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AN EFFECTIVE CHIRAL MESON LAGRANGIAN AT $O(p^6)$ FROM THE NJL MODEL[1]

A.A.Bel'kov[†], A.V.Lanyov[†], A.Schaale[‡], S.Scherer[¶]

[†] *Particle Physics Laboratory, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia*

[‡] *Deutsches Elektronen-Synchrotron DESY, Institut für Hochenergiephysik, Platanenallee 6, D-15735 Zeuthen, Germany*

[¶] *Institut für Kernphysik, Johannes Gutenberg-Universität, J. J. Becher-Weg 45, D-55099 Mainz, Germany*

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In this work we present a strong chiral meson Lagrangian up to and including $O(p^6)$ in the momentum expansion. It is derived from the Nambu–Jona-Lasinio (NJL) model using the heat-kernel method. Identities related to the properties of covariant derivatives of the chiral matrix U as well as field transformations have been used to obtain a minimal set of linearly independent terms.

1. The Bosonization of the NJL Model

The effective four-quark interaction of the NJL model [1] is a low-energy approximation of QCD, the standard model of the strong interactions of quarks and gluons. The bosonization of the NJL model generates an effective chiral meson Lagrangian which results from the quark determinant (see [2] and references therein). Applying the heat-kernel techniques [3, 4] for the analytical calculation of the quark determinant one can derive a momentum expansion of the effective meson Lagrangian. In particular, the terms of $O(p^2)$ lead to the kinetic and mass parts of the Lagrangian, and the terms of $O(p^4)$ can be brought into the general form which was introduced by Gasser and Leutwyler [5]. A phenomenological analysis of the chiral coefficients L_i shows a good agreement with the predictions of the NJL model. It is reasonable to expect the same of the next order in the momentum expansion, where precise experimental data are not yet available.

In previous works [4, 6] we have presented the heat-kernel expansion of the quark determinant up to the order h_6 of the heat coefficients, containing the complete information about the $O(p^6)$ terms of the effective meson Lagrangian in the NJL model.

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As far as precision experiments are becoming more sensitive, it will be possible to observe effects which are related to the higher order of the momentum expansion (see for example [7] and [8]). Here we derive the $O(p^6)$-Lagrangian from the NJL model. A systematic study of the most general chiral Lagrangian at $O(p^6)$ can be found in [9].

The starting point of our consideration is the effective four-quark interaction of the NJL model with the Lagrangian

$$\mathcal{L}_{NJL} = \bar{q}(i\hat{\partial} - m_0)q + \mathcal{L}_{int}, \tag{1}$$

where

$$\mathcal{L}_{int} = 2G_1\left\{\left(\bar{q}\frac{\lambda^a}{2}q\right)^2 + \left(\bar{q}i\gamma^5\frac{\lambda^a}{2}q\right)^2\right\} - 2G_2\left\{\left(\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\right)^2 + \left(\bar{q}\gamma^\mu\gamma^5\frac{\lambda^a}{2}q\right)^2\right\}.$$

Here G_1 and G_2 are empirical coupling constants, m_0 is the current quark mass matrix, and λ^a are the generators of the $SU(n)$ flavor group, normalized according to $\mathrm{tr}(\lambda^a\lambda^b) = 2\delta^{ab}$. Using a standard quark bosonization approach based on path integral techniques one can derive an effective meson action from the NJL Lagrangian (1).

First, one has to introduce colorless collective fields for the scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) mesons associated with the quark bilinears,

$$S^a = -4G_1\bar{q}\frac{\lambda^a}{2}q,\quad P^a = -4G_1\bar{q}i\gamma^5\frac{\lambda^a}{2}q,\quad V^a_\mu = i4G_2\bar{q}\gamma_\mu\frac{\lambda^a}{2}q,\quad A^a_\mu = i4G_2\bar{q}\gamma_\mu\gamma^5\frac{\lambda^a}{2}q,$$

where a is a flavor index. After substituting these expressions into $\mathcal{L}_{NJL}$ the interaction part of the Lagrangian becomes bilinear in the quark fields:

$$\mathcal{L} = \bar{q}i\widehat{\mathbf{D}}q$$

with $\widehat{\mathbf{D}}$ being the Dirac operator in the presence of the collective meson fields,

$$i\widehat{\mathbf{D}} = [i(\hat{\partial} + \widehat{A}_R) - (\Phi + m_0)]P_R + [i(\hat{\partial} + \widehat{A}_L) - (\Phi^\dagger + m_0)]P_L.$$

Here $\Phi = S + iP$, $\widehat{V} = V_\mu\gamma^\mu$, $\widehat{A} = A_\mu\gamma^\mu$; $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ are chiral projectors; $\widehat{A}^{R/L} = \widehat{V} \pm \widehat{A}$ are right and left combinations of fields, and

$$S = S^a\frac{\lambda^a}{2},\quad P = P^a\frac{\lambda^a}{2},\quad V_\mu = -iV^a_\mu\frac{\lambda^a}{2},\quad A_\mu = -iA^a_\mu\frac{\lambda^a}{2}$$

are the matrix-valued collective fields.

After integration over the quark fields the generating functional, corresponding to the effective action of the NJL model for collective meson fields, can be presented in the following form:

$$\mathcal{Z} = \int \mathcal{D}\Phi\,\mathcal{D}\Phi^\dagger\,\mathcal{D}V\,\mathcal{D}A\,\exp[i\mathcal{S}(\Phi,\Phi^\dagger,V,A)],$$

where

$$\mathcal{S}(\Phi,\Phi^\dagger,V,A) = \int d^4x\left[-\frac{1}{4G_1}\mathrm{tr}(\Phi^\dagger\Phi) - \frac{1}{4G_2}\mathrm{tr}(V_\mu^2 + A_\mu^2)\right] - i\,\mathrm{Tr}'\,[\log(i\widehat{\mathbf{D}})] \tag{2}$$

is the effective action for scalar, pseudoscalar, vector and axial-vector mesons. The first term in Eq. (2), quadratic in the meson fields, arises from the linearization of the four-quark interaction. The second one is the quark determinant describing the interaction of mesons. The quark determinant can be evaluated using the heat-kernel technique with proper-time regularization [3, 4]. Then, the real part of $\log(\det i\widehat{\mathbf{D}})$ contributes to the even intrinsic parity part of the effective Lagrangian while the imaginary part gives the odd intrinsic parity effective Lagrangian which at $O(p^4)$ is related to the anomalous action of Wess and Zumino [10, 11].

