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CHARGE DENSITY OF VIRTUAL QUARKS IN EXTERNAL STATIC QUARK FIELDS

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We evaluate the correlation function $C(r) = \langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle$ within perturbative QCD at the leading order in QCD running coupling constant expansion. The results are in quantitative agreement with the data obtained within lattice QCD by the Vienna group.

Recently [1,2] the charge density of dynamical quark-antiquark pairs around a static quark represented by the Polyakov loop $L(0)$ [3] was investigated within lattice QCD. In Ref.1 the correlation function

$$C(r) = \langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle \tag{1}$$

was computed by means of path-integral in Euclidean space-time. The results are depicted in Fig.1. One observes a net charge excess of opposite sign around the source which approaches zero with increasing distance from the source.

In this paper we are trying to confirm analytically the results of Faber et al. [1] by using the continuum space-time formulation of QCD instead of the lattice regularization. The possibilities of such evaluations are restricted by the use of perturbation theory.

We compute the quantity $\langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle$ at the leading order in the coupling constant expansion of perturbative QCD. The Polyakov loop can be written in the form

$$L(0) = \frac{1}{N}\mathrm{tr}_C\left[\mathrm{T}\ \exp\left\{i\int_{-\infty}^{\infty} dt t^a g_S G_0^a(0,t)\right\}\right]. \tag{2}$$

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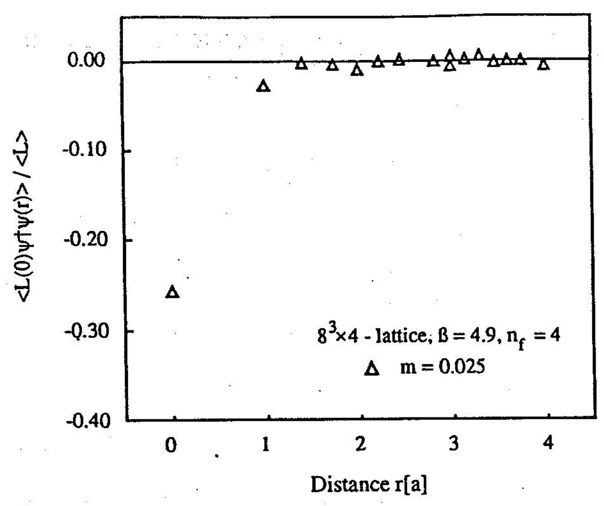

Fig. 1.Lattice QCD data for four flavors of dynamical fermions with mass $ma = 0.025$ in lattice units [1,2] for the virtual charge density around a static quark $C(r)$. This investigation indicates that there is a net excess of virtual antiquarks in the vicinity of the source which goes to zero with increasing distance approaching the undisturbed vacuum.

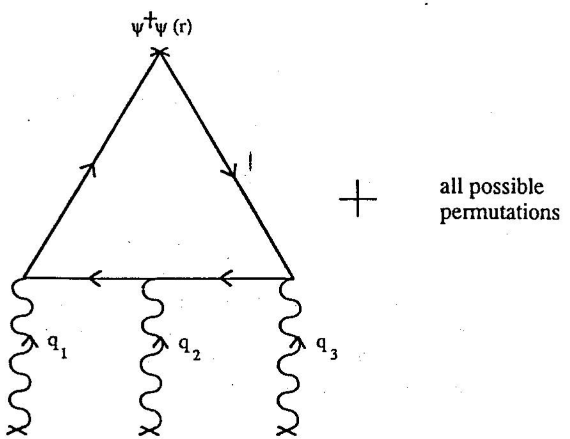

Fig. 2.Diagrams describing $\langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle$ at leading order $O(g_S^6)$ in the QCD coupling constant expansion.

Here N is the number of quark color degrees of freedom (in our case $N = 3$), $G_0^a(0,t)$ denotes the time component of the gluon field ($a = 1,\ldots,N^2-1$), g_S is the QCD coupling constant and t^a are traceless color matrices normalized by $\mathrm{tr}_C(t^a t^b) = \delta^{ab}/2$. Since the Gell-Mann matrices t^a are traceless and the charge density operator $\psi^\dagger(\vec{r},0)\psi(\vec{r},0)$ is the time component of a 4-vector $\bar{\psi}(\vec{r},0)\gamma^\mu\psi(\vec{r},0)$

the lowest non-vanishing contribution to the correlation function $C(r)$ in (1) is of order of $O(g_S^6)$. The corresponding diagrams are shown in Fig. 2. The analytic expression for these graphs takes the form [4,5]

