

STATISTICAL NEURODYNAMICS: A MODEL FOR UNIVERSAL PROPERTIES OF EEG - DATA AND PERCEPTION¹Hans - Otto Carmesin²*Institut für Theoretische Physik, Universität Bremen, PO Box 330440,
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The formation of percepts through the brain is characterized by four 'universal properties': disambiguation of ambiguous sensory stimuli, hysteresis, P300 - EEG - brain potentials and binding of sensory stimuli according to 'Gestalt' laws. Novel neurostatistical network models are proposed, containing fast neurons, slower Hebb - type couplings, prestabilized coupling factors, fluctuations and, optionally, a comparison mechanism; in order to illustrate the physiological plausibility, subnetworks with fixed couplings are explicated that instantiate fast couplings in effect. The network models exhibit a quenched disorder type free energy that characterizes a self - organization process in coupling space. The networks bind stimuli by forming locally stable collective states, so called Hebb cell assemblies. The formation of these collective states exhibits four properties that correspond to the four 'universal properties' of perception.

1. Introduction

The human brain can be regarded as a highly complex physical system that exhibits a variety of observable and measurable phenomena. Sensory stimuli are processed by the brain on various levels; thereby the brain exhibits states that have been characterized scientifically by psychologists and are called *percepts*. A neural network is presented that models the emergence of such percepts; thereby a percept corresponds to an order parameter of a self - organization process. Empirical physiological and psychological data and the network model exhibit four corresponding 'universal properties'.

To begin with, the phenomenon of *perception of motion* is characterized in a preliminary manner: The nervous system is stimulated externally by light projected onto the retina. The task for the nervous system is to interpret this external stimulation in terms of objects and their motion. To this end, the nervous system instantiates several levels of processing. For the present purpose, it is adequate to characterize these levels

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according to the event related brain - potentials that can be measured as EEG after events of stimulation. These potentials exhibit up to five prominent stereotypic peaks of few microvolts at roughly determined times after the external stimulation. The peaks are: a negative one after 100 milliseconds, a positive one after 200 milliseconds followed by a negative one shortly later, a large positive peak after 300 milliseconds, and a negative one after 400 milliseconds [1]. The peaks are denoted by N100, P200, N200, P300 and N400. The presently modeled motion percepts correspond to P300 peaks.

One of the main ideas used in the present network model was proposed by Hebb [2]. He suggested that a percept is established by the fast formation of a cell assembly. ... *What I have in mind, in emphasizing half a second or so as the duration of a reverberatory activity, is the observed duration of a single content in perception. ... This then is the cell assembly.*

How do the couplings of a cell assembly form? Two different mechanisms have been proposed, by which couplings between inner neurons and peripheral neurons should form: According to the Hebb rule, such couplings increase, whenever the presynaptic and the postsynaptic neuron fire coincidentally; whereas I suggest that the Hebb rule should be applied combined with a mechanism of comparison, Maturana [3] suggested a similar idea. Both mechanisms of coupling formation as well as the cell assembly are studied here in terms of neural network models.

2. Method

In a first subsection, four phenomena of perception are made precise in terms of certain experiments about motion percepts (see Fig. 1); these four phenomena are typical for perception in general. In a second subsection, the above qualitative ideas are made precise in terms of neural network models. In the third section, these networks are applied to model the experiments. As results, qualitative and quantitative properties of these network models are derived. In particular, it is shown that the networks exhibit order parameters [4] corresponding to percepts. In the fourth section, these results are discussed and compared with each other.

2.1. Modelled Experiments

Perception exhibits its rich inherent properties especially in situations of ambiguity [5]. Nowadays, such situations can be prepared under precisely determined conditions in the laboratory [6]. Accordingly, such laboratory situations are studied: A square of dots is defined on a screen, see Fig. 1. At each instant of time, two non - neighbouring dots are elicited. So one of two possible patterns is shown at every time, see Fig. 1. More precisely, these two patterns alternate with constant frequency. An observer perceives either a vertical alternating motion or a horizontal alternating motion. So the percept is bistable³ On a longer time scale of seconds, the percept switches randomly. That is, the nervous system 'disambiguates' the ambiguous external stimulation at any instant of time, but it switches on a longer time scale.

³More precisely, clockwise percepts and counter clockwise percepts are also produced by persons. This occurs only seldom and is omitted in the following, because such generality is covered by the generalized model without any additional effort.

* ○ * → ○
↓ ↑
○ * ○ ← *

Fig. 1. Stimulation. Stars: first of the two shown patterns. Arrows mark motion percepts. Left: vertical motion percept. Right: horizontal motion percept. In the second pattern dots and stars are exchanged.

In further experiments, the square is changed into a rectangle. As a consequence, the motion percept between the narrower dots becomes more stable than the other one; if one slowly modifies the rectangle, then a nice *hysteresis* can be measured for the two motion percepts.

Moreover, brain - potentials have been recorded in the above experiments [7]. The test person is instructed to push a key when the percept changes from horizontal to vertical motion or vice versa, i.e., when a new percept occurs. It turns out that a P300 - like EEG can be measured, as if the new percept were externally stimulated by an event that occurred 300 milliseconds before (see Fig. 2). Of course, there is no such event. Instead, the new percept changes spontaneously. A control group of test persons was instructed to press the key voluntarily; as a result, no such EEG - signal was measured. This shows that the formation of the percept is preceded by a P300 - like EEG - signal. Similar EEG - measurements have been performed for the Necker cube [8]. More generally, P300 - EEG - signals can be measured whenever test persons generate percepts [1].

For an external stimulation, only certain percepts are possible; for simple stimulations, these can be roughly characterized by 'Gestalt' laws [9]. The characterized empirical findings can be regarded as *four 'universal properties of perception'*: 'disambiguation', 'hysteresis', 'P300 - EEG - signals and binding of external stimuli according to 'Gestalt' rules.

2.2 Network Models

In a first part, a basic neural network is specified in order to model the explicated experiments. The basic neural network contains fast couplings; in a second part, these fast couplings are instantiated by subnetworks. The basic neural network uses the above mentioned Hebb rule together with a comparison mechanism; a neural network with a simple Hebb rule is explicated in a third part. The basic and simplified neural networks contain discrete time steps; in a fourth part it is shown how discrete time steps with an appropriate time interval could be selected. In the fifth and sixth parts, the basic network model as well as the network model with simple Hebb rule are generalized so that they can be applied to perception in general.

