acta physica slovaca vol. 44 No. 1, 13 – 24 February1994

TO THE PROBLEM OF CHIRAL ANOMALIES IN NON-LEPTONIC DECAYS

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Received 2 December 1993, accepted 23 December 1993

The contribution of chiral anomalies to non-leptonic weak decays of K-mesons at the quark level has been investigated by taking the $K^+ \to \pi^+\pi^0\gamma$ decay as an example. The theoretical prediction for the probability of the $K^+ \to \pi^+\pi^0\gamma$ decay with the direct emission of a photon is $B(K^+ \to \pi^+\pi^0\gamma(DE))_{anom} \le (1.1 \pm 0.2) \times 10^{-5}$. This value overlaps itself with the experimental one: $B(K^+ \to \pi^+\pi^0\gamma(DE))_{anom} \le (1.56 \pm 0.35) \times 10^{-5}$ obtained by Abrams et al. There has been presented the critical analysis of the description of non-leptonic weak decays and the account for the chiral anomaly contributions into these processes, performed within chiral perturbation theory at the hadronic level with non-linear realization of chiral symmetry.

The exploration of the influence of chiral anomalies [1] in non-leptonic weak decays of K-mesons is an important business for a deeper understanding of Standard model, embodying Standard electroweak model [2] and QCD. Recently within chiral perturbation theory at the hadronic level $(CHPT)_h$ with the non-linear realization of chiral SU(3)×SU(3) symmetry [3] there were considered the contributions of chiral anomalies to the $K^+ \to \pi^+\pi^0\gamma$ decay amplitude, caused by the direct emission (DE) of a photon. For the definition of the amplitude $A(K^+ \to \pi^+\pi^0\gamma(\mathrm{DE}))$ the authors

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of [3] introduced two new free parameters a_2 and a_3, which can not be determined within (CHPT)$_h$ and must satisfy the restriction [3]

$$|2 + 6a_3 - 2a_2| \leq 5, \qquad (1)$$

imposed by the comparison of theoretical value of the probability $B(K^+ \to \pi^+\pi^0\gamma(\mathrm{DE}))$ with the world-average value of experimental data [4]:

$$B(K^+ \to \pi^+\pi^0\gamma(\mathrm{DE}))_{exp} = (1.8 \pm 0.4) \times 10^{-5}.$$

Since the restriction (1) cannot help to fix these free parameters, so they remain fully undetermined. Thus from the practical point of view the results, obtained in [3], one cannot appraise as a theoretical explanation of the $K^+ \to \pi^+\pi^0\gamma$ decay with a direct emission of a photon. Besides this fact there is another reason compeling us to think, that the results, obtained in ref. [3], are not established good enough. This is concerning the misrepresentation of the transformation and dynamical properties of the penguin operator at the hadronic level.

In this paper we repeat the criticism concerning the bosonization of the penguin operator within (CHPT)$_h$ [5,6]. We analyse the crucial consequences of this bosonization by example of the account for chiral anomalies in non-leptonic weak decays of K-mesons. We give the quark level evaluation of chiral anomaly contributions to the probability of the $K^+ \to \pi^+\pi^0\gamma$ decay with the direct emission of a photon, especially to underscore the twist of the mechanism of the enhancement of the $\Delta I = 1/2$ transitions in nonleptonic decays, provided by the incorrect bosonization of the penguin operator used in ref. [3]

The effective weak Lagrangian, obtained within Standard model and governing transitions with $\Delta S = 1$ and $\Delta I = 1/2$ selection rules, reads [7]:

$$L_{eff}^{\Delta S=1,\Delta I=1/2} = -\frac{G_F}{\sqrt{2}}V^*_{us}V_{ud} : O_{\Delta I=1/2} : + \text{h.c.}, \qquad (2)$$

where

$$O_{\Delta I=1/2} = C_1O_1 + C_2O_2 + C_5O_5 \qquad (3)$$

and C_i $(i = 1, 2, 5)$ are Wilson coefficients being as function of heavy quark masses, Λ_{QCD} and renormalization scale μ. In the Shifman- Vainshtein- Zakharov- basis the operators O_i take the form [7a]:

