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THRESHOLD EFFECTS AND THE RADIATIVELY CORRECTED HIGGS MASS IN THE MSSM

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In this article, we apply the renormalization group techniques to resum the large logarithmic corrections to the Higgs masses which occur when the scale of supersymmetry breaking is far above the electroweak breaking scale. We argue that the calculations made so far missed some terms that, potentially, can be very relevant.

1. Introduction

The Standard Model (SM) is in excellent agreement with all precision electroweak tests [1, 2]. Nevertheless, there are still some aspects that prevent a complete understanding of its structure. The "Higgs" mechanism that is responsible for the generation of the masses of the particles is one example of these and requires the existence of a scalar particle (the Higgs boson). Probably the most important challenge facing present (LEP, LEP-200) and future (LHC,SSC) colliders is the discovery of this elusive particle. Of considerable interest also is the possibility of extensions of the SM. This is so because, despite excellent experimental agreement shown by the SM at low energies, there is still room for extensions at higher scales. The most appealing of those are the supersymmetric extensions of the SM, since they address the hierarchy problem, the origin of the electroweak breaking mass scale [3].

In this letter, we will consider the minimal supersymmetric extension of the standard model (MSSM) and, in particular, the Higgs sector in the MSSM. In the MSSM, at tree-level, the lightest Higgs boson mass is predicted to be smaller that that of the Z (M_Z). Since that would be the range of experiments at LEP-200 [4], a negative result from LEP Higgs searches would apparently imply that the MSSM is excluded. Recently, however, it has been demonstrated that radiative corrections in the MSSM can violate the inequality $M_h \leq M_Z$ (where M_h is the mass of the lightest Higgs), and thus there is a possibility that the Higgs is out of the reach of LEP-200 and could only be seen by the LHC or SSC.

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In this paper, we will re-assess the calculation of the one-loop radiative corrections using the renormalization group equations (RGE) approach, and paying particular attention to the threshold effects. Our analysis applies in the case that the supersymmetry breaking scale, M_{susy}, lies far above the electroweak breaking scale when the RGE method is needed to sum the large logarithms.

2. Radiatively corrected Higgs mass in the MSSM

In the MSSM, the Higgs sector contains two complex Higgs doublets $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ with hypercharges -1/2 and 1/2, respectively.

After including soft supersymmetry breaking terms, the tree-level potential for the neutral components H_1^0 and H_2^0 is given by

$$V = m_1^2|H_1^0|^2 + m_2^2|H_2^0|^2 - m_3^2(H_1^0 H_2^0 + \overline{H_1^0 H_2^0}) + \frac{1}{8}(g_1^2 + g_2^2)(|H_1^0|^2 - |H_2^0|^2)^2 \quad (1)$$

where g_1 and g_2 are the gauge couplings of the SU(2) and $U(1)_Y$ groups respectively.

For M_{susy} much larger than the electroweak breaking scale, the physical Higgs spectrum of three neutral Higgs bosons splits with only the lightest CP-even Higgs left much lighter than M_{susy}. To discuss the expectations for its mass it is convenient to define the field H by

$$H = cos\beta H_1^0 + sin\beta H_2^0 \quad (2)$$

Here, $\tan\beta = v_2/v_1$ and, v_1 and v_2 are the vacuum expectation values (vevs) of the fields H_1^0 and H_2^0 respectively. H is the combination of Higgs fields acquiring a vev ($\langle H\rangle = \frac{v}{\sqrt{2}}, (v^2 = v_1^2 + v_2^2)$) and $h = (\frac{1}{\sqrt{2}})ReH$ contains the lightest CP-even Higgs eigenstate at tree level. The potential for H following eq. (1) may be written as

$$V = -\mu^2|H|^2 + (1/2)\lambda|H|^4 \quad (3)$$

where

$$\lambda = (1/4)(g_1^2 + g_2^2)\cos^2 2\beta \quad (4)$$

and μ^2 is a function of m_1^2, m_2^2, m_3^2, v_1 and v_2.