2.2.1. Basic network model

Each neuron takes the value 1 or -1 at discrete steps $t = 1, 2, 3, \dots$. The four dots are modeled by the 'peripheral' neurons s_1, s_2, s_3 and s_4 , see Fig. 3. The peripheral neurons take values according to a dynamics; these values are 'compared' with the states of the dots, see Fig. 1; the peripheral neurons shall reproduce the following time

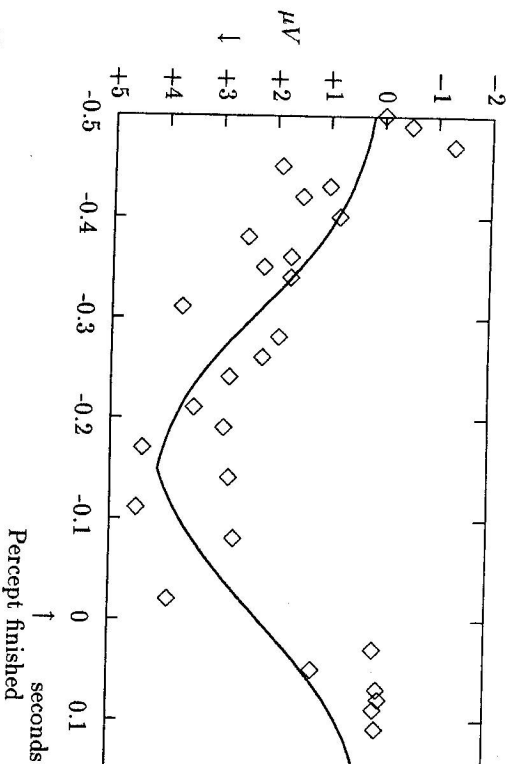


Fig. 2. EEG accompanying spontaneous formation of a new percept. Reference: electrode at left earlobe. Ordinate: brain - potential in microvolt at right parietal position. Abscissa: time in seconds; $t=0$: test person pushes key to indicate a change of the percept. Data: measured EEG. Solid line: modeled voltage U_m .

behaviour of the dots. The network has to generate the first pattern ($\mu = 1$) at $t = 1, 2$, the second ($\mu = 2$) at $t = 3, 4$, the first ($\mu = 1$) at $t = 5, 6$, and so forth. I.e., the network has to produce the alteration of the presented patterns. For this purpose, the network has inner neurons s_5, s_6, s_7 and s_8 that connect the peripheral neurons s_1, s_2, s_3 and s_4 with 16 couplings W_{ij} (from s_j to s_i), see Fig. 3. The comparison is formalized by an indicator τ^μ that is 1 whenever the network performs as desired and 0 otherwise.

The neuronal dynamics is characterized by a transition probability from $\{s_j(t)\}$ to $s_i(t+1)$

$$P[s_i(t+1)] = \frac{\exp[h_i(t+1)s_i(t+1)/T]}{2 \cosh[h_i(t+1)s_i(t+1)/T]}, \quad \text{with } h_i(t+1) = \frac{1}{2} \sum_j W_{ij}^2(t) \zeta_{ij} s_j(t). \quad (1)$$

Here T is a formal temperature that models statistical fluctuations, h_i is a formal local field and ζ_{ij} are prestabilized factors that are characterized by the horizontal and vertical sides of the presented rectangle, i.e., if W_{ij} is horizontal in Fig. 3, then $\zeta_{ij} = \zeta_h$, otherwise $\zeta_{ij} = \zeta_v$; ζ_v is small if the vertical side is large; ζ_h is small if the horizontal side is large. The prestabilized factors might be interpreted with synaptic densities. Their dynamical effect gives rise to the 'Gestalt' laws. The above square of the couplings is unusual, however it can be eliminated by the substitution $K_{ij} = W_{ij}^2$ and $\Delta K_{ij} = 2W_{ij} \Delta W_{ij}$. The coupling dynamics is established by a Hebb term proportional

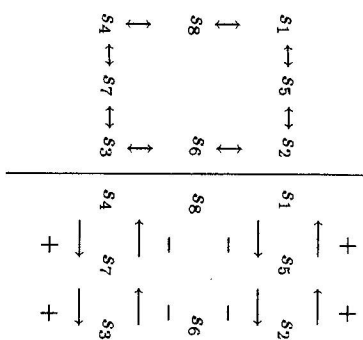


Fig. 3. Network architecture. Left: network; an arrow indicates two couplings, one in each direction. s_1, s_2, s_3 and s_4 : peripheral neurons, corresponding to corners of the square. s_5, s_6, s_7 and s_8 : inner neurons. Right: a coupling state with nonzero horizontal couplings, zero vertical couplings are not explicated.

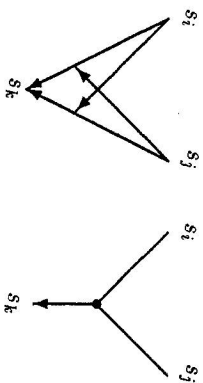


Fig. 4. Network that instantiates a trilinear coupling. LEFT: s_i, s_j : presynaptic neurons. s_k postsynaptic neuron. Long arrows: synapses. Short arrows: inhibitory synapses on synapses. RIGHT: equivalent network with a trilinear coupling J_{kij} .

to a learning rate a , a decay term b and the above indicator τ^μ .

$$W_{ij}(t+2) = W_{ij}(t+1) + \Delta W_{ij}(t+1), \quad (2)$$

$$\Delta W_{ij}(t+1) = \frac{\tau^\mu}{\varepsilon^\mu} [a W_{ij} \zeta_{ij} f - b W_{ij} \sum_{k,l} W_{kl}^2], \quad (3)$$

$$f = \begin{cases} s_i(t+1)s_j(t) & \text{if } s_i \text{ is inner neuron;} \\ s_i(t+2)s_j(t+1) & \text{if } s_j \text{ is inner neuron.} \end{cases} \quad (4)$$

In eqs. (3) and (4) and in the following, the time arguments of couplings W_{ij} , W_{kl} , are $t+1$ and are neglected, for short. Roughly speaking, at the time $t+1$ the couplings are changed according to signals that they transferred to and from the inner neurons at that time. The term ε^μ is 1 plus the number of times at which the network was successful at pattern μ divided by 1 plus the number of times at which the pattern μ occurred. ε^μ can be interpreted as a success rate. According to its dynamical effect, ε^μ can be interpreted as an attention parameter [10], so that the attention is high when the network has little success.

2.2.2. Fast couplings by subnetworks

If a neural network is used to model empirical findings, then it should consist of components that have been observed in the nervous system. For this reason, subnetworks are explicated that instantiate fast couplings effectively. For this purpose, first subnetworks are proposed that instantiate trilinear couplings, then subnetworks with trilinear couplings are suggested that instantiate fast couplings effectively.

Trilinear couplings by synapses on synapses: In nervous systems there occur synapses on synapses [11]. These can inhibit or enhance the signal transmission [11]. In this part a network with such synapses is explicated that instantiates in effect a trilinear coupling, that is, the neuronal dynamics

$$s_k(t+1) = \text{sgn}(J_{kij}s_i(t)s_j(t)) \quad (5)$$

is effectively instantiated, see Fig. 4.

The neuronal dynamics without synapses on synapses is

$$s_k(t+1) = \text{sgn}\left(\sum_i F_{ki}(s_i(t)) - \lambda_k\right) \quad (6)$$

with a synaptic stimulation $F_{ki}(s_i(t)) = J_{ki}s_i(t)$.