$$O_1 = [\bar{s}\gamma_\mu(1-\gamma^5)u][\bar{u}\gamma^\mu(1-\gamma^5)d] - [\bar{s}\gamma_\mu(1-\gamma^5)d][\bar{u}\gamma^\mu(1-\gamma^5)u],$$
$$O_2 = O_1 + 2[\bar{s}\gamma_\mu(1-\gamma^5)d][\bar{q}\gamma^\mu(1-\gamma^5)q],$$
$$O_5 = [\bar{s}\gamma_\mu(1-\gamma^5)\lambda^A_c d][\bar{q}\gamma^\mu(1+\gamma^5)\lambda^A_c q], \qquad (4)$$

where q is a column matrix with elements (u, d, s) such that the every quark flavour possesses N colour degrees of freedom; the matrices of $SU(N)_c$ colour group λ^A_c $(A = 1, ..., N^2 - 1)$ are normalized by the condition $\mathrm{tr}_c(\lambda^A_c\lambda^B_c) = 2\delta^{AB}$. The symbol :...: denotes the normal-ordering operation. It should be stressed that the use of the normal-ordered form of the effective weak Lagrangian (2) is due to the ordinary requirements of quantum field theory [8].

Now we propose to analyse the transformation properties of the $: O_{\Delta I=1/2} :$-operator. For this aim we have to consider the transformation properties of the $: O_i :$-operators. In order to do it, for the sake of convenience, one can introduce the following generalizations of these operators [6]:

$$O_{1,2} = O^{6-i7}_{1,2} = (d^{6bc} - id^{7bc})[\bar{q}\gamma_\mu(1-\gamma^5)t^b q][\bar{q}\gamma^\mu(1-\gamma^5)t^c q] +$$
$$+\xi_{1,2}[\bar{q}\gamma_\mu(1-\gamma^5)t^{6-i7}q][\bar{q}\gamma^\mu(1-\gamma^5)q] \longrightarrow$$
$$\longrightarrow O^a_{1,2} = d^{abc}[[\bar{q}\gamma_\mu(1-\gamma^5)t^b q][\bar{q}\gamma^\mu(1-\gamma^5)t^c q] +$$
$$+\xi_{1,2}[\bar{q}\gamma_\mu(1-\gamma^5)t^a q][\bar{q}\gamma^\mu(1-\gamma^5)q], \qquad (5a)$$

$$O_5 = O^{6-i7}_5 = [\bar{q}\gamma_\mu(1-\gamma^5)t^{6-i7}\lambda^A_c q][\bar{q}\gamma^\mu(1+\gamma^5)\lambda^A_c q] \longrightarrow$$
$$\longrightarrow O^a_5 = [\bar{q}\gamma_\mu(1-\gamma^5)t^a\lambda^A_c q][\bar{q}\gamma^\mu(1+\gamma^5)\lambda^A_c q], \qquad (5b)$$

where $d^{abc} = 2\mathrm{tr}(t^a\{t^b, t^c\})$ and $\xi_1 = -1/3$ and $\xi_2 = 5/3$. The matrices t^a $(a = 1, ..., 8)$ are normalized by the condition $\mathrm{tr}(t^a t^b) = \delta^{ab}/2$.

It is well known that any operator O^a $(a = 1, ..., 8)$, transforming like $(\mathbf{8}_L, \mathbf{1}_R)$, must obey the following commutation relations [5,9]:

$$[Q^a_L(0), O^b(0)] = if^{abc}O^c(0), \qquad [Q^a_R(0), O^b(0)] = 0, \qquad (6)$$

where $f^{abc} = -2i\mathrm{tr}(t^a[t^b, t^c])$ and

$$Q^a_L(0) = \int d^3x : q^+(0, x)\left(\frac{1-\gamma^5}{2}\right)t^a q(0, x) :,$$

$$Q^a_R(0) = \int d^3x : q^+(0, x)\left(\frac{1+\gamma^5}{2}\right)t^a q(0, x) :$$

are the operators of left and right chiral charges, taken in the normal-ordered form [8].