The logarithm of the modulus of the quark determinant is defined in "proper-time" regularization as

$$\log|\det i\widehat{\mathbf{D}}| = -\frac{1}{2}\mathrm{Tr}'\log(\widehat{\mathbf{D}}^\dagger\widehat{\mathbf{D}}) = -\frac{1}{2}\int_{1/\Lambda^2}^{\infty} d\tau\,\frac{1}{\tau}\,\mathrm{Tr}'\exp(-\widehat{\mathbf{D}}^\dagger\widehat{\mathbf{D}}\tau) \tag{3}$$

with Λ being the intrinsic regularization parameter. The "trace" Tr' is to be understood as a space-time integration and a "normal" trace with respect to Dirac, color and flavor matrices, $\mathrm{Tr}' = \int d^4x\,\mathrm{Tr}$, and $\mathrm{Tr} = \mathrm{tr}_\gamma \cdot \mathrm{tr}_C \cdot \mathrm{tr}_F$. The main idea of the heat-kernel method is to expand $< x \mid \exp(-\widehat{\mathbf{D}}^\dagger\widehat{\mathbf{D}}\tau) \mid y >$ around its "free" part

$$< x \mid \exp(-(\Box + \mu^2)\tau) \mid y > = \frac{1}{(4\pi\tau)^2}e^{-\mu^2\tau + (x-y)^2/(4\tau)}$$

in powers of the proper time τ with the so-called Seeley-deWitt coefficients $h_k(x,y)$

$$< x \mid \exp(-\widehat{\mathbf{D}}^\dagger\widehat{\mathbf{D}}\tau) \mid y > = \frac{1}{(4\pi\tau)^2}e^{-\mu^2\tau + (x-y)^2/(4\tau)}\sum_k h_k(x,y)\cdot\tau^k.$$

The new mass scale μ arises as a nonvanishing vacuum expectation value of the scalar field S, and corresponds to the constituent quark mass.

After integration over τ in (3) one gets

$$\log|\det i\widehat{\mathbf{D}}| = -\frac{1}{2}\frac{\mu^4}{(4\pi)^2}\sum_k \frac{\Gamma(k-2,\mu^2/\Lambda^2)}{\mu^{2k}}\mathrm{Tr}\,h_k, \tag{4}$$

where $\Gamma(n,x) = \int_x^\infty dt\,e^{-t}t^{n-1}$ is the incomplete gamma function. Using the definition of the gamma function $\Gamma(\alpha,x)$, one can separate the divergent and finite parts of the quark determinant

$$\frac{1}{2}\log(\det\widehat{\mathbf{D}}^\dagger\widehat{\mathbf{D}}) = B_{\mathrm{pol}} + B_{\log} + B_{\mathrm{fin}},$$

where

$$B_{\mathrm{pol}} = \frac{1}{2}\frac{e^{-x}}{(4\pi)^2}\left[-\frac{\mu^4}{2x^2}\mathrm{Tr}\,h_0 + \frac{1}{x}\left(\frac{\mu^4}{2}\mathrm{Tr}\,h_0 - \mu^2\,\mathrm{Tr}\,h_1\right)\right]$$

has a pole at $x = \mu^2/\Lambda^2 = 0$,

$$B_{\log} = -\frac{1}{2}\frac{1}{(4\pi)^2}\Gamma(0,x)\left[\frac{1}{2}\mu^4\,\mathrm{Tr}\,h_0 - \mu^2\,\mathrm{Tr}\,h_1 + \mathrm{Tr}\,h_2\right]$$

is logarithmically divergent, and the finite part has the form

$$B_{\text{fin}} = -\frac{1}{2}\frac{1}{(4\pi)^2}\sum_{k=3}^{\infty}\mu^{4-2k}\Gamma(k-2,x)\,\mathrm{Tr}\,h_k.$$

The very lengthy calculations of the Seeley-deWitt coefficients h_k can be only performed by computer support. The calculation of the heat-coefficients is a recursive process which can conveniently be done by Computer Algebra Systems such as FORM and REDUCE. In ref.[4] we have calculated the coefficients up to the order $k=6$. After voluminous computations one gets the expressions for heat-coefficients $h_1, \ldots h_6$ up to terms of $O(p^6)$ (terms contributing only at higher orders are dropped)

$$\begin{aligned}
h_0 &= 1, \\
h_1 &= -a, \\
\mathrm{Tr}\,h_2 &= \mathrm{Tr}\left\{\frac{1}{12}(\Gamma_{\mu\nu})^2 + \frac{1}{2}a^2\right\}, \\
\mathrm{Tr}\,h_3 &= -\frac{1}{12}\mathrm{Tr}\left\{2a^3 - S_\mu S^\mu + a(\Gamma_{\mu\nu})^2 - \frac{2}{45}(\Gamma_{\alpha\beta\gamma})^2 - \frac{1}{9}(\Gamma^\alpha{}_{\alpha\beta})^2 \right. \\
&\quad \left. -\frac{2}{45}\Gamma_{\mu\nu}\Gamma^{\nu\alpha}\Gamma_\alpha{}^\mu\right\}, \\
\mathrm{Tr}\,h_4 &= \mathrm{Tr}\left\{\frac{1}{24}a^4 + \frac{1}{12}\left(a^2S^\mu{}_\mu + aS_\mu S^\mu\right) + \frac{1}{720}\left(7(S^\mu{}_\mu)^2 - (S_{\mu\nu})^2\right)\right. \\
&\quad + \frac{1}{30}a^2(\Gamma_{\mu\nu})^2 + \frac{1}{120}(a\Gamma_{\mu\nu})^2 + \frac{1}{144}a[\Gamma^\mu{}_{\mu\nu}, S^\nu] \\
&\quad \left. + \frac{1}{40}a\left(\Gamma_{\mu\nu}S^{\mu\nu} + \frac{11}{9}S_{\mu\nu}\Gamma^{\mu\nu}\right)\right\}, \\
\mathrm{Tr}\,h_5 &= -\mathrm{Tr}\left\{\frac{1}{120}a^2(a^3 - 3S_\mu S^\mu) - \frac{1}{60}(aS_\mu)^2\right\}, \\
\mathrm{Tr}\,h_6 &= \frac{1}{720}\mathrm{Tr}\,a^6.
\end{aligned}$$

Here

$$\Gamma_{\mu\nu} = [d_\mu, d_\nu]\,,\ \ \Gamma_{\lambda\mu\nu} = [d_\lambda, \Gamma_{\mu\nu}]\,,\ \ S_\mu = [d_\mu, a]\,,\ \ S_{\mu\nu} = [d_\mu, S_\nu]$$

are commutators of the operators d_μ and a which are defined by the relations

$$d_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = V_\mu + A_\mu\gamma^5, \quad a(x) = i\widehat{\nabla}H + H^\dagger H + \frac{1}{4}[\gamma^\mu, \gamma^\nu]\Gamma_{\mu\nu} - \mu^2.$$