So, we have the following minimization condition for eq. 3 and expression for the mass of the Higgs.

$$\mu^2 = \frac{\lambda v^2}{2} \quad (5)$$

$$M_h^2 = \lambda v^2 \quad (6)$$

We also have the other usual mass expressions given by:

$$M_W^2 = \frac{1}{4}g_2^2 v^2 \quad (7)$$

$$M_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)v^2 \quad (8)$$

$$M_t^2 = h_t v/\sqrt{2} \quad (9)$$

where h_t is the top-quark Yukawa coupling, and M_W, M_Z, M_t are W, Z, and top masses respectively.

Using eqs. (4), and (6)-(9), one has the tree level inequality $M_h \leq M_Z$ following from the relation $M_h^2 = M_Z^2 \cos^2 2\beta$ which [5] is broken by radiative corrections. We turn now to the inclusion of these radiative corrections. There is now a considerable amount of work estimating these effects [[8] to [20]], using several different methods. One method is to use the effective potential approach (EPA)[8, 9, 10, 11, 14], starting with the potential as given by Coleman-Weinberg [6] and determining the second derivative of this effective potential , calculated at the minimum, to estimate the physical masses for the particles. In [13], a modified version of this technique was used explicitly introducing wave-function and gauge coupling renormalization. (In the EPA, these effects were approximately included by the appropriate choice of the renormalization scale). Both approaches give numerically very similar results.

Another technique used in calculating the one-loop corrections is the so-called "diagrammatic" approach [5, 15, 16, 17, 18]. In this approach one works directly with Feynman diagrams. Thus the physical masses are identified with the propagators poles, according to the standard definition[2] whereas, in the EPA, they were associated with the second derivatives of the effective potential. The approximation one makes, when one uses the EPA instead of the more complete diagrammatic approach, corresponds to neglecting the W-self-energy and to evaluating scalar self-energies at zero momentum rather than on shell [18]. It has been found that the Higgs mass calculations made with the EPA are a good approximation to the ones made with the more complete diagrammatic approach, both for the neutral and charged cases [18].

Finally, the renormalisation group has been used [10, 19, 20] to sum the radiative corrections involving the logs of the ratio of the supersymmetry breaking scale to the Fermi scale [we will refer to this as the RG approach]. Since we are interested in this paper in the case that this ratio and the associated logs are large we will use this method in what follows. However, as we will discuss, it is essential correctly to include the boundary conditions to the renormalisation group equations if the method is to yield reliable results.

The starting point of thr RG approach is the effective potential, eq. (3), but with running mass $\mu(t)$ and running coupling $\lambda(t)$ evaluated, via the RG equations, at the field dependent scale $t = \ln(h/Q)$ (Q is the renormalisation scale)[6].

$$V_{eff.} = -\frac{1}{2}\mu^2(t)h^2(t) + \frac{1}{8}\lambda(t)h^4(t) \quad (10)$$

Here $h(t)$ is the renormalized field

$$h(t) = hexp(-\int_0^t \frac{\gamma}{1-\gamma}dt') \quad (11)$$

where γ is the anomalous dimension.

[2] physical masses are, by definition, the zeroes of the real part of the inverse propagator

Following a procedure similar to that of the case of the tree-level potential, the minimization condition is normally taken to be

$$\mu^2(t) = \frac{1}{2}\lambda(t)\langle h^2(t)\rangle|_{t=\ln(\frac{\langle h\rangle}{Q})} \quad (12)$$

where $\langle h^2(t)\rangle = v^2$ is fixed experimentally ($v^2 = (\sqrt{2}G_F)^{-1}$), and the coupling is evaluated at the minimum (we will consider below the effect of keeping the t dependence in μ and λ when minimizing the potential).