The effect of the inhibitory synapses on synapses can be expressed with a function analogous to the above F_{ki} as follows. The neuronal dynamics with inhibitory synapses on synapses is

$$s_k(t+1) = \text{sgn}\left(\sum_i G_{kij}(s_i, s_j) - \lambda_k\right)$$

with a synaptic stimulation

$$G_{kij}(s_i, s_j) = \begin{cases} J_{ki} & \text{if } s_i = 1 \text{ and } s_j = -1; \\ -J_{ki} & \text{else.} \end{cases} \quad (7)$$

That is, the postsynaptic neuron is stimulated by the coupling J_{ki} if the presynaptic neuron s_i fires and if the neuron s_j with the inhibitory synapse does not fire. The performance of the network in Fig. 4. with couplings and threshold

$$J_{ki} = J_{kj} = 1 \text{ and with } \lambda_k = -1.5 \quad (8)$$

is adequately discussed with a Table 1. The four rightmost columns correspond to the four possible values of the two presynaptic neurons. The first two rows represent the values of the presynaptic neurons; the 3rd and 4th row represent the values of the synaptic stimulations G (see eq. (7)); the 5th row represents the value of the postsynaptic neuron; the 6th row represents the value of the postsynaptic neuron in a network with a negative trilinear coupling (see eq. (5)); the 7th row represents the value of the postsynaptic neuron in a network with a positive trilinear coupling (see eq. (5)); the lowest row indicates the value s'_k of the postsynaptic neuron for the analogous network but with couplings and threshold

$$J_{ki} = J_{kj} = -1 \text{ and with } \lambda_k = 1.5. \quad (9)$$

1	$s_i(t)$	-	+	-	+
2	$s_j(t)$	-	-	+	+
3	G_{kij}	-	+	-	-
4	G_{kji}	-	-	+	-
5	$s_k(t+1)$	-	+	+	-
6	$s_k^{\text{negative trilinear coupling}}(t+1) = -s_i(t)s_j(t)$	-	+	+	-
7	$s_k^{\text{trilinear coupling}}(t+1) = s_i(t)s_j(t)$	+	-	-	+
8	$s'_k(t+1)$	+	-	-	+

Table 1.

Because the 5th and 6th Table 1. rows contain the same configurations of signs, the network in Fig. 4. with eq. (8) instantiates a negative trilinear coupling (see Fig. 4.). Analogously, since the 7th and 8th rows contain the same configurations of signs, the network in Fig. 4. with eq. (9) instantiates a negative trilinear coupling (see Fig. 4.).

Altogether, the present simple network models show, how an effective trilinear coupling might be instantiated by synapses and synapses on synapses.

Fast couplings: In nervous systems there may occur changes of synaptic weights within 4 minutes, while the membrane potential of a neuron may change on the time scale of milliseconds [11]. Moreover, the nervous system contains neurons that change their membrane potential smoothly rather than according to an action potential [11]. Such a smoothly varying neuron is modeled with the following difference equation

$$\Delta s_w(t) = h_w(t) - b s_w(t) \quad (10)$$

where $h_w(t)$ is the formal local field and $s_w(t+1) = s_w(t) + \Delta s_w(t)$. Here b is a decay coefficient.

In this part a network with such a smooth neuron $s_w(t)$ is explicated that instantiates in effect a coupling $K_{ij}(t)$ that changes on the time scale of milliseconds, that is, the local excitation $h_j(t)$ and the coupling $K_{ij}(t)$ are

$$h_j(t+1) = K_{ij}(t)s_i(t) \quad (11)$$

with a prestabilized weight ζ_{ij} and with a fast weight $W_{ij}(t)$ as factors $K_{ij}(t) = \zeta_{ij}W_{ij}(t)$ and with a fast weight dynamics $\Delta W_{ij}(t) = \zeta_{ij}s_i(t)s_j(t) - bW_{ij}(t)$ and $W_{ij}(t+1) = W_{ij}(t) + \Delta W_{ij}(t)$.

The equivalence of the two networks in Fig. 5. is shown as follows: By definition of trilinear couplings, the trilinear coupling J_{jwi} in the left network gives rise to the local formal field

$$h_j^{\text{left}}(t+1) = J_{jwi}s_i(t)s_w(t). \quad (12)$$

By construction (see Fig. 5.), the trilinear coupling is the same as the factor ζ_{ij} ; so one gets

$$h_j^{\text{left}}(t+1) = \zeta_{ij}s_i(t)s_w(t). \quad (13)$$

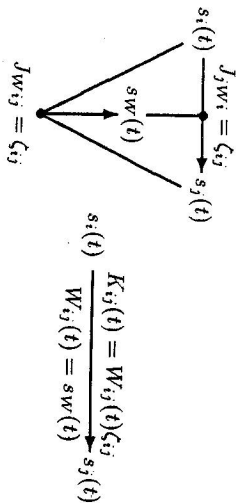


Fig. 5. Network that instantiates a fast coupling dynamics. LEFT and RIGHT: $s_i(t)$: presynaptic neuron, $s_j(t)$: postsynaptic neuron. LEFT: $s_w(t)$: smoothly varying neuron. Arrows: trilinear constant couplings $J_{wi} = J_{wi} = \zeta_{ij}$. RIGHT: equivalent network with a bilinear fast coupling $K_{ij}(t) = \zeta_{ij}W_{ij}(t)$. Thereby $W_{ij}(t)$ is the membrane potential of the smooth neuron $s_w(t)$ of the left (equivalent) network.

By construction (see Fig. 5.) $s_w(t) = W_{ij}(t)$ and $s_w(t)$ and $W_{ij}(t)$ evolve according to the same dynamics (see eqs. (10) and (11)). Thereby and by identifying the local formal field $h_j^{right}(t+1)$ of the network in the right part of Fig. 5, one obtains

$$h_j^{left}(t+1) = W_{ij}(t)\zeta_{ij}s_i(t) = h_j^{right}(t+1). \quad (14)$$

So the local formal fields $h_j^{left}(t+1)$ and $h_j^{right}(t+1)$ at the postsynaptic neuron s_j are the same in the two networks in Fig. 5.; in this sense, these two networks are equivalent. I.e., the considered equivalence relation is the equality of formal local fields at the postsynaptic neuron.

The present models for fast synapses contain a neural dynamics with a signum function, in contrast to the neural dynamics with a transition probability (see eq. (1)). However, the neural dynamics with the above transition probability becomes that with the signum function in the zero temperature limit; conversely, the dynamics with the signum function can be generalized to that with the above transition probability.

Altogether, the present simple network models show, how an effective fast coupling $K_{ij}(t)$ might be instantiated by a network with constant trilinear couplings and a smoothly varying neuron $s_w(t)$.