Moreover,

$$H = P_R(\Phi + m_0) + P_L(\Phi^\dagger + m_0) = S + m_0 + i\gamma_5 P\,,$$

and

$$\Gamma_{\mu\nu} = [d_\mu, d_\nu] = \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu + [\Gamma_\mu, \Gamma_\nu] = F^V_{\mu\nu} + \gamma^5 F^A_{\mu\nu}\,,$$

where $F^{V/A}_{\mu\nu}$ are the field strength tensors,

$$F^V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] + [A_\mu, A_\nu],$$

$$F^A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [V_\mu, A_\nu] + [A_\mu, V_\nu],$$

and

$$\nabla_\mu H = \partial_\mu H + [V_\mu, H] - \gamma^5\{A_\mu, H\}\,.$$

2. The Chiral Lagrangian

We will consider here a nonlinear parameterization of chiral symmetry corresponding to the representation $\Phi = \Omega\Sigma\Omega$. The matrix of scalar fields $\Sigma(x)$ belongs to the diagonal flavor group, while the matrix $\Omega(x)$ represents the pseudoscalar degrees of freedom φ living in the coset space $SU(n)_L\times SU(n)_R/SU_V(n)$. It can be parameterized by the $SU(n)$ matrix

$$\Omega(x) = \exp\left(\frac{i}{\sqrt{2}F_0}\varphi(x)\right)\,, \quad \varphi(x) = \varphi^a(x)\frac{\lambda^a}{2}\,,$$

with F_0 being the bare π decay constant. Under chiral rotations

$$q \to \widetilde{q} = (P_L\xi_L + P_R\xi_R)\,q$$

the fields Φ and $A^{R/L}_\mu$ are transforming as

$$\Phi \to \widetilde{\Phi} = \xi_L\Phi\xi_R^\dagger\,,$$

and

$$A^R_\mu \to \widetilde{A}^R_\mu = \xi_R(\partial_\mu + A^R_\mu)\xi_R^\dagger, \quad A^L_\mu \to \widetilde{A}^L_\mu = \xi_L(\partial_\mu + A^L_\mu)\xi_L^\dagger.$$

The effective meson Lagrangian in terms of the collective fields is obtained from the quark determinant by calculating the trace over Dirac indices in $\mathrm{Tr}\,h_i(x)$. The "divergent" part of the effective meson Lagrangian is defined by the coefficients h_0, h_1 and h_2 of the expansion (4)

$$\begin{aligned}
\mathcal{L}_{div} &= \tfrac{N_c}{16\pi^2}\mathrm{tr}\left\{\Gamma\left(0, \tfrac{\mu^2}{\Lambda^2}\right)\left[D^\mu(\Phi + m_0)\,\overline{D}_\mu(\Phi + m_0)^\dagger - \mathcal{M}^2\right.\right. \\
&\quad \left.\left. + \tfrac{1}{6}\left((F^L_{\mu\nu})^2 + (F^R_{\mu\nu})^2\right)\right] + 2\left[\Lambda^2 e^{-\mu^2/\Lambda^2} - \mu^2\Gamma\left(0, \tfrac{\mu^2}{\Lambda^2}\right)\right]\mathcal{M}\right\}, \qquad (5)
\end{aligned}$$

where $\mathcal{M} = (\Phi + m_0)(\Phi + m_0)^\dagger - \mu^2$ and $F^{R/L}_{\mu\nu} = F^V_{\mu\nu} \pm F^A_{\mu\nu}$. The covariant derivatives D_μ and $\overline{D}_\mu$ are defined as

$$D_\mu * = \partial_\mu * + (A^L_\mu * - * A^R_\mu)\,, \quad \overline{D}_\mu * = \partial_\mu * + (A^R_\mu * - * A^L_\mu)\,,$$

where it is understood that D_μ and $\overline{D}_\mu$ act upon expressions transforming as $\xi_L \cdots \xi_R^\dagger$ and $\xi_R \cdots \xi_L^\dagger$, respectively. Assuming $\Sigma \approx \mu$ and therefore $\Phi = \mu\Omega^2 \equiv \mu U$, the Lagrangian of $O(p^2)$ can be written in the form

$$\mathcal{L}^{(2)}_{eff} = -\frac{F_0^2}{4}\,\mathrm{tr}\,(L_\mu L^\mu) + \frac{F_0^2}{4}\,\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger),$$

where $L_\mu = D_\mu U\, U^\dagger$. The bare constant F_0 and the matrix $\chi = \mathrm{diag}\,(\chi_u^2, \chi_d^2, \chi_s^2)$ are given by $F_0^2 = yN_c\mu^2/(4\pi^2)$ and $\chi_i^2 = m_0^i\mu/(G_1F_0^2) = -2m_0^i < \overline{q}q > F_0^{-2}$, where $y = \Gamma(0,\mu^2/\Lambda^2)$ and $< \overline{q}q >$ is the quark condensate.

The terms of $O(p^4)$ of the effective Lagrangian result from the logarithmically divergent part of the quark determinant and from the coefficients h_3 and h_4 contributing to the finite part. Using properties of the derivatives (see appendix, Eqs.(9,10)) the finite contribution of $O(p^4)$ can be written as

$$\begin{aligned}
\mathcal{L}^{(p^4)}_{fin} \;=\; & \frac{N_c}{32\pi^2\mu^4}\mathrm{tr}\Big\{\frac{1}{3}\Big[\mu^2 D^2(\Phi+m_0)\,\overline{D}^2(\Phi+m_0)^\dagger - (D^\mu(\Phi+m_0)\,\overline{D}_\mu(\Phi+m_0)^\dagger)^2\Big] \\
+ \; & \frac{1}{6}(D_\mu(\Phi+m_0)\,\overline{D}_\nu(\Phi+m_0)^\dagger)^2 \\
- \; & \mu^2(\mathcal{M}D_\mu(\Phi+m_0)\,\overline{D}^\mu(\Phi+m_0)^\dagger + \overline{\mathcal{M}}\,\overline{D}_\mu(\Phi+m_0)^\dagger D_\mu(\Phi+m_0)) \\
+ \; & \frac{2}{3}\mu^2\Big(D^\mu(\Phi+m_0)\,\overline{D}^\nu(\Phi+m_0)^\dagger\, F^L_{\mu\nu} + \overline{D}^\mu(\Phi+m_0)^\dagger\, D^\nu(\Phi+m_0)\,F^R_{\mu\nu}\Big) \\
+ \; & \frac{1}{3}\mu^2 F^R_{\mu\nu}(\Phi+m_0)^\dagger F^{L\,\mu\nu}(\Phi+m_0) - \frac{1}{6}\mu^4\Big[(F^L_{\mu\nu})^2 + (F^R_{\mu\nu})^2\Big]\Big\},
\end{aligned} \tag{6}$$

where $\overline{\mathcal{M}} = (\Phi+m_0)^\dagger(\Phi+m_0) - \mu^2$. We will assume the approximation $\Gamma(k,\mu^2/\Lambda^2) \approx \Gamma(k)$, valid for $k \geq 1$, and $\mu^2/\Lambda^2 \ll 1$.