The masses of the Higgs and the top are given by

$$M_h^2 = \lambda(M_h)v^2 \quad (13)$$

$$M_t = h_t(M_t)v/\sqrt{2} \quad (14)$$

By using the renormalization group equations (RGE) (and the associated boundary conditions) one may evolve the running couplings (principally λ) to the appropriate scale. This technique includes the leading logarithmic radiative corrections to the mass of the Higgs (eq. (13)).

In the analyses so far performed, the RGEs have been used to determine the h-dependence below the SUSY mass threshold, using t=ln(h/Q) with Q=M_{susy} and with the t dependence of $\mu^2(t)$ and $\lambda(t)$ given by the standard model contributions only. It has been argued that this is an acceptable procedure for the effect of the SUSY states decouples[10, 20]. However we will show that this is not the case for the SUSY particles themselves acquire electroweak breaking masses which introduces a h-dependence in the effective potential of the same order of the terms considered before. The correct procedure is to use the SUSY values for scales above the relevant SUSY mass threshold and the non-SUSY ones below this threshold. Doing this introduces a dependence on log(f(h)/Q) where Q may be chosen for above the SUSY scale and f(h) takes account of the correct SUSY mass threshold.

Keeping now the h-dependence in both $\mu^2(h)$ and $\lambda(h)$ the minimization condition, eq. (12), must be modified to

$$(dV/dh) = -\mu^2 h + \frac{\lambda h^3}{2} - \frac{1}{2}h^2(d\mu^2/dh) + \frac{h^4}{8}(d\lambda/dh) = 0 \quad (15)$$

and the second derivative is given by

$$\begin{aligned}(d^2V/dh^2) &= h^3(d\lambda/dh) - 2h(d\mu^2/dh) - \mu^2 + (3/2)\lambda h^2 \\ &- (h^2/2)(d^2\mu^2/dh^2) + (h^4/8)(d^2\lambda/dh^2)\end{aligned} \quad (16)$$

both equations evaluated at $h = \langle h\rangle$.

From eqs. (15) and (16) above, we can see that the final expression for the Higgs mass is

$$M_h^2 = \left(\lambda h^2 + (\frac{7h^3}{8}\frac{d\lambda}{dh} + \frac{h^4}{8}\frac{d^2\lambda}{dh^2}) - (\frac{3}{2}h\frac{d\mu^2}{dh} + \frac{h^2}{2}\frac{d^2\mu^2}{dh^2})\right)\Bigg|_{h^2=\langle h^2\rangle} \quad (17)$$

Note that the second and third terms above are the ones which take the h-dependence of λ and μ^2 into account.

In order to calculate the derivatives of $\lambda(h)$ and $\mu^2(h)$ present in eq. (17) above, we have to determine the correct h-dependence of $\lambda(h)$ and $\mu^2(h)$. To do this, we use the Coleman-Weinberg expression for the effective potential at one-loop order. This is given by the tree-level potential plus a one-loop correction. The expression for the potential is then given by:

$$V_1(Q) = V_0(Q) + \Delta V(Q) \quad (18)$$

where [7, 6]

$$\Delta V(Q) = \frac{1}{64\pi^2}StrM^4(log(\frac{M^2}{Q^2} - \frac{3}{2}). \quad (19)$$

where M^2 is the field-dependent generalized squared mass matrix for the MSSM and Str denotes a supertrace over all fields, see 20 below.

$$Str f(M^2) = \sum(-1)^{2j_i}(2j_i+1)f(m_i^2) \quad (20)$$

where m_i^2 is the field-dependent mass eigenvalue of the i-th particle of spin-j_i.