2.2.3. Simplified model

The basic network model is now modified so as to obtain a network model with a simple Hebb rule. For this purpose, the architecture remains unchanged (see Fig. 3) as well as the transition probability (see eq. (1)). However in this simplified model, the peripheral neurons take the values as the dots on the screen (see Fig. 1.). In the coupling dynamics, the factor τ^μ / ϵ^μ and the sum are omitted, so one gets

$$W_{ij}(t+2) = W_{ij}(t+1) + \Delta W_{ij}(t+1) \text{ with} \quad (15)$$

$$\Delta W_{ij}(t+1) = aW_{ij}\zeta_{ij}f - bW_{ij}g \text{ with} \quad (16)$$

$$f = \begin{cases} s_i(t+1)s_j(t) & \text{if } s_i \text{ is inner neuron;} \\ s_i(t+2)s_j(t+1) & \text{if } s_j \text{ is inner neuron.} \end{cases} \quad (17)$$

$$g = \begin{cases} \sum_k W_{ik}^2 & \text{if } s_i \text{ is peripheral neuron;} \\ \sum_k W_{kj}^2 & \text{if } s_j \text{ is peripheral neuron.} \end{cases} \quad (18)$$

The factor g is motivated only qualitatively. This factor models a finite capacity for signal transfer at each peripheral neuron. Such a property might be motivated by the empirical finding that the number of synaptic and the number of dendritic contacts of a particular neuron is constant [12]. Formally, one may interpret this condition as a local analogue of the global interaction of couplings proportional to b (see eq. (3)) in the basic model.

2.2.4. Selection of appropriate time intervals

In the above basic and the simplified network model, the time discretization has been chosen so that a time step is half the time interval between the switching of the patterns on the screen (see Fig. 1). This can be modeled as follows. The couplings in Fig. 3 are provided for a set of time intervals, one of which is the one explicated above. It will be shown later as a result, that only the couplings with the appropriate time intervals become nonzero.

2.2.5. Generalized basic network model

The basic network model is generalized so as to obtain a network model that performs the binding of stimuli. The generalization can be expressed with three steps. First, an inner neuron with four couplings is introduced for any pair of peripheral neurons; for instance, in the case of Fig. 3 this yields two more inner neurons with four couplings each, connecting s_1 with s_3 and s_2 with s_4 . Second, the external stimulation is established by a set of dots and a sequence of p patterns μ , each specifying what dots are elicited in the pattern and each occurring at a certain time. Third, one peripheral neuron is assigned to each such dot, i.e., the network consists of these peripheral neurons together with the above specified inner neurons with their couplings. Thereby, the couplings shall obey the normalization condition $\sum_{ij} \zeta_{ij}^2 = 1$.

2.2.6. Generalized model with simple Hebb rule

The simplified network model is generalized so as to obtain a network model that performs the binding of stimuli. The generalization is expressed by the above three steps.

3. Results

3.1. Analysis of the basic model

The combined neuronal and coupling dynamics exhibits [13] ergodicity properties [14]. As a consequence, the network is characterized by a mean coupling change $\langle \Delta W_{ij} \rangle$ together with a mean neuronal change. The neurons are regarded as fast variables and the mean neuronal change is eliminated in the adiabatic limit [4]. The remaining average is taken over the two patterns ($\mu = 1, 2$) and the neuronal configurations of inner neurons. So $\langle \Delta W_{ij} \rangle$ is a function of W_{ij} , i.e. it is a vector field. It turns out that the vector field $\langle \Delta W_{ij} \rangle(W_{ij})$ is the gradient of a scalar potential $V(W_{ij})$. These results are made precise as follows.

Potential Theorem: In the adiabatic limit

$$\langle \Delta W_{ij}(t) \rangle = -\frac{\partial V}{\partial W_{ij}} \quad (19)$$

with a potential

$$V = \frac{b}{4} \left(\sum_{ij} W_{ij}^2 \right)^2 - \frac{aT}{2} \sum_{\mu=1}^2 \ln Z^\mu \quad (20)$$

with partition functions

$$Z^\mu = \sum_{s_i(t+1)=\pm 1}^{i=5,6,7,8} \tau^\mu \exp[-H(t)/T] \quad (21)$$

and a formal energy function

$$H(t) = -\sum_{i=5}^8 h_i(t+1)s_i(t+1) - \sum_{i=1}^4 h_i(t+2)s_i(t+2). \quad (22)$$

The locally stable (steady) states exhibit $\langle \Delta W_{ij}(t) \rangle = 0$ and are the local minima of the potential V . A proof is presented in the appendix. Networks with nonzero couplings form below a critical temperature. The potential is similar to a free energy of a system with quenched disorder [15], because it contains a sum of partition functions. The local minima of the potential V can be interpreted in terms of order parameters [4].

The potential V has to be studied in the 16 - dimensional space of couplings. To make this study transparent, a symmetry is used that is expected to be realized at least approximately: The couplings that are necessary for a successful coupling state take the same absolute value; thereby a successful coupling state achieves $\tau^\mu = 1$ at zero fluctuations. A first coupling state that is successful consists of the 8 horizontal couplings (see Fig. 3), with the homogeneous weight $|W_h|$ (used symmetry). Analogously, a second coupling state that is successful consists of the 8 vertical couplings, with the

homogeneous weight $|W_v|$. Next, coupling states that are linear combinations of these two states are studied, while a general analysis is included in the section 'generalization'. τ^μ is (see Fig. 1)

$$\begin{aligned} \tau^1 = 1 & \text{ is equivalent to } (s_1, s_2, s_3, s_4)(t) = (1, -1, 1, -1) = -(s_1, s_2, s_3, s_4)(t+2) \\ \tau^2 = 1 & \text{ is equivalent to } (s_1, s_2, s_3, s_4)(t) = (-1, 1, -1, 1) = -(s_1, s_2, s_3, s_4)(t+2). \end{aligned} \quad (23)$$

Next one may calculate the energy $H_{\text{top}}(t)$ of the couplings W_{15}, W_{51}, W_{52} and W_{25} at the top of Fig. 3. A successful configuration of signs of the nonzero horizontal (or vertical) couplings is so, that clockwise couplings are negative and other couplings are positive (see Fig. 3). So

$$H_{\text{top}}(t) = \frac{1}{2} \zeta_h W_h^2 [-s_1(t) - s_2(t+2) + s_2(t) + s_1(t+2)] s_5(t+1). \quad (24)$$

Analogously holds:

$$\begin{aligned} H_{\text{right}}(t) &= \frac{1}{2} \zeta_v W_v^2 [-s_2(t) - s_3(t+2) + s_3(t) + s_2(t+2)] s_6(t+1), \\ H_{\text{bottom}}(t) &= \frac{1}{2} \zeta_h W_h^2 [-s_3(t) - s_4(t+2) + s_4(t) + s_3(t+2)] s_7(t+1), \\ H_{\text{left}}(t) &= \frac{1}{2} \zeta_v W_v^2 [-s_4(t) - s_1(t+2) + s_1(t) + s_4(t+2)] s_8(t+1). \end{aligned} \quad (25)$$

Together with eq. (23) one gets the corresponding partition functions

$$Z_{\text{top}}^1 = \sum_{s_i(t+1)=\pm 1} \exp[2\zeta_h W_h^2 s_5(t+1)/T] = 2 \cosh[2\zeta_h W_h^2/T] = Z_{\text{top}}^2 \quad (26)$$

