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TWO-BODY RADIATIVE RECOMBINATION OF SLOW POSITRON AND ANTIPROTON INTO ANTIHYDROGEN

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The two-step radiative recombination process is found to provide an additional channel over the well-known spontaneous photorecombination channel, to the two-body radiative recombination of slow positron and antiproton into antihydrogen. The rate of the two-step radiative recombination process, which consists of (a) capture in an excited state and (b) subsequent decay to the ground state with the emission of a photon, is calculated using field theory in Coulomb gauge. The relative importance of the two channels on the rate for cold antihydrogen production at various low energy range, is discussed.

1. Introduction

The importance of antihydrogen as a new form of space propulsion is gradually being recognised. Antihydrogen is necessary to verify some fundamental properties of matter. The low enegry antihydrogen can be used to measure the 2s - 2p Lambshift, the hydrogen-antihydrogen atomic interaction and for detection of gravitational effect on antimatter.

Antihydrogen formation at LEAR has been first considered by Herr et al [1], and subsequently, Gabrielse et al [2] have discussed about the possibility of antihydrogen formation by merging cold trapped plasmas of antiprotons and positrons. In the experiment [1] circulating antiproton beam of low divergence and momentum spread is merged with positron beam, in a straight section of a storage ring. After radiative recombination, the antihydrogen emerge from the cooling section of the storage ring tangentially. In a merged beam the radiative recombination of antiproton and positron is possible in a three-body (one antiproton and two positrons) as well as in a two-body (one antiproton and one positron) encounters. The radiative recombination reactions are shown in the equations below. Recombination is said to be complete when the antihydrogen is formed in the ground state.

$$P^- + e^+ + e^+ \;\rightarrow\; \overline{H}_\downarrow(nl) + e^+$$
$$\overline{H}(1s) + h\nu \qquad (1)$$

$$P^- + e^+ \rightarrow \overline{H}(1s) + \gamma \tag{2}$$

$$P^- + e^+ \rightarrow \overline{H}(nl) \rightarrow \overline{H}(1s) + h\nu \tag{3}$$

The rates of the three-body radiative recombination in (1) is obtained [3] using the principle of detailed balancing and the classical cross section for collision ionisation of Thomson [4] and Bohr [5] from an excited state n. The recombination rate is then multiplied by the transition probability to the ground state.

The cross section for two-body electron capture from continuum into the low lying Coulomb bound states of hydrogen-like systems with spontaneous photon emission were calculated [6,7] in dipole approximations. This is a spontaneous photo recombination (SPR) of electron and proton into hydrogen. The process (2) gives SPR for antihydrogen formation. SPR is also studied as a time reversed photoionisation process [8]. The antihydrogen production rate by the three-body radiative recombination in (1) was, however, many orders of magnitude higher [2] than the SPR rate(2).

In this paper we are interested in the two-body radiation recombination via the process (3) which provides an alternative path for the two-body radiative recombination. In the case (3) a positron and an antiproton in the merged beam experiment, experience Coulombic attraction to from antihydrogen in a higher orbit, which subsequently decays to the ground state with the emission of a photon. Momentum conservation in the final state is taken care of by the emitted photons. Eventually it is essential for the intermediate state to be an excited state from which radiative transition is possible. A contribution from the process (3) towards radiative recombination of antiproton with positron is not yet computed. In this paper cross section for the two-step radiative recombination (TSRR) process is obtained by calculating the matrix element for the second order interaction consisting of the Coulomb attraction and dipole transition. The state vectors and the interaction Hamiltonian are taken in a field theoretic way and in the Coulomb gauge. Among all the excited intermediate states 2p-state has the maximum probability for radiative decay to the ground state. We have computed TSRR Cross section with 2p as the intermediate state and compared the contribution with that from SPR [7] for the formation of antihydrogen to the ground state. The comparative study reveals importance of TSRR in the two-body radiative recombination process.