In our case, we are going to consider only the loops involving quarks and squarks plus the ones involving Higgs and Higgsinos . So, the one-loop correction will be given by:

$$\begin{aligned}\Delta V(Q) &= (\frac{3}{16\pi^2}) \times (-m^4 ln(m^2/Q^2) + (m^2 + M_{susy}^2)^2 ln((m^2 + M_{susy}^2)/Q^2)) \\ &+ (\frac{1}{32\pi^2}) \times (m'^4 ln(m'^2/Q^2) - (m'^2 + M_{susy}^2)^2 ln((m'^2 + M_{susy}^2)/Q^2))\end{aligned} \quad (21)$$

where $m^2 = h_t^2h^2/2$, $m'^2 = (-\mu_0^2 + (3/2)\lambda_0 h^2)$ and M_{susy} is the supersymmetry-breaking contribution to scalar masses.[3]

It may be seen from eq. (21) that one cannot ignore the field dependence introduced by the supersymmetric loops. For example, the second term in eq. (21) gives a contribution to $\lambda(t)$ of the form $\frac{3h_t^4}{16\pi^2}ln((h_t^2h^2/2 + M_{susy}^2)/Q^2)$ and a contribution to $\mu(t)$ of the form $\frac{3h_t^2M_{susy}^2}{16\pi^2}ln((h_t^2h^2/2 + M_{susy}^2)/Q^2)$. From eq. (21) we see the $\lambda(t)$ dependence introduces a correction to M_h^2 proportional to h^4/M_{susy}^2, negligibly small. However the $\mu^2(t)$ dependence introduces a correction proportional to $h_t^2h^2$, *unsuppressed* by inverse powers of M_{susy}^2 and of the same order as the terms that come from the non-supersymmetric loops. These terms should be kept showing it is not sufficient just to determine the h-dependence of $\mu^2(t)$ using the RGE contribution below the SUSY threshold. The procedure we adopt is straightforward: we use the one-loop Coleman-Weinberg equations to include the important SUSY threshold effects just mentioned, i.e., the additional contributions to $d\mu^2/dt$ and $d^2\mu^2/dh^2$ in eq. (21) (since there are no large logs involved with $Q^2 = M_{susy}^2$ there is no need to use the full RGE in determining

[3] λ_0 and μ_0 are the tree-level value for these quantities

these contributions). The remaining h dependence in $\mu^2(t)$ and $\lambda(t)$ is calculated using the RG eqs. below the SUSY threshold with boundary conditions at M_{susy}.

To see how this procedure works more explicitly, remember that we are going to be interested in the case where M_{susy}^2 is $\gg m^2$ or m'^2. In this limit, after we have disregarded the negligibly small terms, eq. (21) above can be put in the following form:

$$\begin{aligned}\Delta V(Q) &= \frac{3}{16\pi^2}(ln(M_{susy}^2/Q^2)) + 2m^4 \\ &+ 2m^2 M_{susy}^2 ln(M_{susy}^2/Q^2) + M_{susy}^2 m^2 + M_{susy}^4 ln(M_{susy}^2/Q^2)) \\ &+ \frac{-1}{32\pi^2}(m'^4(ln(M_{susy}^2/Q^2)) + 2m'^4 \\ &+ 2m'^2 M_{susy}^2 ln(M_{susy}^2/Q^2) + M_{susy}^2 m'^2 + M_{susy}^4 ln(M_{susy}^2/Q^2))\end{aligned} \quad (22)$$

To determine the large logs below the SUSY threshold we may set $Q^2 = M_{susy}^2$. From eq. (22) one can now see that the terms that will be relevant if one wants to consider the threshold effects discussed above are:

$$\Delta V(Q) = \frac{3}{16\pi^2}(2m^4) - \frac{1}{32\pi^2}(2m'^4) + ... \quad (23)$$

from which the above mentioned non-suppressed contribution to $\mu^2(t)$ comes.