Thereby, the configurations of the peripheral neurons are determined through eq. (23) and the partition functions Z^μ are products $Z^\mu = Z_{\text{top}}^\mu Z_{\text{right}}^\mu Z_{\text{bottom}}^\mu Z_{\text{left}}^\mu$; Z_{top}^μ is explicited above. Analogously one calculates the other partition functions, so one gets the partition function

$$Z^\mu = 16 [\cosh(2\zeta_h W_h^2/T) \cosh(2\zeta_v W_v^2/T)]^2. \quad (27)$$

With polar coordinates $W_h = r \cos \varphi$, $W_v = r \sin \varphi$ one gets

$$V(r, \varphi) = 16b r^4 - 2aT \left[\ln \cosh\left(\frac{2}{T} r^2 \zeta_h \cos^2 \varphi\right) + \ln \cosh\left(\frac{2}{T} r^2 \zeta_v \sin^2 \varphi\right) \right]. \quad (28)$$

A term $aT \ln(16)$ is irrelevant and has been omitted. The first term establishes a radial potential $V_1(r)$, while the last term is an angular dependent potential $V_2(r, \varphi)$ that determines the vertical versus horizontal couplings. For sufficiently small fluctuations there occurs a nonzero radius r . The rescaled angular potential

$$U(r, \varphi) := \frac{1}{2aT} V_2(r, \varphi) = -\ln \cosh\left(\frac{2}{T} r^2 \zeta_h \cos^2 \varphi\right) - \ln \cosh\left(\frac{2}{T} r^2 \zeta_v \sin^2 \varphi\right) \quad (29)$$

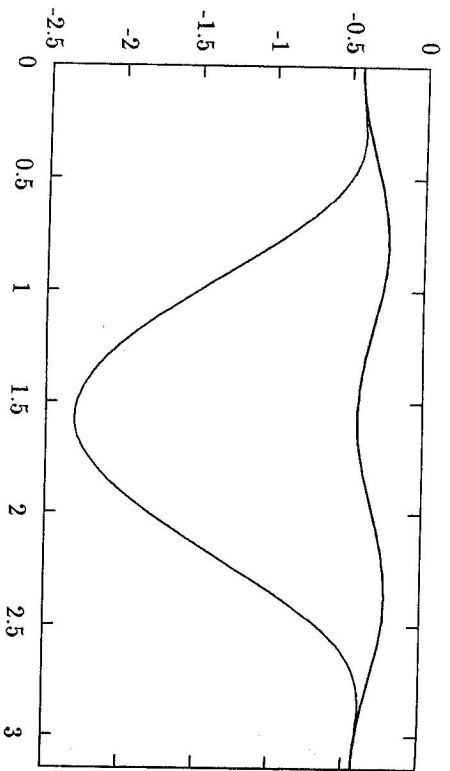


Fig. 6. **Disambiguation.** Rescaled potential U as a function of the angular variable φ . U is π -periodic. Upper curve: one of two equally stable coupling states occurs for $C_v = C_h = 1$ at one of the minima at 0 and $\pi/2$, due to a spontaneous (via random fluctuations) breaking of symmetry. Coupling states correspond to percepts, so no mixed percepts emerge. Lower curve: local minima are not equally stable for $C_h = 1$ and $C_v = 3$.

is shown in Fig. 4, for $\frac{2}{\pi}r^2 = 1$ and two pairs of values of rectangular parameters C_h and C_v .

(1) For $C_h = C_v = 1$, the minima are at $\varphi = 0$ (i.e., $W_v = 0$) and at $\varphi = \frac{\pi}{2}$ (i.e., $W_h = 0$), see Fig. 6, upper curve. If $\varphi = 0$, then the vertical couplings are 0 . This is interpreted as the horizontal motion percept. If $\varphi = \frac{\pi}{2}$, then the horizontal couplings are 0 . This is interpreted as the vertical motion percept. Both percepts are equally stable. This finding is in agreement with experimental data [6], namely, the subjects perceive either horizontal or vertical motion. The symmetry of the two motion percepts is broken spontaneously, i.e., as a consequence of a stochastic dynamics with two possible steady states. The neurons with nonzero couplings establish *cell assemblies*, see introduction.

(2) For $C_h = 1$ and $C_v = 3$, the minima are still at $\varphi = 0$ and $\varphi = \frac{\pi}{2}$, see Fig. 6, lower curve. However, the minimum at $\varphi = \frac{\pi}{2}$ is more pronounced. I.e., the state at $\varphi = \frac{\pi}{2}$ is more stable and has a larger basin of attraction. I.e., the vertical motion percept is preferred. This is in qualitative agreement with experiments [6] and corresponds to a 'Gestalt' law.

(1 and 2) The values W_h and W_v characterize local minima of the potential V in coupling space and can be regarded as *order parameters* [4] corresponding to percepts. The ambiguous stimulation is *disambiguated*, because no mixed order parameters occur. Spontaneous switching between percepts and *hysteresis* at rectangular length variations occur for systems with such potentials [4].

Next, the EEG - measurements are modeled. A *test person* observes the alternating patterns (see Fig. 1). Thereby it perceives at any instant of time either the horizontal or the vertical motion percept; additionally, the percept changes spontaneously at some instances of time, i.e., without extra stimulation. When the percept switches, then the test person pushes a key. Simultaneously the *EEG of the person is measured*, averaged and drawn in such a manner, that the time is zero when the test person pushes the key, i.e., immediately after a new percept emerged (see Fig. 2). In a control group, test persons push the key voluntarily (i.e., without percept); as a result there is no EEG - signal (in contrast to Fig. 2); this indicates that the EEG - signal (see Fig. 2) is due to the percept. The EEG - signal is modeled as follows. The spontaneous breaking of symmetry is used; accordingly eq. (29) is used with $\sin \varphi = 0$, for the special case $C_h = 1$ and it is expressed with W_h (for simplicity)

$$V(W_h) = 16bW_h^4 - 2aT \ln \cosh\left(\frac{2}{T}W_h^2\right), \quad (30)$$

so

$$\Delta W_h = -64bW_h^3 - 8aW_h \tanh\left(\frac{2}{T}W_h^2\right). \quad (31)$$

Here the equation of motion for W_h is obtained from the potential theorem by taking the derivative. The parameters a , b and T are chosen so that a qualitative agreement is provided as shown in Fig. 2; a systematic fit appears inadequate because the observed data are insufficient. The following values have been used: $a = 1s^{-1}$, $b = 9/32 s^{-1}$, $T = 0.0088$. The modeled voltage U_m is proportional to the formal local field (see eq. (1)), i.e., to W_h^2 ; additionally, a subtrahend is modeled, it is interpreted as the decay of W_h due to another (unexplicated) percept that emerges 0.4 s later (initiated by a higher level of processing); so the modeled voltage is $U_m(t) = 10\mu V[W_h^2(t) - W_h^2(t - 0.4 s)]$. (This unexplicated percept might be initialized at another level of processing, see introduction.) Altogether, the above *four 'universal properties of perception'* have been modeled: 'disambiguation', hysteresis, P300 - signals and 'Gestalt' rules. Next the present network is generalized.