The effective meson Lagrangian of $O(p^4)$, Eq.(6), can be brought into the standard form introduced by Gasser and Leutwyler in ref.[5] (see appendix, Eq.(12)) After using the field transformations [12], which are at $O(p^4)$ equivalent to the application of the classical equation of motion (EOM) (see appendix 13), the NJL model gives the following predictions for the chiral coefficients

$$\begin{aligned}
& L_1 = \frac{N_c}{16\pi^2}\frac{1}{24}, \quad L_2 = \frac{N_c}{16\pi^2}\frac{1}{12}, \quad L_3 = -\frac{N_c}{16\pi^2}\frac{1}{6}, \\
& L_4 = 0, \quad L_5 = \frac{N_c}{16\pi^2}x(y-1), \quad L_6 = 0, \\
& L_7 = -\frac{N_c}{16\pi^2}\frac{1}{6}\Big(xy - \frac{1}{12}\Big), \quad L_8 = \frac{N_c}{16\pi^2}\Big[\Big(\frac{1}{2}x - x^2\Big)y - \frac{1}{24}\Big], \\
& L_9 = \frac{N_c}{16\pi^2}\frac{1}{3}, \quad L_{10} = -\frac{N_c}{16\pi^2}\frac{1}{6}, \\
& H_1 = -\frac{N_c}{16\pi^2}\frac{1}{6}\Big(y - \frac{1}{2}\Big), \quad H_2 = -\frac{N_c}{16\pi^2}\Big[(x+2x^2)y - \frac{1}{12}\Big],
\end{aligned} \tag{7}$$

where $x = -\mu F_0^2/(2 <\overline{q}q>)$ and $y = 4\pi^2F_0^2/(N_c\mu^2)$.

Analogous as for the $O(p^4)$ Lagrangian we present the p^6 Lagrangians in a "minimal" form, avoiding redundant terms. The identities and relations which we have used in order to keep the number of terms as small as possible can be found in the appendix. It is important to realize that the field transformations used to bring the Lagrangian of $O(p^4)$ into the form of Gasser and Leutwyler also result in contributions at $O(p^6)$ and higher [12]. Furthermore, we eliminated (see app.) terms at $O(p^6)$ using field transformations.

The final effective p^6-Lagrangian has the form (see appendix for details)

$$\begin{aligned}
\mathcal{L}^{(6)}_{eff} \;=\; & \frac{N_c}{32\pi^2\mu^2}\mathrm{tr}\Big\{-\frac{1}{10}(L_\mu L_\nu L^\nu)^2 \\
& + \frac{5}{18}(L_\mu L^\mu)^3 - \frac{1}{45}L_\alpha L^\alpha(L_\mu L_\nu)^2 + \frac{1}{30}(L_\mu L_\nu L_\alpha)^2 - \frac{1}{10}(L_\mu L_\nu L^\mu)^2 \\
& - \frac{1}{30}\Big(L_\mu L_\nu D_\alpha D^\nu U\,\overline{D}^\alpha\overline{D}^\mu U^\dagger + R_\mu R_\nu \overline{D}_\alpha \overline{D}^\nu U^\dagger\, D^\alpha D^\mu U\Big) \\
& + \frac{1}{30}\Big(L_\mu L_\nu D_\alpha D^\mu U\,\overline{D}^\alpha\overline{D}^\nu U^\dagger + R_\mu R_\nu \overline{D}_\alpha \overline{D}^\mu U^\dagger\, D^\alpha D^\nu U\Big) \\
& + \Big[4c + \frac{1}{2}\Big(\frac{73}{90} - x\Big)\Big] L_\mu L_\nu L^\nu L^\mu(\chi U^\dagger + U\chi^\dagger) \\
& + \Big[2c - \frac{47}{180}\Big](L_\mu L_\nu)^2(\chi U^\dagger + U\chi^\dagger) \\
& + \Big[-2c + \frac{1}{6}\Big(x + \frac{1}{15}\Big)\Big](L_\mu L^\mu)^2(\chi U^\dagger + U\chi^\dagger) \\
& + \Big[c + \frac{1}{60}\Big]\Big(\chi R^\mu U^\dagger(D_\mu D_\nu U + D_\nu D_\mu U)U^\dagger L^\nu \\
& \qquad + \chi^\dagger L^\mu U(\overline{D}_\mu\overline{D}_\nu U^\dagger + \overline{D}_\nu\overline{D}_\mu U^\dagger)U R^\nu\Big) \\
& - \Big[3c - \frac{1}{3}\Big(x - \frac{1}{20}\Big)\Big]\Big(\chi(\overline{D}_\mu\overline{D}_\nu U^\dagger L^\mu L^\nu + R^\nu R^\mu U\overline{D}_\mu\overline{D}_\nu U^\dagger) \\
& \qquad + \chi^\dagger(D_\mu D_\nu U R^\mu R^\nu + L^\nu L^\mu D_\mu D_\nu U)\Big) \\
& - \Big[2c(1-6x) + \frac{1}{3}\Big(2x^2 - \frac{1}{2}x + \frac{1}{20}\Big)\Big]\chi^\dagger L_\mu \chi R^\mu \\
& - \Big[c(12x+1) - \frac{1}{3}\Big(4x^2 + \frac{1}{4}x - \frac{1}{40}\Big)\Big]\Big(\chi^\dagger\chi R_\mu R^\mu + \chi\chi^\dagger L_\mu L^\mu\Big) \\
& - \Big[c + \frac{x}{3}\Big(2x - \frac{1}{4}\Big)\Big]\Big(U\chi^\dagger U\chi^\dagger L_\mu L^\mu + U^\dagger\chi U^\dagger\chi R_\mu R^\mu\Big) \\
& + \Big[c(1+6x) - \frac{x}{3}\Big(x + \frac{1}{4}\Big)\Big]\Big((\chi U^\dagger L_\mu)^2 + (\chi^\dagger U R_\mu)^2\Big) \\
& + x\Big(6c - x + \frac{1}{12}\Big)\Big(D'_\mu(\chi U^\dagger + U\chi^\dagger)(\chi U^\dagger L^\mu - L^\mu U\chi^\dagger) \\
& \qquad - \overline{D}'_\mu(\chi^\dagger U + U^\dagger\chi)(\chi^\dagger U R^\mu - R^\mu U^\dagger\chi)\Big) \\
& - \frac{x}{12}\Big(D'^2(\chi U^\dagger + U\chi^\dagger)L_\mu L^\mu + \overline{D}'^2(\chi^\dagger U + U^\dagger\chi)\,R_\mu R^\mu\Big)
\end{aligned}$$

