PAIRING IN NEUTRON MATTER WITH COGNY FORCE

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We investigate the pairing properties of neutron matter with the D1 parametrization of the Gogny force. Gogny's effective interaction gives among other things a reasonable pairing properties for finite nuclei. It is interesting to study the pairing properties of this force for pure neutron matter. Features obtained are discussed in comparison to other forces and to the results for the same force in symmetric nuclear matter.

1. Introduction

Superfluid properties of nuclear and neutron matter are of considerable interest in the theoretical study of the basic nucleon-nucleon forces as well as for the physics of neutron stars. In most investigations devoted to this issue finite range interactions are used, because a zero-range force brings a divergence in the pairing–energy gap [1]. Thus, pairing properties should be investigated with forces which are quite different from the case of finite nuclei, where zero-range Skyrme forces give good results for the radii, condensation energies and excited states. In the case of nuclei, the use of a delta force for pairing calculations is justified by a cut in the phase space. On the other hand, this procedure is not possible for infinite matter, unless to introduce an extra cut-off parameter, which simulates a finite range [2] and the problem should be solved from the very beginning. The Gogny force [3] has several advantages: it is rather simple to handle for the infinite system, has a finite range, and gives a good description of the behavior of pairing phenomena in finite nuclei [4]. This force has also been used for calculations in nuclear matter [1,5] where it gives pairing gaps which are compatible with results obtained with more complicated functional forms for the force [6], but whose values are too large at saturation [5].

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2. Method

In this paper, we apply the pairing equations to the calculation of the pairing gap in pure neutron matter using the Gogny interaction. The procedure we follow is essentially the one of ref. [1]. The functional form of the Gogny force is

$$v(1,2) = \sum_{i=1,2} (v_1^i + v_2^i P^\sigma + v_3^i P^\tau + v_4^i P^\sigma P^\tau) \exp\left(-(\vec{r}_1 - \vec{r}_2)^2/r_i^2\right) + t_3(1+P^\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho^{1/3}\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right) + w_{so}(\vec{r}_1 - \vec{r}_2) \quad (1)$$

where 1 and 2 indicate space coordinates, spin and isospin. The v_i are a set of parameters (see below), r_i are the radii of the two gaussians and the P^σ, P^τ are spin an isospin exchange operators, respectively, ρ is the nuclear density and t_3 is a parameter. While the first term represent the sum of the two gaussians with proper spin-isospin exchange coefficients, the second term is essentially a density-dependent part (necessary to reproduce ground-state energies in finite nuclei), and w_{so} denotes a spin-orbit interaction.

The pairing gap for singlet pairing ($^1S_0$) is a function of the wavenumber k and satisfies the BCS integral equation

$$\Delta(p) = -\frac{1}{(2\pi)^3}\int d^3k\, v(\vec{p}-\vec{k})\frac{\Delta(k)}{2E(k)} \quad (2)$$

where

$$E(k) = \sqrt{\left(h(k)-\mu\right)^2 + \Delta^2(k)} \quad (3)$$

is the quasi-particle energy, and

$$h(k) = \frac{\hbar^2 k^2}{2m} + V_{HF}(k) \quad (4)$$

represents the single-particle energy expressed in terms of the wavenumber k as the sum of the kinetic part and of the potential calculated in Hartree-Fock approximation, and $v(\vec{q})$ is the Fourier transform of the neutron-neutron interaction in the superfluid channel.

