acta physica slovaca vol. 44 No. 6, 497 – 501 December 1994

DISTURBANCE IN A PIEZO-ELECTRIC SLAB BOUNDED BY AN ELASTIC MEDIUM AND COATED WITH A THIN FILM

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Received 22 December 1993, accepted 19 April 1994

The problem of a mechanical disturbance in a piezoelectric slab bounded by an elastic medium on both sides and the bounding surfaces of which are coated with a thin film of conducting material has been investigated in the present paper. The problem involves the interaction of two fields viz., electrical and mechanical. The variation of the mechanical disturbances with time ranging from 0 to 10s exhibits a linear relationship but significantly with a constant disturbance of the order of 10^{-8}m at $t = 0$.

1. Introduction

The problems of disturbances in a piezo-electric media owing to electrical or mechanical excitations or both have engaged the attention of many researchers [1–4]. The relevant problems are considered to be important in view of various practical applications in different branches of science and technology. Piezo-electric slab problems have been investigated by Chatterjee [5], Kundu [6], Banno [8], Kirichok [9], etc. In particular, Chatterjee [5], Baranski [10], etc. considered some problems of piezo-electric slab sandwiched between and elastic and visco-elastic media under different input signals. However, the present study seeks to investigate a more interesting problem of piezo-electric slab bounded by an elastic medium on both sides and the bounding surfaces are coated with a thin film of conducting material. The problem involves the interaction of two fields, viz., mechanical and electrical. Maxwell's equations and the equations of elasticity have been used and the solution is obtained with the aid of operational calculus. The variation of mechanical disturbance with time ranging from 0 to 10s gives out a linear relation whereas at $t = 0$, significantly a disturbance of the order of 10^{-8}m persists.

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0323-0465/94

2. Problem fundamental equations and boundary conditions

Let us consider a piezoelectric slab bounded by an elastic medium on both sides of its bounding faces. Let $x = 0$ and $x = d$ represent the bounding faces of the piezoelectric slab. The bounding faces are covered with a thin film of a perfect conductor. The face $x = 0$ which is taken to be mechanically stress-free is excited in the direction of y-axis by a electric field

$$E_y = E_0 sin\omega t \tag{1}$$

We now proceed to obtain the disturbances proceeding in the direction of x-axis.

Following Redwood [4], Kyame [11], the piezoelectric equations of the medium are given by

$$\begin{aligned} T_{ij} &= c^E_{ijkm} S_{km} + e_{lij} E_l \\ D_l &= -e_{lij} S_{ij} + \varepsilon^S_{lm} E_m \end{aligned} \tag{2}$$

where T is the stress, S - the strain, E - electric field intensity, c - the elastic compliance at constant electric field, e - the piezoelectric coefficient, D - the electric displacement, and ε - the clamed dielectric permittivity. All indices, $i, j, \ldots$ run from 1 to 3 and the Einstein summation convention has been employed. The stress-tensor S_{ik} is given by

$$S_{ik} = \frac{1}{2}(\partial u_i/\partial x_k + \partial u_k/\partial x_i) \tag{3}$$

where u is the mechanical displacement.

The mechanical displacement u in the direction of x-axis and the electric field E in the direction of y-axis satisfy the following equations

$$\partial^2 u/\partial x^2 - \rho_1/c\, \partial^2 u/\partial t^2 = -e_1/c\, \partial E/\partial x \tag{4}$$

$$\partial^2 E/\partial x^2 - \mu\varepsilon \partial^2 E/\partial t^2 = -\mu e_1 \partial^2/\partial t^2\, \partial u/\partial x \tag{5}$$

where, ρ_1 is the density of the piezo-electric material.
In the elastic medium, the mechanical displacement satisfies the equation

$$\rho_2 \partial^2 u/\partial t^2 = (\lambda + 2\mu)\partial^2 u/\partial x^2 \tag{6}$$

where, ρ_2 is the density of the elastic material and λ, μ are Lame's constant. The boundary conditions applied are as follows:

$$E_1 = E_0 \sin\omega t \quad \text{at} \quad x = 0 \tag{7}$$
$$T_1 = 0 \quad \text{at} \quad x = 0 \tag{8}$$
$$T_1 = T_2 \quad \text{at} \quad x = d \tag{9}$$
$$u_1 = u_2 \quad \text{at} \quad x = d \tag{10}$$
$$E_1 = 0 \quad \text{at} \quad x = d \tag{11}$$

where the suffixes 1 and 2 stand for the entities of the piezoelectric and elastic media respectively.