From eq. (23), we have

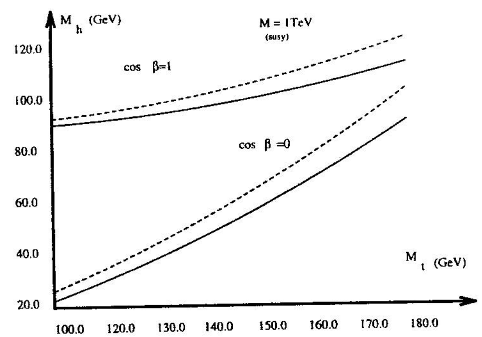
$$\mu^2 = \mu_0^2 - (3h_t^4 h^2/16\pi^2) + (9\lambda_0^2 h^2/64\pi^2) \quad (24)$$

Since we have now taken care of the threshold effects connecting the SUSY and the non-SUSY regimes, we are at the stage of applying the RGE in order to evolve the quantities in the expressions above to the desired scales. The evolution of these quantities is governed by

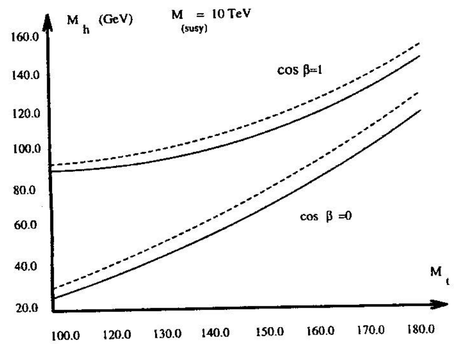
$$\begin{aligned}\frac{d\lambda}{dt} &= \frac{\beta^\lambda}{16\pi^2} \\ \frac{dh_t}{dt} &= \frac{\beta^{h_t}}{16\pi^2} \\ \frac{dg_i}{dt} &= \frac{\beta^i}{16\pi^2}(i = 1, 2, 3)\end{aligned} \quad (25)$$

where β^{i,λ,h_t} are the standard model β-functions given by [12]

$$\begin{aligned}\beta^\lambda &= 12\lambda^2 - (3g_1^2 + 9g_2^2 - 12h_t^2)\lambda + \frac{9}{4}(\frac{1}{3}g_1^4 + \frac{2}{3}g_1^2 g_2^2 + g_2^4) - 12h_t^4 \\ \beta^{h_t} &= h_t[\frac{9}{2}h_t^2 - (\frac{17}{12}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)] \\ \beta^1 &= g_1^3(\frac{20}{9}N_G + \frac{1}{6}) \\ \beta^2 &= g_2^3(\frac{4}{3}N_G - \frac{43}{6}) \\ \beta^3 &= g_3^3(\frac{4}{3}N_G - 11)\end{aligned} \quad (26)$$



(1a)



(1b)

Fig. 1 The relation of M_h to M_t for different values of $\cos^2 2\beta$ and for $M_{susy} = 1TeV(a)$ and $M_{susy} = 10TeV(b)$. In each figure two pairs of curves are displayed. The top pair is related to $\cos^2 2\beta = 1$ and the bottom one is related to $\cos^2 2\beta = 0$. In each of these pairs, the upper dotted curve is the one obtained by considering the h-dependence of λ and μ^2.

where N_G is the number of generations.

To solve eq. (25) above, one has to provide 5 boundary conditions. We have used, at $M_Z = 91.177$ [1, 21].

$$\alpha_E^{-1}(M_Z) = 127.9 \quad (27)$$

$$\sin^2\theta_W(M_Z) = 0.230 \quad (28)$$

$$\alpha_s(M_Z) = 0.12 \quad (29)$$

and

$$\lambda(M_{susy}) = \frac{1}{4}[g_1^2(M_{susy}) + g_2^2(M_{susy})]\cos^2(2\beta) \quad (30)$$