3.2. Analysis of the generalized basic model

Analogously to the above particular model, the possible percepts are characterized by a potential V . For short, 'peripheral neurons' are abbreviated by 'p.n.' and 'inner neurons' by 'i.n.'. *Generalized potential theorem:* In the *adiabatic limit* the averaged synaptic change is determined by a potential function as follows.

$$\langle \Delta W_{ij}(t) \rangle = -\frac{\partial V}{\partial W_{ij}} \quad \text{with } V = \frac{b}{4} \left(\sum_{ij} W_{ij}^2 \right)^2 - \frac{dT}{p} \sum_{\mu=1}^p \ln Z^\mu \quad (32)$$

with

$$Z^\mu = \sum_{s_i(t), s_i(t+2)}^{p.n.} \tau^\mu \sum_{s_i(t+1)=\pm 1}^{i.n.} \exp[-H(t)/T] \quad (33)$$

and

$$H(t) = - \sum_i^{\text{i.n.}} h_i(t+1) - \sum_i^{\text{p.n.}} h_i(t+2) s_i(t+2). \tag{34}$$

The locally stable (steady) states exhibit $\langle \Delta W_{ij}(t) \rangle = 0$ and are the local minima of the potential V . The proof is analogous to that of the potential theorem, see above. Next, it is shown that disambiguation is produced by the network model via the phenomenon of spontaneous breaking of symmetry. For that purpose, one may consider two locally stable (steady) coupling states P_1 with couplings $W_{ij}(1)$ and P_2 with couplings $W_{ij}(2)$. The question is, whether a superposition $W_{ij} = \alpha W_{ij}(1) + \beta W_{ij}(2)$ can be locally stable; any binarily mixed percept would be such a locally stable superposition. *Disambiguation theorem: Superpositions of P_1 and P_2 that do not vary with T do not occur.* A proof is presented in the appendix. A possible superposition of P_1 and P_2 can be destabilized by T -variations, in contrast to P_1 and P_2 . Mixed percepts are unstable in this sense. Next, P300 - signals are modeled. For this purpose, the form of the potential V is studied. *Potential calculation lemma: The potential V takes the following form.*

$$V = \frac{b}{4} \left(\sum_{ij} W_{ij}^2 \right)^2 - \frac{\alpha T}{p} \sum_{\mu=1}^p \sum_i^{\text{i.n.}} \ln \cosh \left[\frac{1}{2T} \sum_j^{\text{p.n.}} W_{ji}^2 \zeta_j s_j^{\mu}(t+2) + \frac{1}{2T} \sum_k^{\text{p.n.}} W_{ik}^2 \zeta_k s_k^{\mu}(t) \right]. \tag{35}$$

A proof is given in [13]. Next, the time course of the formation of couplings is studied. For that purpose, a radius is introduced as follows $r^2 = \sum_{ij} W_{ij}^2$ and $W_{ij} = r w_{ij}$. Using this radius, the potential (see eq. (35)) can be rewritten as follows

$$V = \frac{b}{4} r^4 - \frac{\alpha T}{p} \sum_{\mu} \sum_i^{\text{i.n.}} \ln \cosh \left[\frac{r^2}{2T} H_i^{\mu} \right],$$

with the formal local angular energy function

$$H_i^{\mu} = \sum_j^{\text{p.n.}} w_{ji} \zeta_j s_j^{\mu}(t+2) + \sum_k^{\text{p.n.}} w_{ik} \zeta_k s_k^{\mu}(t). \tag{36}$$

Lemma about equation of motion: If one changes differences into differentials, then one obtains the following differential equation for the radius $\frac{dr}{dt} = -\frac{\partial V}{\partial r}$. A proof is given in [13]. Next one may insert the potential (see eq.(36)) into the above eq. to obtain

$$\frac{dr}{dt} = -br^3 + \alpha r \frac{1}{p} \sum_{\mu} \sum_i^{\text{i.n.}} H_i^{\mu} \tanh \left(\frac{r^2}{2T} H_i^{\mu} \right). \tag{37}$$

In the formal local energy function (see eq. (36)) there are only normalized quantities; so the average $\langle H_i^{\mu} \rangle_{i\mu} = \frac{1}{p} \sum_{\mu} \sum_i^{\text{i.n.}} H_i^{\mu}$ is in the interval $[-1,1]$ and does not vary very

much for different percepts'. Accordingly one may approximate H_i^{μ} in the hyperbolic tangens (see eq.(37)) by its averaged value $\langle H_i^{\mu} \rangle_{i\mu}$, so as to get

$$\frac{dr}{dt} = -br^3 + \alpha r \langle H_i^{\mu} \rangle_{i\mu} \tanh \left(\frac{r^2}{2T} \langle H_i^{\mu} \rangle_{i\mu} \right). \tag{38}$$

This approximation is known as a mean field type approximation. Eq.(38) can be mapped to eq. (31) as follows. The radius r corresponds to W_n ; the average $\langle H_i^{\mu} \rangle_{i\mu}$ corresponds to $4.64 b$ in eq. (31) corresponds to b in this eq.; $2 a$ in eq. (31) corresponds to a in this eq. So the curve in Fig. 2 is expected for any formation of couplings, that is, the present network model yields an explanation for the formation of percepts accompanied by P300 - EEG - signals. *Altogether*, the generalized model instantiates the four 'universal properties of perception' as follows: Disambiguation occurs via destabilization of mixed states; hysteresis occurs due to local minima of a potential; a relatively uniform dynamics corresponds to P300 - signals; the binding of stimuli according to prestabilized couplings corresponds to 'Gestalt' rules.

3.3. Analysis of the generalized simplified model

Analogously as for the basic model, the averaged changes of couplings and the emerging networks can be characterized by a potential as follows. *Generalized potential theorem for the simplified network: In the adiabatic limit*

$$\langle \Delta W_{ij}(t) \rangle = -\frac{\partial V}{\partial W_{ij}} \text{ with } V = \frac{b}{4} \sum_k^{\text{p.n.}} \left(\sum_l^{\text{i.n.}} [W_{lk}^2 + W_{kl}^2] \right)^2 - \frac{\alpha T}{p} \sum_{\mu=1}^p \ln Z^{\mu} \tag{39}$$

where

$$Z^{\mu} = \sum_{s_i(t), s_i(t+2)}^{\text{p.n.}} \sum_{s_i(t+1)}^{\text{i.n.}} \exp[-H(t)/T], \tag{40}$$

$$H(t) = - \sum_i^{\text{i.n.}} h_i(t+1) s_i(t+1) - \sum_i^{\text{p.n.}} h_i(t+2) s_i(t+2). \tag{41}$$

The locally stable (steady) states exhibit $\langle \Delta W_{ij}(t) \rangle = 0$ and are the local minima of the potential V . The proof is analogous to that of the first potential theorem; the above eqs. and eqs. (32) - (34) differ only in two details: The partition functions Z^{μ} do not contain an indicator r^{μ} here, because no comparison is processed. The decay term proportional b is of a local nature here.

