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INFLATION DRIVEN BY HIGHER CURVATURE TERMS AND ENERGY AND CURVATURE DEPENDANT BULK VISCOSITY

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We study inflationary cosmologies when higher curvature terms are present in the lagrangian of gravitation and the matter content admits both curvature and energy dependent bulk viscosity. Our analysis demonstrates that two epochs of inflation or "double inflation" can result when higher curvature dependent bulk viscosity for matter are present.

I. Introduction

Inflationary cosmology has replaced the standard big bang cosmology [1] and theories admitting the continuous creation of matter [2] because it resolves the flatness, horizon and magnetic monopole problems of conventional cosmology. It was Coleman's [3] penetrating insight in observing that the GUT (Weinberg- Coleman) potential which acquired a long flat structure due to radiative corrections, could serve as an effective cosmological constant to drive exponential expansion in the false vacuum. Guth [4] inspired by Coleman's observation studied the evolution of the scalar field (Higgs field) and scale factor to find that the universe could evolve so rapidly from a minuscule size to a size wherein the transition to the true vacuum occurred in such a manner so that questions of matter creation would not be subject to restrictions of initial causal contact between different parts of the universe that in the conventional big bang cosmology was necessary to produce the homogeneous universe now observed. Actually, before Guth's (ref. [4]) original paper, Starobinsky [5] had shown that inflation resulted from the Einstein theory with matter represented by one-loop contributions of conformally covarient matter fields of arbitrary spin. Starobinsky's result is notable for it demonstrates that inflation might be a generic property enjoyed by the Einstein theory and the single assumption of conformal invariance of matter without any detailed specific modelling and initial conditions. Since the Einstein theory has been shown by Adler [6], Sakharov [7], Zee [8] to be induced by path integrating out heavy fields in the action of matter without any assumptions except general covariance we might speculate that inflation might be a generic consequence of conformal invariance in the early universe. After Guth's original proposal of inflation

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which failed to produce a homogeneous universe ("graceful exit problem") because the true vacuum bubbles were produced to few number forbidding collisions of bubbles and thus leading to an inhomogeneous universe, Linde [9] proposed the "new inflationary scenario" which throught fine tuning allowed for sufficient inflation to resolve the horizon, flatness and monopole problems of standard cosmology and in addition produced the presence of matter in a homogeneous and isotropic fashion. In the early eighties following the fundamental work of Guth [4], Starobinsky [5], Linde [9] and Albrecht and Steinhardt [10], questions were asked concerning of whether or not different parts not causally connected would develop different inflationary scenarios.

In Linde's historic paper "on chaotic inflation" [11] he demonstrated that if the potential in the symmetric phase was sufficiently flat and $\partial^\mu \Phi \partial_\mu \Phi < M_p{}^4$ (the energy density due to the scalar field gradient is less than the Planck energy density) then for chaotic conditions on the scalar field, the universe breaks up into a collection of mini-universes with each one being homogeneous and isotropic and each one will inflate to the size of our present universe after sufficient time and thus our present universe may be realizes as the final state of a initial homogeneous and isotropic mini-universe. Linde's "chaotic inflation" suggest that our universe can be thought of as a consequence of an initial random or chaotic distribution of the scalar field driving inflation. Because of the unnatural fine tuning in both new inflation and chaotic inflation recently a new inflationary scenario named "extended inflation" [12] has been discussed which makes use of a Brans Dicke type scalar to serve as a catalyst to give sufficient inflation at the outset to solve the horizon and flatness problems and a slowing down of the scale factor evolution for late times allowing the true vacuum to percolate. Without any recourse to a potential, Verwimp and Callebaut [13] have suggested bulk viscosity driven inflation and Swen [14] has discussed higher-curvature driven inflation... Bulk viscosity can phenomenologically represent the conversion of massive string modes to massless modes as well as a general form of dissipation associated with particle creation [16]. In fact, Barrow [16] has pointed out that particle production due to the non-adiabatic decaying of the field driving slow-rollover inflation can be in a macroscopic sense described by the viscous cosmological model of Murphy [17] with a bulk viscosity coefficient depending on the energy density ($\xi = C\varepsilon$). To model string loop production Turok [18] has proposed a bulk viscosity coefficient proportional to $\varepsilon^{3/2}$. Generalizing Turok's work, Barrow [19] has discussed the general case $\xi = C\varepsilon^n$ (ξ= bulk viscosity coefficient, ε = energy density). with the general result that for $n > 1/2$ the solutions of a Friedman Walker flat universe start out in the de-Sitter phase and evolve into a power law expansion in t at late tames. In a somewhat different direction, Guroivich and Starobinsky [20] have modeled vacuum polarization in a gravitational field by a bulk viscosity coefficient dependent on the curvature squared and we have shown that this proposition leads to inflation in any number of dimensions for a flat universe [21].