Now let us evaluate the equal-time commutation relations (6) for the $:O_i:$-operators. By applying the canonical anticommutation relations for current quark fields one gets [5,6]

$$[Q^a_L(0), : O^b_{1,2} :] = if^{abc} : O^c_{1,2} :, \qquad [Q^a_R(0), : O^b_{1,2} :] = 0, \qquad (7a)$$

$$[Q^a_L(0), : O^b_5(0) :] = if^{abc} : O^c_5(0) :$$
$$-4\left(1 - \frac{1}{N^2}\right)\langle 0|\bar{q}q|0\rangle : \bar{q}(0)(1-\gamma^5)t^a t^b q(0) : +$$
$$+4\left(1 - \frac{1}{N^2}\right)\langle 0|\bar{q}q|0\rangle : \bar{q}(0)(1+\gamma^5)t^b t^a q(0) :,$$
$$[Q^a_R(0), : O^b_5(0) :] = 4\left(1 - \frac{1}{N^2}\right)\langle 0|\bar{q}q|0\rangle : \bar{q}(0)(1-\gamma^5)t^a t^b q(0) : -$$
$$-4\left(1 - \frac{1}{N^2}\right)\langle 0|\bar{q}q|0\rangle : \bar{q}(0)(1+\gamma^5)t^b t^a q(0) : . \qquad (7b)$$

It is seen that the normal-ordered penguin operator does not transform like $(\mathbf{8}_L, \mathbf{1}_R)$, and only the $:O_{1,2}:$-operators have the indicated transformation properties.

In spite of this the authors of [3] insist that the effective Lagrangian (2) "has a unique realization at the mesonic level to lowest order in CHPT first given by Cronin"

$$L_{eff}^{\Delta S=1,\Delta I=1/2} = -\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} g_8 O_{Cr} + \text{h.c.}, \tag{8}$$

where

$$O_{Cr} = F_0^4 \mathrm{tr}(t^{6-i7}\partial_\mu U \partial^\mu U^+) = \mathrm{tr}(t^{6-i7} L_\mu L^\mu) \tag{9}$$

is the Cronin's operator [11], transforming like $(\mathbf{8}_L, \mathbf{1}_R)$ under chiral SU(3)×SU(3) rotations

$$U \to U' = g_L U g_R^+ .$$

By definition [11] we have: $L_\mu = -iF_0^2 U \partial_\mu U^+$, being the (V-A) hadronic current and transforming like $(\mathbf{8}_L, \mathbf{1}_R)$, i.e. $L_\mu \to L'_\mu = g_L L_\mu g_L^+$, then $U = \exp(-2i\Phi/F_0)$ such that $\Phi = t^a\Phi^a$ is the octet of pseudoscalar meson fields, and g_L and g_R are the matrices of the $SU(3)_L$ and $SU(3)_R$ chiral rotations respectively.

By assumption [10,11] the phenomenological constant g_8 must be fixed from the experimental data on the $K_S^0 \to \pi^+\pi^-$ decay, i.e. $|g_8| = 5.1$.

Since the operators $C_1 : O_1 : +C_2 : O_2 :$ and $C_5 : O_5 :$ have different transformation properties, so they cannot be bosonized to the same form with the common factor g_8. The detailed confirmation of this affirmation, performed within current algebra approach, soft-pion technique and low-energy theorems, one can find in [5,6].

As a result the effective Lagrangian (8) can be at most the hadronic version of the Lagrangian

$$L_{eff}^{\Delta S=1,\Delta I=1/2} = -\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud}(C_1 : O_1 + C_2 : O_2 :). \tag{10}$$

In this case the constant g_8, expressed in terms of Wilson coefficients [6]

$$g_8 = C_1 + C_2, \tag{11}$$

is of order unity, and there is not any hidden dynamics being able to provide the unexpectable increasing of the g_8-value up to $|g_8| = 5.1$.