2. Field-theoretic Cross section

We use the field theoretic formalism of reference [9] to write the state-vectors of the interacting systems. Positron-antiproton state vectors in the initial state and that after recombination to the intermediate state are written respectively as

$$|\Psi_i>= \exp(-iE_it/\hbar)\int g_i(\vec{q}_1\vec{l}_1)a^+_{q_1}B^+_{l_1}|o\rangle d^3\vec{q}_1 d^3\vec{l}_1 \tag{4}$$

$$|\Psi_I>= \exp(-iE_It/\hbar)\int g_I(\vec{q}_2,\vec{l}_2)a^+_{q_2}B^+_{l_2}|o\rangle d^3\vec{q}_2 d^3\vec{l}_2 \tag{5}$$

where a_{q_1} and B_{l_1} are the annihilation operators for positron and antiproton respectively. $g(\vec{q}_1,\vec{l}_1)$ and $g_I(\vec{q}_2,\vec{l}_2)$ are the Fourier transforms of the free and the bound state solutions of the unperturbed equations

$$(H_0+V)\phi_i(\chi_1,\chi_2) = E_i\phi_i(\chi_1,\chi_2) \tag{6}$$

and

$$(H_0+V)\phi_I(\chi_1,\chi_2) = E_I\phi_I(\chi_1,\chi_2) \tag{7}$$

respectively, with

$$H_0 = H_{e^+} + H_{p^-}, \quad V = \int\{\varrho(\chi)\sigma(\chi')/|\vec{\chi}-\vec{\chi}'|\}d^3\vec{\chi}d^3\vec{\chi}' \tag{8}$$

H_{e^+}, H_{p^-} are the free particle Hamiltonians for the suffixed particles. $\varrho(\chi)$ and $\sigma(\chi)$ are the charge densities for positron and antiproton respectively. V is the Coulomb interaction so that $\phi_i(\chi_1,\chi_2)$ and $\phi_I(\chi_1,\chi_2)$ contain respectively free particle distorted plane wave and positiron-antiproton bound wave in an excited state.

The final state contains an antiproton-posittron bound state and a photon. Let C_k be the annihilation operator for the photon with momentum $\vec{K}$. The final state vector, with $|o>$ as the vacuum state for particles and photon, is written as

$$|\psi_f>= \exp(-iE_ft/\hbar)\int g_f(\vec{q}_3,\vec{l}_3)a^+_{q_3}B^+_{l_3}C^+_k|o\rangle d^3\vec{l}_3 d^3\vec{q}_3 \tag{9}$$

where $g_f(\vec{q}_3,\vec{l}_3)$ is the Fourier transform in momentum space of the unperturbed solution of the equation

$$(H_0+H_K+V)\phi_f(\chi_1,\chi_2) = E_f\phi_f(\chi_1,\chi_2), \tag{10}$$

where $H_K = \hbar\omega_K C^+_K C_K$ is the Hamiltonian for the emitted photon. $\phi_f(\chi_1,\chi_2)$ is the wave function for positron and antiproton in bound state. Charge densities for positron and antiproton are respectively

$$\varrho(\chi) = e\phi^*(\chi)\phi(\chi) \tag{11}$$

and

$$\sigma(\chi) = -e\Theta^*(\chi)\Theta(\chi) \tag{12}$$

where $\phi(\chi)$ and $\Theta(\chi)$, the respective field operators in the non-relativistic case, are written as

$$\phi(\chi) = \sum_r\int \chi_r a_S \exp(i\vec{S}\vec{\chi})d^3\vec{S} \tag{13}$$

and

$$\Theta(\chi) = \sum_{r'}\int \lambda_{r'} B_{S'} \exp(i\vec{S}'\vec{\chi})d^3\vec{S}' \tag{14}$$

χ_r and $\lambda_{r'}$ are the Pauli-spinors for e^+ and p^- respectively.

The interaction Hamiltonian for the Coulombic attraction between e^- and p^+ is given by

$$H_1 = \int [\varrho(\chi)\sigma(\chi')/\mid \vec{\chi} - \vec{\chi}' \mid] d^3\vec{\chi} d\vec{\chi}'. \tag{15}$$

The interaction between antiatom and the electromagnetic radiation field necessary for photon emission, is given by [10]

$$H_2 = H' + H''. \tag{16}$$

where

$$H' = (e/mc)\vec{P}.\vec{A}(\chi), \qquad H'' = (e^2/2mc^2)\vec{A}^2(\chi) \tag{17}$$

$\vec{p}$ is the momentum operator and $\vec{A}(\xi)$ is the electromagnetic field operator, which at a fixed time, is given by [10]

$$\vec{A}(\chi) = \sum_{K'} (2\pi\hbar c^2/(\Omega\omega_{K'}))^{1/2} u_{K'_\sigma}[C_{K'}\exp(i\vec{K}'.\vec{\chi}) + C^+_{K'}\exp(-i\vec{K}'.\vec{\chi})] \tag{18}$$