After a decade of research it has become clear that inflation results from a wide variety of cosmological models independent of initial conditions, and in fact this has been the main intent in demonstrating the existence of a cosmic no-hair theorem [22,23,24]. Along the same line of thought it has been speculated that inflationary solutions might be "attractors in initial condition space" [25] suggesting that initial conditions possibly given to us from the quantum-gravitational Planck era show up in

an unexpected phenomenological way such as a bulk a viscosity coefficient dependent on energy density and curvature. In what follows we discuss a cosmology admitting a higher-order curvature term in the lagrangian of gravity with a fluid source admitting energy density dependent, curvature dependent and energy density times curvature squared dependent bulk viscosity. Since the low energy effective action of gravity derived from the superstring [26] contains higher order curvature terms and since string interconversions and gravitational vacuum polarization can be modelled by energy density and curvature dependent bulk viscosity our model can apply to the period just following the Planck era when the universe is still highly symmetric and because of the high energy the massless modes of the superstring can be described by the equation of state

$$P = \varepsilon/3.$$

It is well known that these massless modes are in fact the quarks, leptons and gauge bosons and Higgs particles of particle theory.

II. Inflation Drive by Higher Order Curvature Terms and Bulk Viscosity Dependent on Energy Density and Curvature Squared

We begin our analysis by writing the lagrangian of gravitation and matter as

$$\mathcal{L} = \frac{c^4}{16\pi G}(R_S + l_0^4 R_S^3)\sqrt{-g} + \mathcal{L}_M \tag{1}$$

where l_0 is empirical constant of dimension of length, $R_s = R_{\mu\nu} g^{\mu\nu}$ is the curvature scalar, and $\mathcal{L}_M$ is matter lagrangian.

Eq. (1) could be thought as inspired by the superstring low energy effective action or as a result of imposing renormalizability on a quantum gravitational model [27]. Varying (1) we have

$$\frac{C^4}{16\pi G}\Big[(R_{\mu\nu} - \frac{1}{2}R_S g_{\mu\nu}) + l_0^4(3R_S^2 R_{\mu\nu} - \frac{1}{2}R_S^3 g_{\mu\nu}) \\ -g_{\mu\nu}\Box(3R_S^2 l_0^4) + \nabla_\mu\nabla_\nu(3l_0^4 R_S^2)\Big] + \frac{1}{\sqrt{-g}}\frac{\partial \mathcal{L}_M}{\partial g^{\mu\nu}} = 0. \tag{2}$$

We have omitted the quadratic term in R_S since it will give no contribution to inflation in an anisotropic and homogeneous universe. Eq. (2) gives

$$R_{\mu\nu} - \tfrac{1}{2}R_S g_{\mu\nu} + l_0^4\Big(3R_S^2 R_{\mu\nu} - \tfrac{1}{2}R_S^3 g_{\mu\nu}\Big) - g_{\mu\nu}\Box(3R_S^2 l_0^4) + \nabla_\mu\nabla_\nu(3l_0^4 R_S^2) \\ = \tfrac{8\pi G}{C^4}T_{\mu\nu}, \tag{3}$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial \mathcal{L}_M}{\partial g^{\mu\nu}} \qquad (k = \frac{8\pi G}{C^4}, \quad C = 1).$$

We now consider a flat ($K = 0$) R.W. metric (R is the scale factor)

$$(dS)^2 = dt^2 - R^2\left(\frac{dr^2}{1 - Kr^2} + r^2(d\Theta)^2 + r^2\sin^2\Theta(d\Phi)^2\right)$$

$$R_{00} = \frac{3\ddot{R}}{R},$$
$$R_{ij} = \left(\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2\right) g_{ij} \tag{4}$$