3. Solution of the problem

Applying the Laplace transform of parameter $p[\Re p > 0]$ to Eqs. (4), (5) and (6) we get

$$\begin{aligned} (D^2 - \rho_1 p^2/c)\bar{u}_1 + e_1 D\bar{E}_1/c &= 0 \quad (12) \\ \mu e_1 p^2 D\bar{u}_1 + (D^2 - \mu\varepsilon p^2)\bar{E}_1 &= 0 \quad (13) \\ \partial^2 \bar{u}_2/\partial x^2 - \rho_2 p^2 \bar{u}_2/(\lambda + 2\mu) &= 0 \quad (14) \end{aligned}$$

where $D = \partial/\partial x$ and $D^2 = \partial^2/\partial x^2$. Solving Eqs. (12) and (13), we get

$$\bar{u}_1 = C_1 \exp(-m_1 x) + C_2 \exp(-m_2 x) + C_3 \exp(m_1 x) + C_4 \exp(m_2 x) \tag{15}$$

where C_1, C_2, C_3, C_4 are constants to be determined by applying the boundary conditions and m_1^2 and m_2^2 are the roots of the equation

$$m^4 - m^2 p^2(\rho_1/c + \mu\varepsilon + \mu e_1^2/c) + \mu\rho_1\varepsilon p^4/e_1 = 0 \tag{16}$$

The solution of Eq. (14) subject to the condition $\bar{u}_2 \to 0$ as $x \to \infty$ is given by

$$\bar{u}_2 = C_5 \exp(-m_1^{'} x) \tag{17}$$

where, C_5 is a constant to be determined by boundary conditions and $m_1^{'}$ is a root of the equation

$$m_2 - \rho_2 p^2/(\lambda + 2\mu) = 0 \quad . \tag{18}$$

From Eq. (15), we get

$$\bar{u}_1|_{x=0} = C_1 + C_2 + C_3 + C_4 \tag{19}$$

The values of the constants C_1, C_2, C_3 and C_4 are obtained by putting the boundary conditions, Eqs. (7) - (11) in Eqs. (15) and (17).
From Eqs. (16) and (18) as in Sinha [12], Petrov [7], we get

$$m_1 = \theta_1 p, \quad m_2 = \theta_2 p, \quad m_1^{'} = \gamma p$$

where, θ_1, θ_2 and γ are positive constants involving material parameters of the problem. Substituting C_1, C_2, C_3 and C_4 in Eq. (19), to a first approximation as in Redwood [4], Banno [8], etc. we get after taking inverse transformation

$$\begin{aligned} u = {} & t[(A - 2B)/2A^2(\theta_1 + \theta_2) - d/A + e_1 E_0/\omega c(\theta_1 + \theta_2)] \\ & + \exp(-At/B)[d/A + B/A^2(\theta_1 + \theta_2)] - e_1 E_0 \cos\omega t/\omega c(\theta_1 + \theta_2) \end{aligned} \tag{20}$$

where,

$$A = 2(\lambda\gamma + 2\mu\gamma) + \rho(\theta_2 - \theta_1)/\theta_1\theta_2; \qquad B = \rho d(\theta_1 + \theta_2)/\theta_1$$

Eq. (20) shows the response in a piezoelectric slab bounded by an elastic medium on both sides and bounding surfaces are coated with a thin film of conducting material.

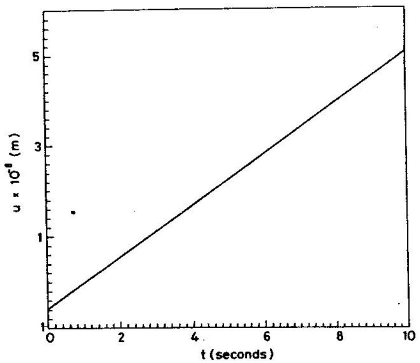

Fig. 1 Variation of the mechanical disturbance of a piezo-electric slab with time.

T [s]	$u \times 10^{-8}$ m
0	- 0.56
1	- 0.0002
2	0.56
3	1.12
4	1.68
5	2.24
6	2.80
7	3.37
8	3.39
9	4.49
10	5.06

Table 1. Numerical values of the mechanical disturbances of a piezoelectric slab (u) Vs. time (t).

5. Discussion

For numerical calculations, the standard values of the material constants have been taken from [13–16] while values like E_0, d, ω have been chosen suitably to facilitate the numerical calculations as follows: $E_0 = 300$ v, $d = 0.05$ m, $\omega = 1.5$ rad/s. The variation of the mechanical disturbance with time is shown in table 1.

The response as given out by Eq. (20) is partly linear, partly transient and partly periodic. It is clear that the contribution of transient part is insignificant as compared to linear and periodic terms. Obviously, the variation of mechanical disturbances with time ranging from 0 to 10 s exhibit a linear relationship (Fig. 1). Interestingly at $t = 0$, a disturbance of the order of 10^{-8}m persists and the results are found to be valid only within the investigated range of time.

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