ + 1.04\%[615]\frac{11}{2}^+ + 2.1\%[743]\frac{7}{2}^- Q_{32} + 2.5\%[725]\frac{11}{2}^- Q_{30}$

analogous to the quasiparticle contribution in the matrix element of eq. (11) of Ref. [23] entering the alpha-decay rate of axially deformed odd-A nuclei. The centrifugal factor F_l is defined as follows (see Ref. [10]): $F_l = P_l(Q)/P_o(q)$ where $P_l(Q)$ stands for the penetrability and Q the energy release of the studied decay. Within the JWKB approximation, F_l has the following form

$$F_l = \exp \frac{2}{\hbar} \int_{R_C}^{r_o} [q_{l=o}(r) - q_l(r)]\mathrm{d}r, \tag{5}$$

in which r_o and R_C stand for the outer and inner turning points, respectively, and

$$q_l(r) = \sqrt{2m_o A_{red}(V^{coul+nucl} - Q)}. \tag{6}$$

Here, $V^{coul+nucl}$ is the sum of the Coulomb and nuclear one-body potential acting between the a-cluster and the daughter nucleus when studying the radial part of the Schrödinger equation. Usually the Coulomb part of this potential is replaced by point like Coulomb potential while the nuclear part by a Saxon-Woods one [7,10]. The nucleon mass is m_o and the reduced mass of the a-cluster and the daughter nucleus A is $A_{red} = aA/(a+A)$. Within such an approximation, we calculated the hindrance factors

for the ^{229}Th → ^{20}O+ ^{209}Pb cluster transitions. In Table 1, the intrinsic spectroscopic amplitude ratios have been estimated to be ≈ 0.52. In calculating the ^{229}Th ground state structure (see Table 2), the used enlarged superfluid model parametrs with the pairing coupling strengths G_p = 0.14 MeV, G_n = 0.10 MeV and the four-nucleon interaction constant G_4 = 0.26 keV. The parameters of the average field (see Ref. [19], [22]) are: V_{op} = 55.537 MeV, r_{op} = 1.30975 fm, a_p = 0.70071, $k_{s-o,p}$ = 5.56479 MeV; V_{on} = 37.787 MeV, r_{on} = 1.39628 fm, a_n = 0.70071, $k_{s-o,n}$ = 7.31907 MeV. The used deformation parameters are β_{20} = 0.15, β_{40} = 0.11. The used particle-hole quadrupole and octupole parameters (see Ref. [5]) are: $k_{n\sigma}^{\lambda\mu} = k_{o\sigma}^{2\mu}$ = 0.67 keV fm^{-4}; $k_{n\sigma}^{\lambda\mu} = k_{o\sigma}^{2\mu}$ = 0.06 keV fm^{-4}; $k_{n\sigma}^{\lambda\mu} = k_{o\sigma}^{3\mu}$ = 0.01 keV fm^{-6}; $k_{n\sigma}^{\lambda\mu} = k_{l\sigma}^{3\mu}$ = 1. eV fm^{-6}. The particle-particle quadrupole parameter (see Ref. [5]) is $G_{n\sigma}^{\lambda\mu} = G_{\sigma}^{2\mu}$ = 15 eV fm^{-4}.

A few more comments may be put here. First of all, our hybrid model with a spherical core and only one deformed orbital, when calculating the spectroscopic amplitudes is not to be taken too seriously for very complex structures. This should be not true even for structures close to single quasiparticle states, because the assumption of the axial deformed core is not realistic [20]. On the other hand, when having realistic structures for both the initial and final states, the calculations within shell model codes with realistic residual interactions [4] are practically not feasible for nowadays computers. Therefore simple schematic models like the above presented one would be useful. In the presented calculations, we estimated the core spectroscopic factor as in the case of the favored cluster decays, i.e. the magnitude of the core spectroscopic factor has been mainly evaluated by the the overlap integral between the spherical wave functions describing the valence odd neutron in the mother and daughter nuclei, which does not participate in the cluster decay. This overlap integral is less than unity due to the fact that the two above orbitals are oscillator orbitals with different frequencies [4].

3. Conclusion

In this work we reported calculations performed within the enlarged superfluid model [5] for some selected ^{20}O -transitions of the ^{229}Th nucleus. In this case, difficulties arise due to unknown structure of the ^{229}Th ground state and due to impossibility to calculate truly microscopically the spectroscopic amplitude. A schematic model has been applied to understand the heavy ^{20}O-cluster decay of the ^{229}Th.

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