For the scalar curvature we have

$$R_S = 6\frac{\ddot{R}}{R} + 6\left(\frac{\dot{R}}{R}\right)^2. \tag{5}$$

For the energy momentum tensor we assume that the net effect of all particles and fields can be represented by the energy E.M. tensor of a perfect fluid with an effective bulk viscosity dependent energy density, curvature squared and energy density times curvature squared [15-21]. We thus write [28]

$$T_{\mu\nu} = (\overline{P} + \varepsilon)U_\mu U_\nu - g_{\mu\nu}\overline{P}$$
$$\overline{P} = P - \xi U^a{}_a \tag{6}$$
$$U^a{}_a = \frac{3\dot{R}}{R} = \text{expansion}$$

where [29]

$$\xi = C_1\varepsilon + C_2 R_S^2 + C_3\varepsilon R_S^2$$

and C_1, C_2, C_3 are phenomenological constants of dimensions cm, g.cm^4/s^2 and cm^5, respectively, and ξ is bulk viscosity coefficient. We now proceed to evaluate the terms in Eq (3). Firstly

$$\Box(3R_S^2 l_0^4) = \frac{1}{R^3}\frac{\partial}{\partial t}\left[3l_0^4 R^3 \frac{\partial}{\partial t}(R_S^2)\right]$$

and $\nabla_\mu\nabla_\nu(3l_0^4 R_S^2)$ has components

$$\nabla_0\nabla_0(3l_0^4R_S^2) = \frac{\partial^2}{\partial t^2}(3l_0^4R_S^2)$$
$$\nabla_i\nabla_j(3l_0^4R_S^2) = \frac{\dot{R}}{R}\frac{\partial}{\partial t}(3l_0^4R_S^2)g_{ij}. \tag{7}$$

The components of Eq. (6) are

$$T_{00} = \varepsilon$$
$$T_{ij} = -g_{ij}\left[P - \frac{3\dot{R}}{R}(C_1\varepsilon + C_2(R_S)^2 + C_3\varepsilon(R_S)^2)\right]. \tag{8}$$

We now choose the radiative equation of state

$$P = \varepsilon/3$$

to represent the highly relativistic state of matter shortly after the Planck era when any particles created from either gravitational vacuum polarization [20] or stringy

interconversions are highly relativistic and approximately massless relative to the scale of energy during this period [15]. Writing out Eq. (3) for the (00) and (ij) components using Eqs (4,5,7) and (8) we have

$$3\frac{\ddot{R}}{R} - \frac{1}{2}\left(6\left(\frac{\dot{R}}{R}\right)^2 + 6\frac{\ddot{R}}{R}\right)(1)$$
$$+l_0^4\left[3\left(\frac{6\ddot{R}}{R} + 6\left(\frac{\dot{R}}{R}\right)^2\right)^2\left(\frac{3\ddot{R}}{R}\right) - \frac{1}{2}\left(\frac{6\ddot{R}}{R} + 6\left(\frac{\dot{R}}{R}\right)^2\right)^3\right]$$
$$-\frac{1}{R^3}\frac{\partial}{\partial t}\left(R^3\frac{\partial}{\partial t}(3R_S^2l_0^4)\right) + \frac{\partial^2}{\partial t^2}(3l_0^4(R_S^2))$$
$$= -k(\varepsilon). \tag{9}$$

Eq. (9) reduces upon inserting the inflantionary solution $R = R_0 e^{\alpha t}$ to

$$k\varepsilon = 3\alpha - l_0^4(3(144)\alpha^6)$$

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{2}\left(\frac{6\ddot{R}}{R} + 6\left(\frac{\dot{R}}{R}\right)^2\right)$$
$$+l_0^4\left[3\left(\frac{6\ddot{R}}{R} + 6\left(\frac{\dot{R}}{R}\right)^2\right)^2\left(\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2\right) - \frac{1}{2}\left(\frac{6\ddot{R}}{R} + 6\left(\frac{\dot{R}}{R}\right)^2\right)^3\right]$$
$$-\frac{1}{R^3}\frac{\partial}{\partial t}\left[R^3 l_0^4\frac{\partial}{\partial t}(3R_S^2)\right] + \frac{\dot{R}}{R}\frac{\partial}{\partial t}(3l_0^4R_S^2)$$
$$= -k\left[\cdot\cdot\,\varepsilon/3 + 3\frac{\dot{R}}{R}(C_1\varepsilon + C_2(R_S)^2 + C_3\varepsilon(R_S)^2)\right]. \tag{10}$$