$$
\begin{aligned}
&-6cxy\Big(D_\mu(\chi - U\chi^\dagger U)\,\overline{D}^\mu\chi^\dagger + \overline{D}_\mu(\chi^\dagger - U^\dagger\chi U^\dagger)\,D^\mu\chi\Big)\\
&-cD_\mu(U\chi^\dagger U - \chi)\overline{D}^\mu(\chi^\dagger - U^\dagger\chi U^\dagger)\\
&-\frac{9}{2}c^2y\Big(D'_\mu(\chi U^\dagger - U\chi^\dagger)\Big)^2\\
&-\frac{9}{2}c^2yD'_\mu(\chi U^\dagger - U\chi^\dagger)[(\chi U^\dagger - U\chi^\dagger), L^\mu]\\
&-\frac{x^2}{6}\Big((D'_\mu(\chi U^\dagger + U\chi^\dagger))^2 + (\overline{D}'_\mu(\chi^\dagger U + U^\dagger\chi))^2\Big)\\
&+\Big[\frac{9}{4}yc^2 + 6cx(1-2xy) + \frac{1}{3}x\Big(2x^2 - 3x + \frac{1}{8}\Big)\Big]\Big((\chi U^\dagger)^3 + (\chi^\dagger U)^3\Big)\\
&+\Big[-\frac{9}{4}c^2y - 6cx(1-2xy) + 2x^3 + x^2 - \frac{1}{24}x\Big](U^\dagger\chi\chi^\dagger\chi + U\chi^\dagger\chi\chi^\dagger)\\
&-\frac{1}{45}\Big(F^L_{\mu\nu}\{L_\alpha L^\alpha, L^\mu L^\nu\} + F^R_{\mu\nu}\{R_\alpha R^\alpha, R^\mu R^\nu\}\Big)\\
&+\frac{5}{36}\Big(F^L_{\mu\nu}(L^\mu L_\alpha L^\nu L^\alpha + L_\alpha L^\mu L^\alpha L^\nu) + F^R_{\mu\nu}(R^\mu R_\alpha R^\nu R^\alpha + R_\alpha R^\mu R^\alpha R^\nu)\Big)\\
&-\frac{3}{20}(F^L_{\mu\nu}L_\alpha L^\mu L^\nu L^\alpha + F^R_{\mu\nu}R_\alpha R^\mu R^\nu R^\alpha)\\
&-\frac{7}{15}(F^L_{\mu\nu}L^\mu L_\alpha L^\alpha L^\nu + F^R_{\mu\nu}R^\mu R_\alpha R^\alpha R^\nu)\\
&-\frac{2}{15}\Big(F^{L\,\alpha}{}_{\alpha\nu}L_\mu L^\nu L^\mu - F^{R\,\alpha}{}_{\alpha\nu}R_\mu R^\nu R^\mu\Big)\\
&+\frac{1}{5}\Big(F^{L\,\alpha}{}_{\alpha\nu}\{L^\nu, L_\mu L^\mu\} - F^{R\,\alpha}{}_{\alpha\nu}\{R^\nu, R_\mu R^\mu\}\Big)\\
&+\frac{x}{6}\Big((\chi U^\dagger + U\chi^\dagger)\{F^L_{\mu\nu}, L^\mu L^\nu\} + (\chi^\dagger U + U^\dagger\chi)\{F^R_{\mu\nu}, R^\mu R^\nu\}\Big)\\
&-\Big[c + \frac{1}{60}\Big]\Big((\chi U^\dagger - U\chi^\dagger)[F^L_{\mu\nu}, L^\mu L^\nu] + (\chi^\dagger U - U^\dagger\chi)[F^R_{\mu\nu}, R^\mu R^\nu]\Big)\\
&+\Big[2c - \frac{1}{2}x\Big]\Big(\chi(\overline{D}^\mu\overline{D}^\nu U^\dagger F^L_{\mu\nu} - F^R_{\mu\nu}\overline{D}^\mu\overline{D}^\nu U^\dagger)\\
&\qquad\qquad + \chi^\dagger(D^\mu D^\nu U F^R_{\mu\nu} - F^L_{\mu\nu}D^\mu D^\nu U)\Big)\\
&+\Big[2c - \frac{1}{6}x\Big]\Big(\chi(U^\dagger F^L_{\mu\nu}\,D^\mu D^\nu U\,U^\dagger - U^\dagger D^\mu D^\nu U\,F^R_{\mu\nu}U^\dagger)\\
&\qquad\qquad + \chi^\dagger(UF^R_{\mu\nu}\,\overline{D}^\mu\overline{D}^\nu U^\dagger U - U\overline{D}^\mu\overline{D}^\nu U^\dagger\,F^L_{\mu\nu}U)\Big)\\
&+\Big[-2c + \frac{1}{2}x + \frac{1}{20}\Big]\Big(F^{L\,\alpha}{}_{\alpha\mu}(L^\mu U\chi^\dagger + \chi U^\dagger L^\mu)\\
&\qquad\qquad - F^{R\,\alpha}{}_{\alpha\mu}(R^\mu U^\dagger\chi + \chi^\dagger U R^\mu)\Big)\\
&+\Big[2c - \frac{1}{6}x - \frac{1}{20}\Big]\Big(F^{L\,\alpha}{}_{\alpha\mu}(L^\mu\chi U^\dagger + U\chi^\dagger L^\mu) - F^{R\,\alpha}{}_{\alpha\mu}(R^\mu\chi^\dagger U + U^\dagger\chi R^\mu)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{2}{15}\Big(F^{L\,\mu\alpha}{}_{\alpha\nu}[L^\nu, L_\mu] + F^{R\,\mu\alpha}{}_{\alpha\nu}[R^\nu, R_\mu]\Big)\\
&-\frac{1}{180}F^L_{\mu\nu}D_\alpha U\,F^{R\,\mu\nu}\overline{D}^\alpha U^\dagger\\
&-\frac{1}{5}F^L_{\alpha\mu}D_\nu U\,F^{R\,\alpha\nu}\overline{D}^\mu U^\dagger\\
&-\frac{1}{5}F^L_{\alpha\mu}D^\mu U\,F^{R\,\alpha\nu}\overline{D}_\nu U^\dagger\\
&+\frac{83}{360}\Big((F^L_{\mu\nu})^2 L_\alpha L^\alpha + (F^R_{\mu\nu})^2 R_\alpha R^\alpha U\Big)\\
&-\frac{1}{15}\Big(L_\alpha L^\alpha U F^R_{\mu\nu}U^\dagger F^{L\,\mu\nu} + R_\alpha R^\alpha U^\dagger F^L_{\mu\nu}UF^{R\,\mu\nu}\Big)\\
&-\frac{1}{2}\Big(F^L_{\mu\alpha}F^{L\,\alpha\nu}L^\mu L_\nu + F^R_{\mu\alpha}F^{R\,\alpha\nu}R^\mu R_\nu\Big)\\
&-\frac{11}{30}\Big(F^L_{\mu\alpha}F^{L\,\alpha\nu}L_\nu L^\mu + F^R_{\mu\alpha}F^{R\,\alpha\nu}R_\nu R^\mu\Big)\\
&+\frac{1}{6}\Big(F^L_{\mu\nu}(D^\mu D_\alpha U + D_\alpha D^\mu U)F^{R\,\nu\alpha}U^\dagger\\
&\qquad\qquad + F^R_{\mu\nu}(\overline{D}^\mu\overline{D}_\alpha U^\dagger + \overline{D}_\alpha\overline{D}^\mu U^\dagger)F^{L\,\nu\alpha}U\Big)\\
&-\frac{1}{6}\Big(F^{L\,\alpha}{}_{\alpha\nu}[L_\mu, UF^{R\,\mu\nu}U^\dagger] - F^{R\,\alpha}{}_{\alpha\nu}[R_\mu, U^\dagger F^{L\,\mu\nu}U]\Big)\\
&-\frac{1}{15}\Big(F^{L\,\alpha}{}_{\alpha\nu}[L_\mu, F^{L\,\mu\nu}] - F^{R\,\alpha}{}_{\alpha\nu}[R_\mu, F^{R\,\mu\nu}]\Big)\\
&+\frac{1}{15}F^{L\,\mu}{}_{\mu\alpha}UF^{R\nu\alpha}_{\nu}U^\dagger\\
&+\frac{1}{6}(x-5)\Big((\chi U^\dagger + U\chi^\dagger)(F^L_{\mu\nu})^2 + (\chi^\dagger U + U^\dagger\chi)(F^R_{\mu\nu})^2\Big)\\
&+\Big[c - \frac{1}{6}\Big(x - \frac{1}{5}\Big)\Big](\chi U^\dagger F^L_{\mu\nu}UF^{R\,\mu\nu}U^\dagger + \chi^\dagger UF^R_{\mu\nu}U^\dagger F^{L\,\mu\nu}U)\\
&-\Big[c - \frac{1}{6}\Big(x - \frac{1}{5}\Big)\Big](\chi F^R_{\mu\nu}U^\dagger F^{L\,\mu\nu} + \chi^\dagger F^L_{\mu\nu}UF^{R\,\mu\nu})\\
&+\frac{41}{540}\Big((F^L_{\mu\nu\alpha})^2 + (F^R_{\mu\nu\alpha})^2\Big)\\
&-\frac{7}{135}\Big((F^{L\,\mu}{}_{\mu\alpha})^2 + (F^{R\,\mu}{}_{\mu\alpha})^2\Big)\\
&+\frac{1}{3}\Big(F^L_{\mu\nu}F^{L\,\mu\alpha}F^{L\,\nu}{}_\alpha + F^R_{\mu\nu}F^{R\,\mu\alpha}F^{R\,\nu}{}_\alpha\Big)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
+\quad &\frac{N_c}{32\pi^2\mu^2}\,\mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger)\frac{1}{180}\,\mathrm{tr}\,\Big(\{L_\mu, L_\nu\}(U\overline{D}_\mu\overline{D}_\nu U^\dagger - D_\mu D_\nu U\,U^\dagger)\Big)\\
&-\Big[c\Big(\frac{1}{3} + 2x - 2xy\Big) - \frac{1}{18}\Big(x - \frac{1}{10}\Big)\Big]\,\mathrm{tr}\,(L_\mu L^\mu(\chi U^\dagger - U\chi^\dagger))
\end{aligned}
$$