$$\langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle_{g_S^6} = -\frac{1}{2^{10}\pi^8}\left(1-\frac{4}{N^2}\right)\int\frac{d^3q_1}{\vec{q}_1^{\,2}}\alpha_S(-\vec{q}_1^{\,2})e^{-i\vec{q}_1\vec{r}}\times$$
$$\times\int\frac{d^3q_2}{\vec{q}_2^{\,2}}\alpha_S(-\vec{q}_2^{\,2})e^{-i\vec{q}_2\vec{r}}\int\frac{d^3q_3}{\vec{q}_3^{\,2}}\alpha_S(-\vec{q}_3^{\,2})e^{-i\vec{q}_3\vec{r}}\,J_{0000}(\vec{q}_1,\vec{q}_2,\vec{q}_3), \qquad (3)$$

where $J_{0000}(\vec{q}_1,\vec{q}_2,\vec{q}_3)$ denotes the time-component of the gauge invariant tensor $J_{\mu_1\mu_2\mu_3\mu_4}$ which determines the amplitude of the low-energy $\gamma\gamma$ scattering [5]. We calculate this quantity in the limit of Ref. 6 which means that we restrict the region of space where the resulting formulae can be applied from below, $r \geq 2a \simeq 2/3$ fm, where $a \simeq 1/3$ fm is the approximate value for the lattice spacing in the corresponding numerical investigation [2]. Therefore we obtain

$$J_{0000}(\vec{q}_1,\vec{q}_2,\vec{q}_3) = -\frac{2\pi^2}{15m^2}(\vec{q}_1\vec{q}_2)(\vec{q}_3^{\,2}+\vec{q}_3\vec{q}_1+\vec{q}_3\vec{q}_2), \qquad (4)$$

with m the dynamical quark mass of the order $O(300\,\mathrm{MeV})$. In (3) we used the expression for the running coupling constant of QCD with four flavors

$$\alpha_s(-\vec{q}^{\,2}) = \frac{12\pi}{25}\frac{1}{\ln(\vec{q}^{\,2}/\Lambda^2)}. \qquad (5)$$

The quantity Λ denotes the QCD scale parameter in the lattice regularization which is connected to the continuum scale $\Lambda_{QCD} \simeq 0.2\,\mathrm{GeV}$ by the relation [8]

$$\Lambda \equiv \Lambda_L \simeq \frac{1}{83}\Lambda_{QCD} \simeq 0.01\,\mathrm{fm}^{-1}. \qquad (6)$$

With the relation (4) the integration over $\vec{q}_i$ ($i = 1,2,3$) in (3) leads to the result

$$\langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle_{g_S^6} = \left(1-\frac{4}{N^2}\right)\frac{9\pi}{312500}\frac{1}{m^4}\times$$
$$\times(\nabla_i\Phi)(\nabla_j\Phi)(\delta_{ij}\Delta\Phi+\nabla_i\nabla_j\Phi), \qquad (7)$$

with

$$\Phi(\vec{r}) = -\frac{1}{2\pi^2}\int\frac{d^3q}{\vec{q}^{\,2}}\frac{1}{\ln(|\vec{q}\,|/\Lambda)}e^{-i\vec{q}\cdot\vec{r}} = \frac{1}{r}\frac{1}{\ln(\Lambda r)}. \qquad (8)$$

In the following step we calculate the derivatives over $\vec{r}$ in (7) and insert $N = 3$. Thus we obtain the following expression for the correlation function between the Polyakov loop which describes the static quark and the virtual charge density

$$C(r) = \langle L(0)\psi^\dagger(\vec{r},0)\psi(\vec{r},0)\rangle = -\frac{\pi}{62500}\frac{1}{m^4}\frac{1}{r^7}\frac{1}{\ln\frac{1}{\Lambda r}}\times$$
$$\times\left[\frac{1}{\ln(\Lambda r)}+\frac{1}{\ln^2(\Lambda r)}\right]^2\left[4+\frac{7}{\ln(\Lambda r)}+\frac{6}{\ln^2(\Lambda r)}\right]. \qquad (9)$$

The region of validity of our final result (9) is restricted by $r \geq 2a \simeq 2/3$ fm (see Refs. 1,2 and 6). In this region the correlation function (9) is in quantitative agreement with the numerical results obtained within lattice QCD and displayed in Fig.1. The negative sign of $C(r)$ shows that virtual antiquarks screen static quarks in analogy to QED.

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