$$h_t(M_t) = \sqrt{2}M_t/v \quad (31)$$

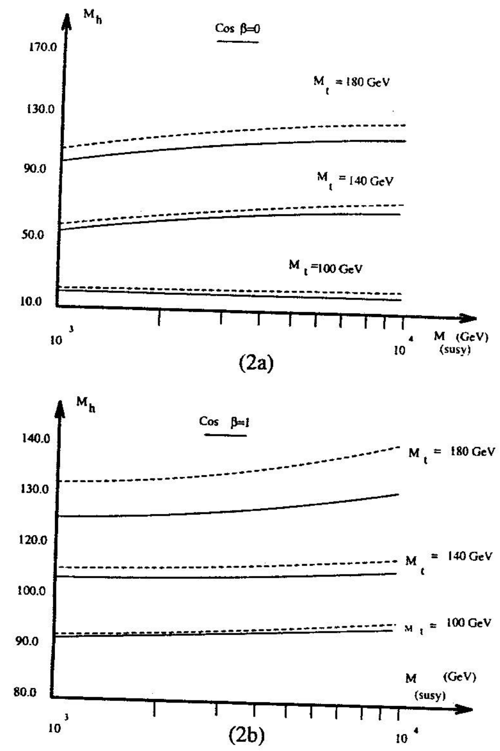


Fig. 2. The relation of M_h to M_{susy} for $\cos^2 2\beta = 0, (a)$ and $\cos^2 2\beta = 1, (b)$ and for three different values of M_{top}. In each figure, we see three pairs of curves, the upper one is for the case where $M_{top} = 180GeV$, the middle one is related to the case where $M_{top} = 140GeV$ and, finally, the bottom one is for the case $M_{top} = 100GeV$. Again, in each pair of curves, the upper dotted curve is related to results obtained when one considers the h-dependence of λ and μ^2.

where α_E^{-1}, $\sin^2\theta_W$ and α_s are the fine structure constant, the electroweak mixing parameter and strong coupling constant respectively. We have numerically solved these equations in order to determine the parameters at the minimum and then used eq. (17) to determine the full radiative corrections to the Higgs mass.

3. Results and Conclusions

The effects of the threshold corrections is to make the Higgs masses somewhat larger than in the case where the calculation is performed disregarding the h-dependence of λ and μ^2. In Figs. 1a, 1b we plot the results for the case where $M_{susy} = 1$ and 10 Tev and $\cos^2(2\beta) = 1, 0$, and M_{susy} is the mass of the stop-quark. Note that the upper limit for the Higgs mass is given by the case where $\cos^2(2\beta) = 1$. We can see from these figures that, as M_t gets bigger, the curves for the different values of $\cos^2(2\beta)$ tend to approach each other. This is so because, in this region, the renormalization effects dominate the initial conditions. In Figs. 2a, 2b we plot the cases corresponding to $\cos^2(2\beta) = 0, 1$, and $M_t = 100, 140, 180$ for M_{susy} varying from 10^3 to 10^4 GeV.

From Fig. 1, we can see that the bigger the value for M_t, the bigger the correction caused by the consideration of the h-dependence in λ and in μ^2. From Fig. 2, one sees that, for the same value for M_{susy} and M_t, the bigger the value of $\cos^2(2\beta)$, the smaller (the percentage in relation to the uncorrected value) the correction one finds.

Before we conclude, we would like to point out that our concern about taking care of the neglected terms mentioned above is a general one. Using [15] as an example of the more complete diagrammatic approach, one can, after careful examination, realize that, in order to obtain the results contained there, one has to use the identity $m^2 = \frac{1}{2}m_Z^2$ (notation of [15]). And that means[4] using the minimization condition disregarding the h-dependence of the parameters in the potential thus falling in the same situation as we have described above, where some terms have been neglected.

To summarize, we have used RG techniques to resum the large radiative logarithmic corrections to Higgs masses which occur when the scale of supersymmetry breaking is far above the electroweak breaking scale. We have showed that if one uses this method it is not sufficient to start the RG evolution below the supersymmetry breaking scale and have derived a straightforward modification of the RG method to include the additional terms which occur from radiative corrections involving virtual SUSY states which couple to the Higgs scalar. The resulting corrections are in the range (2-8) Gev for the range of values for the mass of the top quark suggested by present results [22], and depend sensitively on the top quarks mass.

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