$$
\begin{aligned}
&+\Big[-\frac{3}{2}c^2y - cx(2+y-4xy) + \frac{1}{36}x\Big]\,\mathrm{tr}\left((\chi U^\dagger)^2 - (\chi^\dagger U)^2\right) \\
&+\frac{1}{30}\,\mathrm{tr}\left(F^{L\mu}{}_{\mu\nu}L^\nu + F^{R\mu}{}_{\mu\nu}R^\nu\right) \\
&-\frac{1}{36}\,\mathrm{tr}\left((F^L_{\mu\nu})^2 - (F^R_{\mu\nu})^2\right)\Big\} \\
+\;& \frac{N_c}{32\pi^2\mu^2}\Big\{-\frac{4}{135}\left(\partial_\mu \mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger)\right)\mathrm{tr}\,(L^\mu L_\nu L^\nu) \\
&+\Big[\frac{1}{216}+\frac{3}{2}c^2y\Big]\left(\partial_\mu \mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger)\right)^2\Big\} \\
+\;& \frac{N_c}{32\pi^2\mu^2}\left(\mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger)\right)^2\Big\{\Big[\frac{1}{4}c^2y - \frac{1}{3}cxy + \frac{1}{54}x\Big]\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger) \\
&+\frac{1}{1080}\mathrm{tr}\,(L_\mu L^\mu)\Big\}, \qquad (8)
\end{aligned}
$$

where $R_\mu = U^\dagger D_\mu U$, $D'_\mu * = \partial_\mu * + [A^L_\mu, *]$, $\overline{D}'_\mu * = \partial_\mu * + [A^R_\mu, *]$, $F^R_{\alpha\mu\nu} = \overline{D}'_\alpha F^R_{\mu\nu}$ and $F^L_{\alpha\mu\nu} = D'_\alpha F^L_{\mu\nu}$. Terms proportional to a factor $c = \frac{1}{6}\left(x - \frac{1}{6y}\right)$ are related to the $O(p^4)$ part of the field transformations (20) and those proportional to c^2 arise from Eq.(18) after equivalent transformations.

The P and C symmetries of the strong interaction allow at $O(p^6)$ structures which are proportional to $\varepsilon_{\alpha\beta\mu\nu}$ [11, 13, 9] and which do not belong to the Wess-Zumino anomalous action. However, in this approach these contributions disappear, because we have limited our self to calculate only the absolute value of the quark determinant.

3. Conclusion

We have presented an effective chiral meson Lagrangian to $O(p^6)$ in the momentum expansion, obtained from the bosonization of the NJL model. To minimize the number of independent terms in this expression, extensive use of the properties of covariant derivatives and field transformations has been made. In contrast to previous studies of Lagrangians at $O(p^4)$, we had to retain the next-to-leading order terms in the field transformations which gave additional contributions to the $O(p^6)$ Lagrangian in the process of transforming the bosonized p^4-Lagrangian to the canonical Gasser-Leutwyler form.

The Lagrangian obtained at $O(p^6)$ is expected to be important in neutral meson processes, for example, $\eta \to \pi^0\gamma\gamma$, $\gamma\gamma \to \pi^0\pi^0$ and $K^0_L \to \pi^0\gamma\gamma$ where Born contributions from the $O(p^4)$ Lagrangian vanish [14]. However, taking the relevant effects correctly into account is not a simple task since together with the results of bosonization (i.e. transition to collective meson fields through integrating out the quark degrees of freedom) one has to address the question about the influence of heavy vector and axial-vector resonances [15] and also the nonlocal corrections for the usual local version of NJL model [16].