$u_{K'_\sigma}$ is the polarisation vector. S-matrix for the process is

$$S = 1 + (H_1 + H_2) + H_1H_1 + H_2H_1 + \text{higher order term} \tag{19}$$

The radiative recombination (3) through the two-step process is obtained by taking the matrix element of H_2H_1 in eq (19), between the initial and the final states such that the matrix element

$$M_{fi} = \sum_I \langle\psi_f \mid H_2 \mid \psi_I\rangle\langle\psi_I \mid H_1 \mid \psi_i\rangle/(E_i - E_I + i\eta) \tag{20}$$

E_i and E_I are the relative energies of the interacting systems in the initial and intermediate states respectively, and the quantity η is positive infinitesimal. After substituting, from (11) and (12) in (15) and integrating over the coordinate space we get

$$H_1 = -e^2 \int \delta^3(\vec{S}_1 - \vec{S}_2 + \vec{S}'_1 - \vec{S}'_2) \mid \vec{S}_1 - \vec{S}_2 \mid^{-2} a^+_{S_1} a_{S_2} B^+_{S'_1} B_{S'_2} \chi_{r_1}\chi_{r_2} \lambda_{r'_1}\lambda_{r'_2} d^3S_1\, d^3S_2\, d^3S'_1\, d^3S'_2 \tag{21}$$

The single photon emission is due to the interaction term H' of (17), where

$$H' = (e/mc)\vec{P}.\vec{A}(\chi) \tag{22}$$

2.1 Probability for antihydrogen formation in the intermediate state

Matrix element of H_1 between $\langle\psi_I \mid$ and $\langle\psi_i \mid$ is obtained on using (14), (15) and (21)

$$\begin{aligned} M_1 &= \langle\psi_I \mid H_1 \mid \psi_i\rangle \\ &= \int \delta^3(\vec{S}_1 - \vec{S}_2 + \vec{S}'_1 - \vec{S}'_2) \mid \vec{S}_1 - \vec{S}_2 \mid^{-2} \chi^*_{r_2}\chi_{r_1}\lambda^*_{r'_2}\lambda_{r'_1} \\ &\quad \langle 0 \mid a^+_{q_2} B_{l_2} a^+_{S_1} a_{S_2} B^+_{S'_1} B_{S'_2} a^+_{q_1} B^+_{l_1} \mid 0\rangle g^*_I(\vec{q}_2, \vec{l}_2) \\ &\quad g_i(\vec{q}_1, \vec{l}_1) \prod_{i=1,2} d^3\vec{q}_i\, d^3\vec{l}_i\, d^3\vec{S}'_i\, d^3\vec{S}_i \end{aligned} \tag{23}$$

Vacuum expectation value of the field operators gives product of Dirac δ-functions. Integrating out the δ-functions we get

$$M_1 = \int \delta^3(\vec{q}_2 - \vec{q}_1 + \vec{l}_2 - \vec{l}_1) \mid \vec{q}_2 - \vec{q}_1 \mid^{-2} g^*_I(\vec{q}_2, \vec{l}_2) g_i(\vec{q}_1, \vec{l}_1) \chi^*_{q_2}\chi_{q_1}\lambda^*_{l_2}\lambda_{l_1} \prod_{i=1,2} d^3\vec{q}_i\, d^3\vec{l}_i \tag{24}$$

The wave function $\phi_r(x_1, x_2)$ is the expectation value of the product the operators $\phi(x_1)$ and $\Theta(x_2)$ (eqns. 13,14) between vacuum state $\mid o\rangle$ and the state vectors $\mid \psi_r\rangle$

$$\phi_r(\chi_1, \chi_2) = \langle o \mid \phi(\chi_1)\Theta(\chi_2) \mid \psi_r\rangle \tag{25}$$

Using equations (4) and (25) the wave function in momentum space with associated pauli spinor can be written as [9]

$$g_i(\vec{q}_1, \vec{l}_1)\chi_{q_1}\lambda_{l_1} = \int \phi_i(\chi_1, \chi_2)\exp(i\vec{q}_1.\vec{\chi}_1 + i\vec{l}_1.\vec{\chi}_2) d^3\vec{\chi}_1\, d^3\vec{\chi}_2 \tag{26}$$

Changing the integration variables into centre of mass and relative coordinates and neglecting the mass ratio between positron and antiproton we get

$$g_i(\vec{q}_1, \vec{l}_1) = \phi_c(\vec{q}_1)\chi^*_{q_1}\lambda^*_{l_1}\delta^3(\vec{q}_1 + \vec{l}_1 - \vec{P}_c) \tag{27}$$