Eq. (10) gives upon inserting the inflationary solution

$$k\varepsilon = \frac{3\alpha^2 + l_0^4(3(144)\alpha^6) + k(432)C_2\alpha^5}{\frac{1}{3} - 3C_1\alpha - (432)C_3\alpha^5}.$$

Equating these two values of $k\varepsilon$ gives

$$\alpha^9(3(432)(144)l_0^4C_3) + \alpha^5(9C_1(144l_0^4) - 3(432)C_3)$$
$$-\alpha^4(4l_0^4(144)) - \alpha^3(k(432)C_2) - 9C_1\alpha + 4 = 0. \tag{11}$$

In a previous paper (ref. [29]) we have discussed the inflationary cosmology with the above form of the energy momentum components in Eq. (8) but did not consider higher curvature additions to the Einstein action. If $l_0 = 0$ we find that if C_1, C_2, $C_3 \neq 0$ then there is one positive root of Eq. (11) which is the inflationary solution. If, however, $l_0 \neq 0$ there can be two positive roots of Eq. (11) depending on the magnitude of the constants C_1, C_2, C_3, l_0. Thus we have the possibility of double inflation [30] which has been discussed in the literature as a resolution to the problem of how two scales of perturbations (one for the scale galaxies, the other one for the scale of superclusters) can be generated.

In ref. [30] it was shown that if the first epoch of inflation (extended inflation) is driven by Brans Dicke scalar coupled to a minimally coupled scalar field with perturbations arising from quantum fluctuations of the scalar field and the second epoch is a slowrollover phase of duration with the number of "e-folds" less then 65 then the perturbations that survive are not "scale-free" and do not have the Harrison-Zeldovich slope in harmony with the recent observations of the universe on large scales [31]. Said more simply, double inflation offers us a mechanism to generate perturbations from the quantum fluctuations of a minimally coupled scalar field that have the correct power on both the scale of galaxies and the scale of super-clusters and voids.

Our calculation demonstrates that two periods of inflation can exist for a cosmology admitting higher curvature terms plus energy and curvature dependent bulk viskosity but the evolution of perturbations of the energy density in both epochs would have to be studied to ascertain whether the model fits the presently known distribution of galaxies on one scale and super clusters and voids on the second scale. If this could not be achieved a minimally coupled scalar field would have to be added to the model to establish the correct perturbation spectra.

III. Conclusion

As mentioned in Ref. [27], to enforce the renormalizibility of quantum gravity higher order curvature corrections to the action are necessary. Also superstring predicts an infinite set of higher order curvature corrections in the form of the Lovelock lagrangian [32] which certainly will be influential in generating both the rate of inflation following Planck era and the form and structure of perturbations. Barrow [33] has also pointed out that any gravitational lagrangian which is analytic function of R can be shown to be conformally equivalent to general relativity plus a scalar field with a particular self-interacting scalar potential. In the same paper, he points out that a certain class of three potentials lead to powerr law inflation. If we consider the addition of

$$\varepsilon l_0^2 R_S^2 \left(\frac{C^4}{16\pi G} \right)$$

to the Einstein action, we find that it does not contribute to the rate of inflation for the homogeneous and isotropic case, however for stability [34] it is shown that ε must be negative. The fact that higher order gravity plus bulk viscous effects can lead to two epochs of inflation certainly prods us to search deeper into the microphysical origin of higher order curvature terms and bulk viscous effects. We also ask whether or not the inflationary cosmology derived from this model can stimulate the character of both scales od structure (galaxies and superclusters) when perturbations are studied in the background of the inflationary models studied in this note.

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