By completing this discussion we should like to emphasize that the inapplicability of the Cronin's operator to be considered as a bosonized version of the penguin one is not connected with the normal-ordered or normal-unordered form of the later. The main reason of this problem has the dynamical nature [10]. The matter is the Cronin's operator governs the $K^0 \to \pi^+\pi^-$ transition via P-wave-intermadiate states:

$$\langle\pi^+\pi^-|O_{Cr}|K^0\rangle = \langle\pi^+\pi^-|(L_\mu L^\mu)_{23}|K^0\rangle =$$

$$= \langle\pi^+|(L_\mu)_{21}|0\rangle\langle\pi^-|(L^\mu)_{13}|K^0\rangle + \langle\pi^+\pi^-|(L_\mu)_{22}|0\rangle\langle 0|(L^\mu)_{23}|K^0\rangle, \tag{12}$$

where the matrix elements $\langle\pi^-|(L^\mu)_{13}|K^0\rangle$ and $\langle\pi^+\pi^-|(L_\mu)_{22}|0\rangle$ are saturated by P-wave intermediate states. It implies that only the vector part of L_μ gives the contribution. In the contrary the penguin operator has the open S-wave channel, being clearly seen after Fierz transformations in (4)

$$O_5 = -4\left(1 - \frac{1}{N^2}\right)\{[\bar{s}(1+\gamma^5)u][\bar{u}(1-\gamma^5)d] +$$

$$+ [\bar{s}(1+\gamma^5)d][\bar{d}(1-\gamma^5)d] + [\bar{s}(1+\gamma^5)s][\bar{s}(1-\gamma^5)d]\}, \tag{13}$$

and hence [12]

$$\langle\pi^+\pi^-| : O_5 : |K^0\rangle = 4\left(1 - \frac{1}{N^2}\right)\langle\pi^+\pi^-|\bar{d}d|0\rangle\langle 0|\bar{s}\gamma^5 d|K^0\rangle -$$

$$-\langle\pi^+|\bar{u}\gamma^5 d|0\rangle\langle\pi^-|\bar{s}u|K^0\rangle], \tag{14}$$

where the vacuum saturation approximation has been applied [7a,12]. The enhancement of the $\Delta I = 1/2$ transitions in the $K^0 \to \pi^+\pi^-$ decay occurs by virtue the saturation of the matrix elements $\langle\pi^+\pi^-|\bar{d}d|0\rangle$ and $\langle\pi^-|\bar{s}u|K^0\rangle$ of the scalar current quark densities. Within linear realization of chiral SU(3)×SU(3) symmetry these matrix elements are saturated by the σ- and κ-mesons, being the scalar partners of pseudoscalar mesons under chiral transformations [7a,12,13,14]. Since the penguin and the Cronin's operators govern the $\Delta I = 1/2$ transitions in the $K^0 \to \pi^+\pi^-$ decay via different intermediate states, so these operators cannot be identified. Consequently the value of g_8 (11) cannot be increased at expense of the penguin operator contribution. The latter means that if the authors of [3] would prefer to approximate the $K^+ \to \pi^+\pi^0\gamma$ decay amplitude with the direct emmission of a photon by chiral anomaly contributions, so they would have to change the estimate of the a_i-parameter values, i.e.

$$a_i > \frac{5.1}{C_1 + C_2} \tag{15}$$

instead of $a_i \le 1$ [3], admitted in accordance with the bosonization procedure, proposed by Pich and de Rafael [15] and leading to incorrect bosonization of the penguin operator [16].