Where $\vec{P}_c$ is the centre of mass momentum. $\phi_c(\vec{q}_1)$ is the coulomb distorted plane wave of the incident positron (in momentum space). Similarly, the wave function in momentum space for intermediate state

$$g_I(\vec{q}_2, \vec{l}_2) = \phi_{nl}(\vec{q}_2)\chi^*_{\vec{q}_2}\chi^*_{l_2}\delta^3(\vec{q}_2 + \vec{l}_2 - \vec{Q}_I) \tag{28}$$

$\vec{Q}_1$ is the C.M. momentum of the intermediate system and $\phi_{nl}(\vec{q}_2)$ is the nl - state bound positron wave function in momentum space. Substituting (27) and (28) in (24) we get after integration over $\vec{l}_1$ and $\vec{l}_2$

$$M_1 = -e^2\delta^3(\vec{P}_c - \vec{Q}_I) \int \phi^*_{nl}(\vec{q}_2)\phi_c(\vec{q}_1) \mid \vec{q}_1 - \vec{q}_2 \mid^{-2} d^3\vec{q}_1 d^3\vec{q}_2 \tag{29}$$

Using the integral we get

$$M_1 = -e^2\delta^3(\vec{P}_c - \vec{Q}_I)\psi_{nl}(r)F_c(r)|\,\vec{r}\,|^{-1}d^3\vec{r} \quad (30)$$

$F_c(r)$ is Coulomb distorted plane wave for incident positron and $\psi_{nl}(r)$ is the nl-state bound wave function of positron in antihydrogen.

2.2 Radiative decay of antihydrogen from nl-state to ls ground state

H' in (16) connects these states in the first order contribution to the radiative decay. The decay amplitude M_2

$$\begin{aligned} M_2 &= \langle\psi_f \mid H_2 \mid \psi_I\rangle \\ &= (e/mc)\langle\psi_f \mid \vec{P}.\vec{A}(\chi) \mid \psi_I\rangle \\ &= (e/mc)\sum_{K'\sigma}(2\pi\hbar c/\Omega\omega_{K'})^{1/2}C_{K'}\langle\psi_f \mid \vec{P}.\vec{u}_{K'\sigma}\exp(-i\vec{K}'.\vec{\chi}) \mid \psi_I\rangle \end{aligned} \quad (31)$$

Since the wave length of the emitted photon is larger than atomic dimension, one can use dipole approximation. The sum over polarisation directions $\vec{u}_{K_1'}$ and $\vec{u}_{K_2'}$ is

$$\begin{aligned} \textstyle\sum_{\sigma=1,2} & \langle\psi_f \mid \vec{p}.\vec{u}_{K'_\sigma}\exp(-i\vec{K}'.\vec{\chi}) \mid \psi_I\rangle \qquad \text{Since } \theta \text{ is the angle between } \vec{p} \text{ and } \vec{K}' \\ &= \langle\psi_f \mid p\exp(-iK'.\chi) \mid \psi_I\rangle\sin\theta \\ &= \langle\psi_f \mid p \mid \psi_I\rangle\sin\theta \qquad \text{Dipole approximation} \\ &= \langle\psi_f \mid \frac{m}{\hbar}\frac{d\chi}{dt} \mid \psi_I\rangle\sin\theta \\ &= (-\frac{im}{\hbar})\langle\psi_f \mid xH_{op} - H_{op}x \mid \psi_I\rangle\sin\theta \qquad H_{op} = H_o + V \\ &= (-\frac{im}{\hbar})(E_I - E_f)\langle\psi_f \mid x \mid \psi_I\rangle\sin\theta \\ &= (-\frac{im}{\hbar})(\epsilon_{nl} - \epsilon_{1s})\langle\psi_f \mid x \mid \psi_I\rangle\sin\theta \end{aligned} \quad (32)$$