In the following, we adopt the effective mass approximation, consisting in making the following substitution

$$h(k) - \mu \approx \hbar^2 \frac{(k^2 - k_F^2)}{2m^*(k_F)} \quad (5)$$

where the effective mass $m^*(k_F)$ is

$$m^*(k_F) = \frac{m\hbar k_F}{(dh(k)/dk)_{k=k_F}} \quad (6)$$

and k_F denotes the Fermi wavenumber for neutrons. Following ref. [1], we use the same form for the neutron-neutron force, namely the D1 parametrization of the Gogny force. To evaluate the Hartree-Fock potential, we adopt plane wave wavefunctions for the spatial part, times a product of spin and isospin functions. For a uniform system, the spin orbit term gives no contributions. Furthermore, since we adopt the effective mass approximation, we shall discard the density dependent term in (1) which does not contribute to the effective mass. The part of the Gogny force we consider is thus

$$v(1,2) = \sum_{i=1,2}\left(v_1^i + v_2^i P^\sigma + v_3^i P^\tau + v_4^i P^\sigma P^\tau\right)\exp\left(-(\vec{r}_1-\vec{r}_2)^2/r_i^2\right).$$

To calculate the effective mass, we thus need the Hartree–Fock potential to be calculated starting from the elementary interaction (1). Since the direct part of the Hartree-Fock mean field is not dependent on the wavenumber, it does not contribute to the effective mass value, and the only contribution comes from the exchange potential. The exchange potential has the form

$$U_{exc}(p) = -\int \frac{d^3k}{(2\pi)^3}\, v_{n-n}(\vec{p}-\vec{k})n_k(k)$$

where v_{n-n} is the matrix element in the neutron-neutron channel, while n_n is the occupation probability of the neutron state of wavenumber k. Due to superfluidity, the distribution of neutrons is depleted with respect to a sharp distribution. In the first istance, we neglect these superfluidity effects in the calculation of the effective mass. It is thus possible to calculate analytically the effective mass for pure neutron matter at the Fermi surface as

$$\frac{m^*(k_F)}{m} = \left(1 + \frac{mk_F}{4\pi^2\hbar^2}\sum y_i v_i\right)^{-1} \quad (7)$$

with

$$v_i = \left(\frac{2}{(r_i k_F)}\right)^4 \left[2\left(\frac{r_i k_F}{2}\right)^2 - 1 + \left(1 + 2\left(\frac{r_i k_F}{2}\right)\right)^2 \exp(-r_i^2 k_F^2)\right] \quad (8)$$

and

$$y_i = v_3^i - 2v_4^i + v_1^i + 2v_2^i \quad (9)$$

The parameters of spin-isospin mixtures in the D1 representation have the values reported in Table 1. The range of the attractive gaussian is $r_1 = 0.7$ fm and of the repulsive is $r_2 = 1.2$ fm. Finally, the matrix element of the pairing interaction entering the pairing–energy gap equation (1) is [1]

$$v(\vec{q}) = \sum z_i \pi^{3/2} r_i^3 \exp\left(-\frac{r_i^2 q^2}{4}\right) \quad (10)$$

with $z_i = v_1^i - v_2^i + v_3^i + v_4^i$ for pure neutron matter.

Table 1. The parameter set D1 for the Gogny force (values in MeV).

i	v1	v2	v3	v4
1	-402.4	-100.0	496.2	23.56
2	-21.3	-11.77	-37.27	68.81

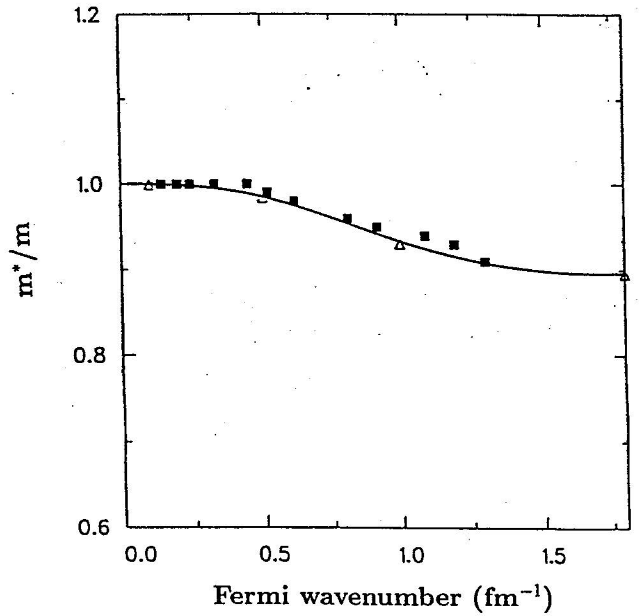

Fig. 1 Neutron effective mass in pure neutron matter. The continuous line is our calculation. Squares denote the values reported by Takatsuka [7]. Points denoted by triangles have been calculated numerically for the superfluid neutron matter.