Now let us proceed to the quark level evaluation of chiral anomaly contributions to the $K^+ \to \pi^+\pi^0\gamma$ decay. The diagrams describing this decay are depicted in Figs.1 and 2. The pole-diagrams in Fig.1 involve the transitions $\pi^+ \to \pi^+\pi^0\gamma$, $K^+ \to K^+\pi^0\gamma$ and $K^+ \to K^0\pi^+\gamma$ being determined fully by the anomalous γPPP-vertices. This is due to the form factors of the VPP-interactions, vanishing at low-energies [17]. The diagrams in Fig.2 contain both anomalous and non-anomalous contributions. We are picking up only the anomalous ones.

First let us regard the contribution of diagrams in Fig.1. By keeping only the chiral anomalies of the γPPP-type, one obtains

$$A(\pi^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{anom} =$$

$$A(K^+(p) \to K^+(p_+)\pi^0(p_0)\gamma(q))_{anom} =$$

$$= \frac{e}{4\pi^2 F_0^3}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta, \quad (\epsilon_{0123} = 1),$$

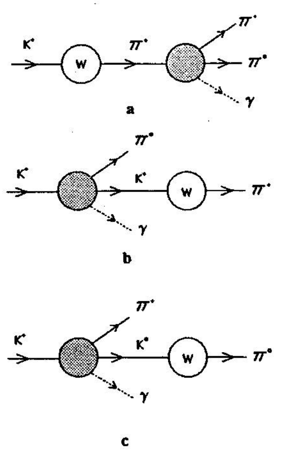

Fig.1. Feynman diagrams, describing the (π, K)-pole contributions to the $K^+ \to \pi^+\pi^-\gamma$ decay amplitude with the direct emission of a photon.

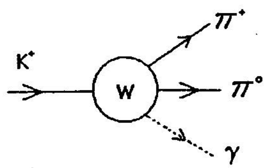

Fig.2. Feynman diagrams of the structure part of the $K^+ \to \pi^+\pi^0\gamma$ decay.

$$A(K^+(p) \to K^0(p_0)\pi^0(p_0)\gamma(q))_{anom} = 0. \tag{16}$$

Now we have to avaluate the matrix elements of the $K^+ \to \pi^+$ transition. By using the results obtained in ref. [18] one gets at the leading order in large N expansion

$$\langle\pi^+(p)| : O_{1,2} : |K^+(p)\rangle = 2F_0^2 p^2,$$
$$\langle\pi^+(p)| : O_5 : |K^+(p)\rangle = -8F_0^2\bar{v}^2\left(1 + \frac{2p^2}{\Lambda_1^2}\right), \tag{17}$$

where $\bar{v} = -\langle 0|\bar{q}q|0\rangle/F_0^2$ and Λ_1 is the slope parameter. In Chiral perturbation theory at the quark level $(\mathrm{CHPT})_q$ [19] these parameters are fixed in terms of chiral symmetry breaking scale $\Lambda_\chi = 0.94$ GeV and the constituent quark mass $m = 0.33$ GeV, calculated in the chiral limit, i.e.

$$\bar{v} = 4mI_1(m)/F_0^2 = 1.92\ \mathrm{GeV}, \qquad \Lambda_1 = (2m\bar{v})^{1/2} = 1.13\ \mathrm{GeV} \tag{18}$$

and

$$I_1(m) = \frac{3}{16\pi^2}\int\frac{d^4k}{\pi^2 i}\frac{1}{m^2-k^2} = \frac{3}{16\pi^2}\left[\Lambda_\chi^2 - m^2\ln\left(1+\frac{\Lambda_\chi}{m^2}\right)\right].$$

As a result we can write down the contribution of the diagrams in Fig.1 to the $K^+ \to \pi^+\pi^0\gamma$ decay amplitude:

$$A(K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{Fig.1a} = \frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\frac{1}{m_K^2 - m_\pi^2}\times$$
$$\times\left[(C_1+C_2)m_K^2 - C_5 4\bar{v}^2\left(1+\frac{2m_K^2}{\Lambda_1^2}\right)\right]\frac{e}{2\pi^2F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta,$$
$$A(K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{Fig.1b} = -\frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\frac{1}{m_K^2 - m_\pi^2}\times$$
$$\times\left[(C_1+C_2)m_\pi^2 - C_5 4\bar{v}^2\left(1+\frac{2m_\pi^2}{\Lambda_1^2}\right)\right]\frac{e}{2\pi^2F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta,$$
$$A(K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{Fig.1c} = 0. \tag{19}$$