The binding of stimuli in the simplified model is not ambiguous in the following sense: *Disambiguation theorem for the simplified model: For a locally stable network that does not vary with T , the following holds. Each peripheral neuron sends a signal via exactly one inner neuron to one peripheral neuron and receives a signal via exactly one inner neuron from one peripheral neuron.* A proof is presented in the appendix.

The selection of an appropriate delay time for the time discretization (see section 2) can be achieved with the same mechanism of disambiguation.

Next, one may model the EEG. For this purpose one may introduce for each peripheral neuron s_i two multidimensional polar coordinate systems, one for the couplings that direct to s_i with radius r_i (see eq. (47)), one for the couplings that direct from s_i with radius q_i (see eq. (48)). Each such coordinate system establishes a local orthogonal coordinate system. Because the set of couplings is expressed with the set of these coordinate systems, this set of coordinate systems establishes a local orthogonal coordinate system for all couplings. As a consequence, the change of a radius vector r_i or q_i can be expressed as the corresponding partial derivative of the potential with respect to that radius. Moreover, if one starts at zero couplings (for simplicity), then exactly one coupling grows in each of these local orthogonal coordinate systems (according to the disambiguation theorem for the simplified model); the growth of this coupling takes place in the same manner as described by eq. (31). As a result, the same modeled brain potentials U_m occur, see Fig. 2.

4. Discussion

It has been shown, that the proposed network models can be used to bind a sequence of stimuli patterns to motion percepts. This includes the case, in which the sequence contains only one pattern, this case corresponds to a usual percept. This binding by networks exhibits *four properties*: disambiguation, hysteresis, a stereotype time behaviour and the binding according to rules that can be encoded in terms of prestabilized coupling factors ζ_{ij} .

The generalized basic network model should be compared with the generalized simplified network model. By definition, these two models differ in two aspects: First, the basic model contains an indicator τ^u , by which a correspondence with the stimuli is achieved; whereas the simplified network model does not achieve a correspondence with the stimulation, in contrast, that correspondence is produced in advance and the peripheral neurons are simply bound via inner neurons. As a consequence, only the basic network model *can fail to produce a correspondence*. Second, the basic model contains a global decay term proportional to b , while the simplified model has a decay term proportional to a sum of the couplings that direct to a peripheral neuron and another sum of the couplings that direct from a peripheral neuron. So the dynamics of the simplified model is *completely local*.

The four 'universal properties' of the binding by the networks can be compared with four corresponding properties of the binding of sensory stimuli through the brain: disambiguation, hysteresis, P300 EEG signals and 'Gestalt' rules. For this purpose, one may characterize the processing time of such binding by the P300 EEG signals. As a result, one may exclude similar processes performed by the brain at different times; for instance, before the brain generates a percept, it processes the related but different data analysis of 'motion detection' during the first 100 milliseconds after the external stimulation [16]. If one applies the networks as models for the EEG signals, then a neuron in the network corresponds to at least 10000 nerve cells of the brain, because signals measured as an EEG origin from at least 10000 nerve cells [1]. 'Gestalt' rules

can be encoded by the prestabilized coupling factors; for instance, one 'Gestalt' rule states that narrow stimuli are preferentially bound, this is achieved by the prestabilized factors ζ_h and ζ_o .

In order to model the fast synapses, one may explicate subnetworks that consist of physiologically observed components only and that exhibit fast couplings effectively. Analogously, in order to model the used time discretization, one may model couplings with various delay times, as a result, a delay time is selected through the coupling dynamics.

Altogether, the correspondence between the network models and the nervous system is explicated for certain measurable phenomena. It can be performed in a rough quantitative manner. In particular, the spontaneity of percept formation is reflected in the network dynamics: the solutions of the equations of motion for the averaged couplings that establish a percept (see eqs. (31) and (37)) exhibit the initial state at infinite negative time, consequently the coupling formation is initialized by a random fluctuation, i.e., spontaneously. According to this overall qualitative and rough quantitative correspondence between the proposed network models and the observed phenomena of the nervous system, one may regard the present network models as promising 'Ansätze' for more precise future theories of percept formation by the brain. The present networks do already model four 'universal properties' of percept formation.

5. Conclusion

The brain binds stimuli received by sensor neurons to certain collective states of inner neurons; an important class of such collective states are the percepts. Percepts exhibit four 'universal phenomena': disambiguation of ambiguous stimulus configurations, hysteresis, P300 EEG brain potentials and binding according to so called 'Gestalt' rules.

Though Hebb formulated already more than fifty years ago the hypothesis that these collective states are due to cell assemblies [2], a theoretical model for these cell assemblies has hardly been developed so far. In the present study, neurostatistical network models have been proposed that operate along the lines of the cell assembly idea. In particular, these networks contain prestabilized coupling factors in order to encode 'Gestalt' rules, fast couplings in order to bind stimuli according to the 'Gestalt' rules and neurons in order to represent the stimuli.

The analysis of these models is performed in terms of vector fields that are gradients of a scalar potential V similar to a free energy. The potential V has the form of a system with quenched disorder, because it contains an average of partition functions. The analysis yields collective states as local minima of the potential V , these collective states can be described by order parameters [4]. The networks exhibit four properties that correspond to the 'four universal' properties of perception. In particular, the P300 EEG brain potentials have been modeled quantitatively by adjusting the three model parameters: learning rate a , decay rate b and formal temperature T .

The network models can be divided into two classes: those with a simple Hebb rule and a completely local dynamics and those with a Hebb rule combined with a comparison mechanism and a global decay term. Both models exhibit the four properties, this

indicates that the models are robust. The network with the comparison mechanism can make errors at the stimulus level, this is possibly in agreement with the nervous system. Altogether, four 'universal properties' of perception have been modeled with relatively simple and robust neurostatistical networks along the lines of the cell assembly idea and the Hebb rule.