3. Results and Conclusions

We have found the values of the effective–to–bare mass ratio to be in all cases close to 0.9 for neutron matter. For symmetric nuclear matter a value as low as 0.65 can be found at large wavenumbers [5]. For asymmetric matter we found values intermediate between the two. In Figure 1 we plot the effective-to bare mass ratio as a function of the Fermi wavenumber of pure neutron matter, calculated with the D1 parametrization of the Gogny force. The full curve represents the analytical formulas (7-9), while numerical calculations that take into account neutron superfluidity have been reported for a few values of the Fermi wavenumber and are represented by open triangles. The data reported here are in agreement with calculations made with different forces (data from ref [7] are shown as black squares). We used the above formalism for a computer iterative calculation of the pairing gap equation. The results are presented in Figure 2. The full line represents our result with the proper

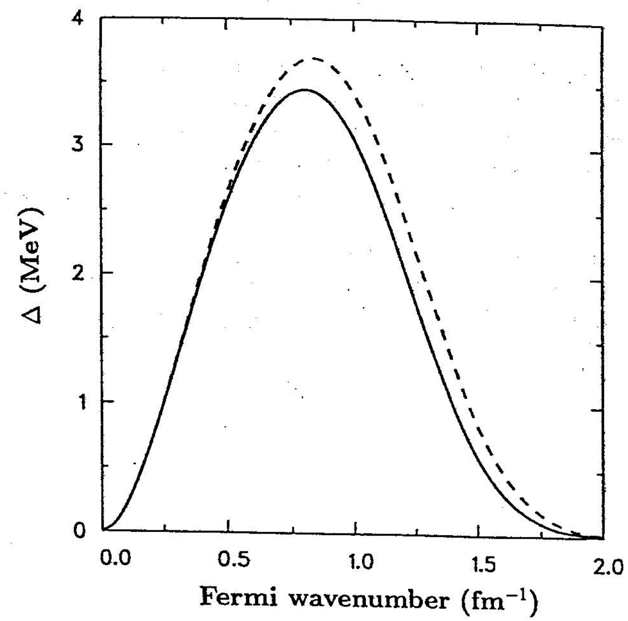

Fig. 2 Neutron pairing gap at the Fermi wavenumber for pure neutron matter calculated with the Gogny force as a function of the Fermi wavenumber (solid line). The calculation has also been done with the effective mass fixed and equal to the bare mass (dashed line).

calculation of the effective mass. The dashed line is the calculation with $m^* = m$. We observe that the maximum value of the energy gap corresponds to a value close to 0.8 fm^{-1} and is $\sim$ 3.4 MeV. This value is about 20% larger than for the symmetric matter case, as an effect of the larger effective masses with pure matter. We also notice that at saturation densities, that roughly corresponds essentially to the density where the nuclei in the inner crust of neutron stars merge together in a uniform medium [7], the pairing gap is still rather large. We notice that, although the results for the pairing gap in pure neutron matter are not unique, the agreement with other calculations is quite reasonable. In particular, the Gogny interaction in the vicinity of saturation density gives results which are intermediate between the results obtained with bare potentials [6] and those obtained with an effective interaction [7]. This may give confidence in the use of the Gogny force in the study of the status of baryonic matter inside the inner crust of neutron stars, where it is believed that neutron rich nuclei disposed in a Coulomb lattice are permeated by a sea of superfluid neutrons at very low temperature.

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