The total contribution of the diagrams in Fig.1 to the $K^+ \to \pi^+\pi^0\gamma$ decay amplitude sums to the following expression

$$A(K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{Fig.1} =$$
$$= \frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\left(C_1 + C_2 - C_5\frac{8\bar{v}^2}{\Lambda_1^2}\right)\frac{e}{2\pi^2F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta. \tag{20}$$

It is necessary to remind that this result is obtained in the leading order of both large N and chiral expansions. Also it is safe to say that this result is the model-independent one, being fully determined by the transformation properties of the starting effective weak Lagrangian (2).

To confirm our affirmation, concerning the incorrect bosonization of the penguin operator within $(\mathrm{CHPT})_h$ which results the incorrect account for the chiral anomaly contributions, let us compare the factor

$$\left(C_1 + C_2 - C_5\frac{8\bar{v}^2}{\Lambda_1^2}\right), \tag{21}$$

produced by the structure of the Lagrangian (2), with the corresponding factor, coming up in the $K^0 \to \pi^+\pi^-$ decay amplitude. The $K^0 \to \pi^+\pi^-$ decay amplitude, evaluated with the help of the Lagrangian (2) in the leading order of large N and chiral expansions, takes the form [18]:

$$\frac{1}{i}A(K^0 \to \pi^+\pi^-) = G_F V^*_{us}V_{ud}F_0\left(C_1 + C_2 - C_5\frac{4\bar{v}^2}{\Lambda_1^2}\right)(m_K^2 - m_\pi^2). \tag{22}$$

It is seen that the factor

$$\left(C_1 + C_2 - C_5\frac{4\bar{v}^2}{\Lambda_1^2}\right) \tag{23}$$

in (22) differs from the factor (21) in (20). It means that in [3] there is incorrectly obtained the effective Lagrangian (see the formula (10) in [3]), having the form

$$L^{\Delta S=1}_{an.} = -\frac{ie}{8\pi^2F_0}\frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}g_8\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}\partial_\alpha\pi^0 K^+\overleftrightarrow{\partial}_\beta\pi^+ + \ldots,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength tensor. In this expression the factor g_8 is just the same, determining the $K^0 \to \pi^+\pi^-$ decay amplitude via the Lagrangian (8)

$$\frac{1}{i}A(K^0 \to \pi^+\pi^-) = G_F V^*_{us} V_{ud} g_8 (m_K^2 - m_\pi^2)$$

in $(\mathrm{CHPT})_h$. This incorrect result is, of course, the consequence of the misrepresentation of the transformation and dynamical properties of the penguin operator at the hadronic level, i.e. the use of the Cronin's operator for the bosonization of the penguin one.

Now let us proceed to the evaluation of the diagrams in Fig.2. These diagrams describe the structure part of the $K^+ \to \pi^+\pi^0\gamma$ decay amplitude, containing the matrix elements $\langle\gamma\pi^0\pi^+| : O_i : |K^+\rangle$. However in this paper, as has been emphasized above, we are keeping only the anomalous parts of these matrix elements, i.e. the quantities $\langle\gamma\pi^0\pi^+| : O_i : |K^+\rangle_{anom}$. By using the vacuum saturation approximation [7a,18] we can present the matrix elements $\langle\gamma\pi^0\pi^+| : O_i : |K^+\rangle_{anom}$ in the form of the following decompositions