To arrive at (32) we use the equation

$$H_{op} \mid \psi_{I,f} >= E_{I,f} \mid \psi_{I,f} >$$

$\mid\psi_I\rangle, \mid\psi_f\rangle$ are the eigenfunctions with eigen values E_I, E_f respectively of the unperturbed Hamiltonian operator H_{op} of the bound positron in the intermediate and final state of the antihydrogen. Substituting (32), (5) and (9) in (31) and taking the vacuum expectation value for the product of field operators, and integrating over the momentum space we arrive at

$$M_2 = -(im/\hbar)\bar{c}(\hbar\omega_K)\int\psi_{nl}(r)r\psi_{1s}(r)\sin\theta d^3\vec{r} \quad (33)$$

where $\hbar\omega_K = \epsilon_{nl} - \epsilon_{1s}$ and $\bar{c} = (e/mc)(2\pi\hbar c^2/\omega_K)^{1/2}$
ϵ_{n1} and ϵ_{1s} are respectively the eneries of positron in nl and ls-bound states.
The decay rate for radiative transition from nl state to ls state is given by (31)

$$\tau^{-1}_{nl\to 1s} = (2\pi/\hbar)\int\delta(E_I - E_f)|\,M_2\,|^2(2\pi)^{-3}d^3\vec{K} \quad (34)$$

2.3 Radiative recombination probability

From (20) the radiative recombination probability M_{fi} with nl as the intermediate state is given by

$$|\,M_{fi}\,| = [\,|\,M_1\,|\,|\,M_2\,|\,/\,|\,E_i - E_I + i\eta\,|\,]_{I\to nl\,state} \quad (35)$$

The radiative recombination cross section for the process (3) through the nl intermediate state becomes

$$\sigma = (2\pi/\hbar)\int\delta(E_i - E_f)(m/\,|\,\hbar\vec{p}\,|)(2\pi)^{-6}|\,M_{fi}\,|^2d^3\vec{K}d^3\vec{p}' \quad (36)$$

$\vec{p}$ and $\vec{p}'$ are the relative momenta of the interacting systems before and after interaction. Using (34) and (35) in (36) the cross section becomes

$$\sigma = \tau^{-1}_{nl\to 1s}\int(m/\,|\,\hbar\vec{p}\,|)/|\,M_1\,|^2|\,E_i - E_I\,|^{-2}(2\pi)^{-3}d^3\vec{p}' \quad (37)$$

Substituting (30) in (37) we arrive at

$$\sigma = \tau^{-1}_{nl\to 1s}(e^4/(2\pi)^3)m|\,\hbar\vec{p}\,|^{-1}|\,E_i - E_I\,|^{-2}|\,I_{nl}\,|^2, \quad (38)$$

where

$$I_{nl} = \int\{\psi_{nl}(r)F_c(r)/\,|\,r\,|\}d^3\vec{r}$$

3. Results and Discussions

The cross section for TSRR, with the $2p$ as the intermediate state which decays to the $1s$-state with the emission of a Lyman photon, is calculated. From (38) the cross section

$$\sigma = [e^4/(2\pi)^3]m|\,\hbar\vec{p}\,|^{-1}\tau^{-1}_{2p\to 1s}|\,E_i - E_I\,|^{-2}\,\left|\int\psi_{2p}(r)F_c(r)|\,\vec{r}\,|^{-1}d^3\vec{r}\,\right|^2 \quad (39)$$

$\psi_{2p}(r)$ is the $2p$ orbital wave function of the antihydrogen. At low relative velocity the effect of distortion on the plane wave of inicident positron in the Coulomb field of the antiproton is obtained by taking

$$F_c(r) = (2\pi\,|\,\xi\,|)^{1/2}\exp(i\vec{p}.\vec{r}) \quad (40)$$

Where $2\pi \mid \vec{\xi} \mid$ is the Sommer-feld factor [11] and

$$\vec{\xi} = -e^2 m \mid \hbar \vec{p} \mid^{-1}. \tag{41}$$

Since $\psi_{2p}(r) = N.r \exp(-r/2a)\cos\theta$,
the integral I_{2p} in (39) becomes

$$I_{2p} = \int \psi_{2p}(r) F_c(r) \mid \vec{r} \mid^{-1} d^3\vec{r} = [2\pi \mid \vec{\xi} \mid]^{1/2} (-4\pi i) N F(p) \tag{42}$$

where a is the Bohr radius, $N = (2/\pi a^3)^{1/2}(8a)^{-1}$ and

$$F(p) = [2a^2 p(p^2 + (2a)^{-2})^2]^{-1}. \tag{43}$$

The required radiative recombination Cross section (39) becomes

$$\sigma = (L/E_i) F^2(p) \mid E_i - E_I \mid^{-2} \tag{44}$$

Where

$$L/E_i = \mid 6\pi^2 N^2 e^4 (2\pi)^{-3} \tau^{-1}_{2p \to 1s} m \mid \hbar \vec{p} \mid^{-1} (2\pi \mid \vec{\xi} \mid). \tag{45}$$