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Appendix A. Identities and Field Transformations

A1 Identities

In order to reduce the number of terms as much as possible we have made use of several identities and relations, which are listed here.

The Lagrangian contains different types of derivatives which satisfy the following rules:

$$
\begin{aligned}
D_\mu(O_1O_2) &= (D_\mu O_1)O_2 + O_1(\overline{D}'_\mu O_2) = (D'_\mu O_1)O_2 + O_1(D_\mu O_2)\,, \\
\overline{D}_\mu(O_1O_2) &= (\overline{D}_\mu O_1)O_2 + O_1(D'_\mu O_2) = (\overline{D}'_\mu O_1)O_2 + O_1(\overline{D}_\mu O_2)\,, \\
D'_\mu(O_1O_2) &= (D'_\mu O_1)O_2 + O_1(D'_\mu O_2) = (D_\mu O_1)O_2 + O_1(\overline{D}_\mu O_2)\,, \\
\overline{D}'_\mu(O_1O_2) &= (\overline{D}'_\mu O_1)O_2 + O_1(\overline{D}'_\mu O_2) = (\overline{D}_\mu O_1)O_2 + O_1(D_\mu O_2)\,. \qquad (9)
\end{aligned}
$$

In order to reduce the number of terms which contain Goldstone bosons only, we applied the following relations

$$
\begin{aligned}
&[D_\mu, D_\nu]O = F^L_{\mu\nu}O - OF^R_{\mu\nu}\,, \qquad [\overline{D}_\mu, \overline{D}_\nu]O = F^R_{\mu\nu}O - OF^L_{\mu\nu}\,, \\
&[D'_\mu, D'_\nu]O = [F^L_{\mu\nu}, O]\,, \qquad [\overline{D}'_\mu, \overline{D}'_\nu]O = [F^R_{\mu\nu}, O]\,. \qquad (10)
\end{aligned}
$$

We have also made use relations arising from the unitarity of the matrix U:

$$
D_\mu U\,U^\dagger = -U\,\overline{D}_\mu U^\dagger\,, \quad D_\mu D_\nu U\,U^\dagger + U\,\overline{D}_\mu \overline{D}_\nu U^\dagger = -\left(D_\mu U\,\overline{D}_\nu U^\dagger + D_\nu U\,\overline{D}_\mu U^\dagger\right).
$$

A2 Field Transformations

The initial Lagrangian also contained redundant terms which can be eliminate with the help of field transformations. At $O(p^4)$ the use of field transformations and a naive application of the classical EOM are equivalent. At the next order in the momentum expansion, $O(p^6)$, the method of field transformations will give rise to contributions which one would miss using the classical EOM only. Before the application of field transformations the most general Lagrangian of $O(p^4)$ typically has the form (see Eq.(6)) [2]

$$\begin{aligned}\mathcal{L}_4 = & L_1'\left(\mathrm{tr}\,(D_\mu U\overline{D}^\mu U^\dagger)\right)^2 + L_2'\mathrm{tr}\,(D_\mu U\overline{D}_\nu U^\dagger)\,\mathrm{tr}\,\left(D^\mu U\overline{D}^\nu U^\dagger\right)\\ & +L_3'\mathrm{tr}\,\left(D_\mu U\overline{D}^\mu U^\dagger D_\nu U\overline{D}^\nu U^\dagger\right) + L_4'\mathrm{tr}\,\left(D_\mu U\overline{D}^\mu U^\dagger\right)\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger)\\ & +L_5'\mathrm{tr}\,\left(D_\mu U\overline{D}^\mu U^\dagger(\chi U^\dagger + U\chi^\dagger)\right) + L_6'\,(\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger))^2\\ & +L_7'\,(\mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger))^2 + L_8'\mathrm{tr}\,(U\chi^\dagger U\chi^\dagger + \chi U^\dagger\chi U^\dagger)\\ & +L_9'\mathrm{tr}\,\left(F^L_{\mu\nu}D^\mu U\overline{D}^\nu U^\dagger + F^R_{\mu\nu}\overline{D}^\mu U^\dagger D^\nu U\right) - L_{10}'\mathrm{tr}\,(UF^R_{\mu\nu}U^\dagger F^{\mu\nu}_L)\\ & -H_1'\mathrm{tr}\,(F^R_{\mu\nu}F^{\mu\nu}_R + F^L_{\mu\nu}F^{\mu\nu}_L) + H_2'\mathrm{tr}\,(\chi\chi^\dagger)\\ & +\lambda_1\mathrm{tr}\,\left(D^2U\overline{D}^2U^\dagger\right) + \lambda_2\mathrm{tr}\,\left(D^2U\chi^\dagger + \chi\overline{D}^2U^\dagger\right). \end{aligned} \tag{11}$$

It contains 2 more structures than the standard Lagrangian of Gasser and Leutwyler. In order to eliminate the two additional terms, one rewrites $D^2U\,U^\dagger$ and $U\overline{D}^2U^\dagger$ applying the chain rule to $UU^\dagger = 1$. After some algebra the Lagrangian of Eq.(11) can be written as

$$\mathcal{L}_4 = \mathcal{L}_4^{G\&L} + c_1\mathrm{tr}\,\left((D^2UU^\dagger - U\overline{D}^2U^\dagger)\mathcal{O}^{(2)}_{EOM}\right) + c_2\mathrm{tr}\,\left((\chi U^\dagger - U\chi^\dagger)\mathcal{O}^{(2)}_{EOM}\right), \tag{12}$$

where $\mathcal{L}_4^{G\&L}$ is the Gasser and Leutwyler Lagrangian defined in [5] and $\mathcal{O}^{(2)}_{EOM}$ has the functional form of the classical EOM of $O(p^2)$:

$$\mathcal{O}^{(2)}_{EOM}(U) = D^2UU^\dagger - U\overline{D}^2U^\dagger - \chi U^\dagger + U\chi^\dagger + \frac{1}{3}\mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger). \tag{13}$$

The unprimed (G&L) and primed coefficients are related through

$$\begin{aligned} & L_1 = L_1',\quad L_2 = L_2',\quad L_3 = L_3' + \lambda_1,\quad L_4 = L_4',\quad L_5 = L_5' - \lambda_2,\\ & L_6 = L_6',\quad L_7 = L_7' + \frac{\lambda_1}{12} + \frac{\lambda_2}{6},\quad L_8 = L_8' - \frac{\lambda_1}{4} - \frac{\lambda_2}{2},\quad L_9 = L_9',\\ & L_{10} = L_{10}',\quad H_1 = H_1',\quad H_2 = H_2' + \frac{\lambda_1}{2} + \lambda_2,\\ & c_1 = -\frac{\lambda_1}{4},\quad c_2 = -\frac{\lambda_1}{4} - \frac{\lambda_2}{2}. \end{aligned} \tag{14}$$