Appendix

First, the potential theorem is proven. The average is taken over the 2^5 neuronal configurations, 2^4 for inner neurons and two for peripheral neurons. So the averaged change of couplings (see eq. (2)) is

$$\langle \Delta W_{ij} \rangle = \frac{1}{2} \sum_{\mu} \sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \frac{\tau^\mu}{c^\mu} \times \prod_{i=5}^8 \frac{\exp[h_i^\mu(t+1)s_i(t+1)/T]}{\exp[h_i^\mu(t+1)s_i(t+1)/T] + \exp[-h_i^\mu(t+1)s_i(t+1)/T]} \times \prod_{i=1}^4 \frac{\exp[h_i^\mu(t+2)s_i(t+2)/T]}{\exp[h_i^\mu(t+2)s_i(t+2)/T] + \exp[-h_i^\mu(t+2)s_i(t+2)/T]} \times [aW_{ij}\zeta_{ij}f - bW_{ij} \sum_{kl} W_{kl}^2]. \quad (42)$$

The probability function for the neuronal states has been determined according to eq. (1). It can be simplified in terms of the above energy function (take the product in the denominator and the numerator, multiply sums according to the distributive law and multiply exponentials). So one gets

$$\langle \Delta W_{ij} \rangle = \frac{1}{2} \sum_{\mu} \sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \frac{\tau^\mu}{c^\mu} \frac{\exp[-H(t)/T]}{\sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \exp[-H(t)/T]} \times [aW_{ij}\zeta_{ij}f - bW_{ij} \sum_{kl} W_{kl}^2]. \quad (43)$$

In the adiabatic limit, c^μ takes its average value $\frac{\sum_{i=5,6,7,8} \tau^\mu \exp[-H(t)/T]}{\sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \exp[-H(t)/T]}$. By inserting it one gets

$$\langle \Delta W_{ij}(t+1) \rangle = \frac{1}{2} \sum_{\mu} \sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \frac{\tau^\mu \exp[-H(t)/T]}{\sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \tau^\mu \exp[-H(t)/T]} \times [aW_{ij}\zeta_{ij}f - bW_{ij} \sum_{kl} W_{kl}^2]. \quad (44)$$

In the adiabatic limit, the couplings are constant on the time scale of neuronal change; so the average can be performed for the above subtrahend

$$\langle \Delta W_{ij} \rangle = -bW_{ij} \sum_{kl} W_{kl}^2 + \frac{a}{2} \sum_{\mu} \frac{\sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \tau^\mu \exp[-H(t)/T][aW_{ij}\zeta_{ij}f]}{\sum_{s_i, (t+1) = \pm 1}^{i=5,6,7,8} \tau^\mu \exp[-H(t)/T]}. \quad (45)$$

By taking the derivative $\frac{\partial V}{\partial W_{ij}}$ in eq. (19) one gets the same eq. as the above, q.e.d.

Next the disambiguation theorem is proven. That is, it is shown that a superposition of P_1 and P_2 is not locally stable or varies with T . Most generally, a mixed state consists of nonzero couplings $W_{ij}(A)$ that are nonzero only in P_1 , of nonzero couplings $W_{ij}(B)$ that are nonzero only in P_2 and of nonzero couplings $W_{ij}(C)$ that are nonzero in P_1 and P_2 . The couplings are expressed in terms of angles as follows. A polar angle ϑ is chosen so that $\cos \vartheta$ is proportional to α and $\sin \vartheta$ is proportional to β . Another angle φ is chosen so that

$$\alpha W_{ij}(1) + \beta W_{ij}(2) = W_{ij} = \cos \vartheta W_{ij}(A) + \sin \vartheta \cos \varphi W_{ij}(B) + \sin \vartheta \sin \varphi W_{ij}(C). \quad (46)$$

At a locally stable state one obtains $\frac{\partial V}{\partial W_{ij}} = 0$. By the chain rule this implies $\frac{\partial V}{\partial \vartheta} \frac{\partial \vartheta}{\partial W_{ij}} = 0$. The second factor is nonzero, because $\frac{\partial W_{ij}}{\partial \vartheta}$ cannot diverge; consequently, the first factor is zero, i.e., $\frac{\partial V}{\partial \vartheta} = 0$. Because the sets of couplings $W_{ij}(A)$, $W_{ij}(B)$ and $W_{ij}(C)$ are disjoint, one obtains $W_{ij}^2 = W_{ij}^2(A) \cos^2 \vartheta + W_{ij}^2(B) \sin^2 \vartheta \cos^2 \varphi + W_{ij}^2(C) \sin^2 \vartheta \sin^2 \varphi$. Because the couplings occur in terms of squares in the potential V , it is adequate to study the derivative with the chain rule as follows $\frac{\partial V}{\partial \vartheta} = \sum_{ij} \frac{\partial V}{\partial W_{ij}^2} \frac{\partial W_{ij}^2}{\partial \vartheta}$. Because $\frac{\partial W_{ij}^2}{\partial \vartheta}$ is proportional to $\sin \vartheta \cos \vartheta$, so is $\frac{\partial V}{\partial \vartheta}$, i.e., $\frac{\partial V}{\partial \vartheta} = \sin \vartheta \cos \vartheta \text{Rest}(T) = 0$; thereby $\text{Rest}(T)$ is a term that depends on the temperature. The above product becomes zero, if $\text{Rest}(T)$ is zero or if $\sin \vartheta \cos \vartheta$ is zero. Only the second case yields coupling states that do not vary with the temperature, so only the second case is considered, thus $\sin \vartheta \cos \vartheta = 0$. So a locally stable state that does not vary with temperature is not a superposition of P_1 and P_2 , q.e.d.

Next the disambiguation theorem for the simplified model is proven. For this purpose, it is convenient to express the couplings W_{ik} that direct to a peripheral neuron s_i in terms of multidimensional polar coordinates [17]

$$\begin{aligned} W_{ik} &= r_i \cos \vartheta_{ik}, \\ W_{il} &= r_i \sin \vartheta_{ik} \cos \vartheta_{il}, \\ W_{im} &= r_i \sin \vartheta_{ik} \sin \vartheta_{il} \cos \vartheta_{im}, \dots \\ W_{iz} &= r_i \sin \vartheta_{ik} \sin \vartheta_{il} \sin \vartheta_{im} \dots \sin \vartheta_{iz}. \end{aligned} \quad (47)$$

At a local minimum of the potential, the partial derivative with respect to an angle ϑ_{in} is zero. Such a derivative takes the form $\frac{\partial V}{\partial \vartheta_{in}} = \cos \vartheta_{in} \sin \vartheta_{in} \text{rest}(T)$, whereby $\text{rest}(T)$ expresses a T -dependent factor. In a locally stable network that is not T -dependent, the condition $\cos \vartheta_{in} \sin \vartheta_{in} \text{rest}(T) = 0$ is fulfilled independently of T , so $\cos \vartheta_{in} \sin \vartheta_{in} = 0$, consequently $\vartheta_{in} = 0$ or $\vartheta_{in} = \pi/2$. Because the multidimensional

polar coordinates establish a locally orthogonal coordinate system, exactly one polar angle ϑ_{in} is nonzero, thus the peripheral neuron receives a signal from exactly one inner neuron. One may introduce multidimensional polar coordinates for the couplings that origin from a peripheral neuron s_i

$$W_{ki} = q_i \cos \vartheta_{ki};$$

$$W_{li} = q_i \sin \vartheta_{li} \cos \vartheta_{li};$$

$$W_{mi} = q_i \sin \vartheta_{mi} \sin \vartheta_{li} \cos \vartheta_{mi}; \dots$$

$$W_{zi} = q_i \sin \vartheta_{zi} \sin \vartheta_{li} \sin \vartheta_{mi} \dots \sin \vartheta_{zi};$$

(48)

With it one may show analogously, that a peripheral neuron sends a signal to exactly one inner neuron. q.e.d.

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