The energy E_I in the intermediate state is now given by

$$E_I = E_i + \epsilon_{2p} \tag{46}$$

and

$$\mid E_i - E_I \mid = \epsilon_{2p}.$$

The TSRR Cross section with $2p$ as the intermediate state finally becomes

$$\sigma = (L/E_i) F(p)^2 \epsilon_{2p}^{-2}. \tag{47}$$

The decay rate of $2p$-state for transition to $1s$ state being [3]

$$\tau^{-1}_{2p \to 1s} = 6.25 \times 10^8 sec^{-1},$$

we compute the Cross section σ from (47). Present TSRR Cross section σ_{TSRR} are shown in Table 1 along with the SPR Cross section σ_{SPR} of ref.7, at relative collision energies E_i.

3.1 Special cases to study dependence of σ on E_i

Case I:

$$\mid \vec{p} \mid \ll \frac{1}{2a} \quad i.e. \quad E_i \ll \epsilon_{2p} \tag{48}$$

Using this condition we get from (43)

$$F(p) = 8a^2 / \mid \vec{p} \mid$$

Table 1. Radiative recombination cross sections (in 10^{-20}cm^2) for the formation of antihydrogen due to Collision between antiproton and positron at relative Collision energies E_i(in10^{-2}eV). σ_{TSRR} is the result for two-step radiative recombination from present work. σ_{SPR} is the result for spontaneous photo recombination from ref.7.

E_i (10^{-2}eV)	σ_{TSRR} (10^{-20}cm^2)	σ_{SPR} (10^{-20}cm^2)
0.68	3.97×10^3	30
1.36	1.003×10^3	18
4.08	1.06×10^2	50
6.8	36.8	3
9.52	18	1
13.6	8.4	1.8
27.2	1.79	0.9
54.4	0.328	0.4
68	0.182	0.3
136	0.0235	0.16
340	8.4×10^{-5}	0.06

and the Cross section (47) becomes

$$\sigma = 32a^4 L\hbar^2 / (m\epsilon_{2p}^2 E_i^2) \tag{49}$$

When the relative collision energy E_i is small compared to the $2p$-state orbital energy ϵ_{2p} the TSRR Cross section varies as E_i^{-2}.
Case II:

$$\mid \vec{p} \mid \gg (2a)^{-1} \quad i.e. \quad E_i \gg \epsilon_{2p} \tag{50}$$

Condition (50) leads to

$$F(p) = (2a^2 p^5)^{-1}$$

and

$$\sigma = (\hbar^2/2m)^5 L(\epsilon_{2p}^2 4a^4 E_i^6)^{-1} \tag{51}$$

When the relative collision energy E_i is high compared to ϵ_{2p} the TSRR Cross section varies as E_i^6.

In a merged beam experiment the relative velocity of collisior (48) should be kept well below $(2ma)^{-1}$ to have a good antihydrogen formation rate. Table 1 provides a basis for the comparative study of the contributions from TSRR and SPR mechanisms towards antihydrogen formation. The SPR Cross section σ_{SPR} varies as E_i^{-1} [7] whereas in the low energy limit the TSRR Cross section σ_{TSRR} varies as E_i^{-2}. Hence the TSRR Cross section is larger than the SPR Cross section in the low energy collision region.

From Table 1, the antihydrogen formation cross section for collision energy below 0.0272 eV, are two orders of magnitude larger by TSRR mechanism as compared to that by the SPR mechanism. Around 0.544 eV., contributions from both the mechanisms are almost of the same order of magnitude, with SPR Cross sections remaining slightly higher than TSRR Cross sections. With the increase of the collision energy above 1.36 eV., SPR Cross section dominates the radiative recombination process, over TSRR Cross section. Near 3.4 eV which is the 2p-state binding energy, the TSRR Cross section is three orders of magnitude smaller than SPR Cross section.

4. Conclusion

Present result predicts higher contribution from the TSRR process to the rate of cold antihydrogen formation, as compared to the result obtained from the SPR process [6,7], at relative collision energies below 0.272 eV. near which experiments [12] are being conducted. The theoretical predictions on the antihydrogen formation rate are available, till now, only from the SPR channel. Present result from TSRR will provide additional contribution to the formation rate over the SPR channel [13].

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