In our NJL-based approach the coefficients λ_i are

$$\lambda_1 = \frac{1}{6}\frac{N_c}{16\pi^2},\quad \lambda_2 = -\frac{N_c}{16\pi^2}xy.$$

Using the field transformation technique [12] we will get rid of the last two terms of Eq. (12). For that purpose we write

$$U(x) = \exp(iS_2(V))V(x), \tag{15}$$

[2] Note our different convention for the definitions of the tensors $F^{L,R}$: $F^{L,R} = -iF^{R,L}_{G\&L}$.

where $S_2(V)$ is given by

$$\begin{aligned} S_2(V) &= -i\frac{\lambda_1}{F_0^2}(D^2V\,V^\dagger - V\overline{D}^2V^\dagger)\\ &\quad -i\left(\frac{\lambda_1}{F_0^2} + \frac{2\lambda_2}{F_0^2}\right)\left(\chi V^\dagger - V\chi^\dagger - \frac{1}{3}\mathrm{tr}\,(\chi V^\dagger - V\chi^\dagger)\right). \end{aligned} \tag{16}$$

If we insert $U = \exp(iS)V$ into $\mathcal{L}_2(U)$ we obtain

$$\mathcal{L}_2(U) = \mathcal{L}_2(V) + \delta^{(1)}\mathcal{L}_2(V,S) + \delta^{(2)}\mathcal{L}_2(V,S) + \ldots \tag{17}$$

In Eq. (17) we have dropped an irrelevant total derivative. The superscripts denote the power of S (or $D_\mu S, \ldots$) and the corresponding expressions are given by

$$\begin{aligned} \delta^{(1)}\mathcal{L}_2(V,S_2) &= \frac{F_0^2}{4}\mathrm{tr}\,\left(iS_2\mathcal{O}^{(2)}_{EOM}(V)\right) = O(p^4),\\ \delta^{(2)}\mathcal{L}_2(V,S_2) &= \frac{F_0^2}{4}\mathrm{tr}\,\left(S_2(D_\mu VV^\dagger D'^\mu S_2 - D'^\mu S_2D_\mu VV^\dagger - D'^2S_2)\right.\\ &\quad \left.-\frac{1}{2}(\chi V^\dagger + V\chi^\dagger)S_2^2\right) = O(p^6),\\ \delta^{(3)}\mathcal{L}_2(V,S_2) &= O(p^2)\times O(S_2^3) = O(p^8). \end{aligned} \tag{18}$$

The last term is only interesting at $O(p^8)$ and thus we do not give its explicit form. With our choice of S_2, eq.(16), the term $\delta^{(1)}\mathcal{L}_2(V,S_2)$ precisely cancels the last two contributions of Eq. (12) ($U \to V$ at $O(p^4)$).

The modification of $\mathcal{L}_4$ has a similar form as that of $\mathcal{L}_2$ in Eq.(17)

$$\mathcal{L}_4(U) = \mathcal{L}_4(V) + \delta^{(1)}\mathcal{L}_4(V,S) + O(p^8), \tag{19}$$

where

$$\delta^{(1)}\mathcal{L}_4(V,S) = \frac{F_0^2}{4}\mathrm{tr}\,\left(iS\mathcal{O}^{(4)}_{EOM}(V)\right) = O(p^6). \tag{20}$$

From the Lagrangian in Eq.(11) we obtain for the $O(p^4)$ contribution to the EOM operator

$$\mathcal{O}^{(4)}_{EOM}(U) = \frac{4}{F_0^2}\left(E_4 - \frac{1}{3}\mathrm{tr}\,(E_4)\right), \tag{21}$$

where

$$\begin{aligned} E_4 &= \left(2L_1' - L_2'\right)\mathrm{tr}\,(D_\mu U\,\overline{D}^\mu U^\dagger)\cdot(D^2U\,U^\dagger - U\,\overline{D}^2U^\dagger)\\ &+ 2L_2'\Big[-U\,\overline{D}_\mu\left(\overline{D}_\nu U^\dagger\,D^\mu U\,\overline{D}^\nu U^\dagger\right) + D_\mu\left(D_\nu U\,\overline{D}^\mu U^\dagger\,D^\nu U\right)U^\dagger\Big]\\ &+ (4L_2' + 2L_3')\Big[-U\overline{D}_\mu\left(\overline{D}^\mu U^\dagger D_\nu U\,\overline{D}^\nu U^\dagger\right) + D_\mu\left(D_\nu U\,\overline{D}^\nu U^\dagger\,D^\mu U\right)\,U^\dagger\Big]\\ &+ L_4'\Big[\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger)\cdot(D^2U\,U^\dagger - U\,\overline{D}^2U^\dagger) + \mathrm{tr}\,(D_\mu U\,\overline{D}^\mu U^\dagger)\cdot(U\chi^\dagger - \chi U^\dagger)\Big] \end{aligned}$$

$$
\begin{aligned}
&+ L_5'\Big[-U\overline{D}_\mu\left(\overline{D}^\mu U^\dagger(\chi U^\dagger + U\chi^\dagger)\right) + D_\mu\left((\chi U^\dagger + U\chi^\dagger)D^\mu U\right)U^\dagger \\
&\qquad + U\chi^\dagger D_\mu U\overline{D}^\mu U^\dagger - D_\mu U\overline{D}^\mu U^\dagger\chi U^\dagger\Big] \\
&+ 2L_6'\mathrm{tr}\,(\chi U^\dagger + U\chi^\dagger)\cdot(U\chi^\dagger - \chi U^\dagger) \\
&- 2L_7'\mathrm{tr}\,(\chi U^\dagger - U\chi^\dagger)\cdot(U\chi^\dagger + \chi U^\dagger) \\
&+ L_8'\Big[(U\chi^\dagger)^2 - (\chi U^\dagger)^2\Big] \\
&+ L_9'\Big[-U\overline{D}^\nu(F^R_{\mu\nu}\overline{D}^\mu U^\dagger) + D^\mu(D^\nu U F^R_{\mu\nu})U^\dagger - U\overline{D}^\mu(\overline{D}^\nu U^\dagger F^L_{\mu\nu}) \\
&\qquad + D^\nu(F^L_{\mu\nu}D^\mu U)U^\dagger\Big] \\
&- L_{10}'\Big[U F^R_{\mu\nu}U^\dagger F^{L\,\mu\nu} - F^L_{\mu\nu}U F^{R\,\mu\nu}U^\dagger\Big] \\
&+ \lambda_1\Big[U\overline{D}^2\overline{D}^2U^\dagger - D^2D^2U\,U^\dagger\Big] + \lambda_2\Big[U\overline{D}^2\chi^\dagger - D^2\chi U^\dagger\Big]\,. \qquad (22)
\end{aligned}
$$

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