$$\langle\gamma\pi^0\pi^+| : O_1 : |K^+\rangle_{anom} = \langle\gamma\pi^+|\bar{u}\gamma_\nu d|0\rangle\langle\pi^0|\bar{s}\gamma^\nu u|K^+\rangle +$$
$$+\langle\pi^0\pi^+|\bar{u}\gamma_\nu d|0\rangle\langle\gamma|\bar{s}\gamma^\nu u|K^+\rangle - \langle\gamma\pi^0|\bar{u}\gamma_\nu u|0\rangle\langle\pi^+|\bar{s}\gamma^\nu d|K^+\rangle,$$
$$\langle\gamma\pi^0\pi^+| : O_2 : |K^+\rangle_{anom} = \langle\gamma\pi^0\pi^+| : O_1 : |K^+\rangle_{anom} +$$
$$+2\langle\gamma\pi^0|\bar{u}\gamma_\nu u + \bar{d}\gamma_\nu d|0\rangle\langle\pi^+|\bar{s}\gamma^\nu d|K^+\rangle,$$
$$\frac{1}{4}\langle\gamma\pi^0\pi^+| : O_5 : |K^+\rangle_{anom} = \langle\pi^+|\bar{u}\gamma^5 d|0\rangle\langle\gamma\pi^0|\bar{s}\gamma^5 u|K^+\rangle +$$
$$+\langle\gamma\pi^0\pi^+|\bar{u}\gamma^5 d|0\rangle\langle 0|\bar{s}\gamma^5 u|K^+\rangle + \langle\pi^0|\bar{d}\gamma^5 d|0\rangle\langle\gamma\pi^+|\bar{s}\gamma^5 d|K^+\rangle. \quad (24)$$

Let us write down the analytical expressions of the matrix elements, incoming to the r.h.s. of the formulae (24) [12,18]:

$$\langle\gamma(q)\pi^+(p_+)|\bar{u}\gamma_\nu d|0\rangle = -\frac{1}{\sqrt{2}}\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\alpha q^\beta,$$
$$\langle\gamma(q)|\bar{s}\gamma_\nu u|K^+(p)\rangle = \frac{1}{\sqrt{2}}\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p^\alpha q^\beta,$$
$$\langle\gamma(q)\pi^0(p_0)|\bar{u}\gamma_\nu u|0\rangle = -\frac{e}{2\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_0^\alpha q^\beta,$$
$$\langle\gamma(q)\pi^0(p_0)|\bar{u}\gamma_\nu u + \bar{d}\gamma_\nu d|0\rangle = -\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_0^\alpha q^\beta; \quad (25a)$$
$$\langle\pi^0(p_0)|\bar{s}\gamma_\nu u|K^+(p)\rangle = -\frac{1}{\sqrt{2}}(p+p_0)_\nu,$$
$$\langle\pi^0(p_0)\pi^+(p_+)|\bar{u}\gamma_\nu d|0\rangle = -\sqrt{2}(p_+ - p_0)_\nu,$$
$$\langle\pi^+(p_+)|\bar{s}\gamma_\nu d|K^+(p)\rangle = -(p+p_+)_\nu; \quad (25b)$$
$$\langle\gamma(q)\pi^0(p_0)|\bar{s}\gamma^5 u|K^+(p)\rangle = -\frac{i\sqrt{2}F_0}{\bar{v}}\frac{\bar{v}^2}{\Lambda_1^2}A(K^+(p) \to K^+(p_+)\pi^0(p_0)\gamma(q))_{anom},$$

$$\langle\gamma(q)\pi^0(p_0)\pi^+(p_+)|\bar{u}\gamma^5 d|0\rangle = -\frac{i\sqrt{2}F_0}{\bar{v}}\frac{\bar{v}^2}{\Lambda_1^2}A(\pi^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{anom};$$
$$\langle\gamma(q)\pi^+(p_+)|\bar{s}\gamma^5 d|K^+(p)\rangle = 0; \quad (25c)$$
$$\langle\pi^+(p_+)|\bar{u}\gamma^5 d|0\rangle = \langle 0|\bar{s}\gamma^5 u|K^+(p)\rangle = -i\sqrt{2}F_0\bar{v}. \quad (25d)$$

By using formulae (25) we can find the following matrix elements

$$\langle\gamma\pi^0\pi^+| : O_1 : |K^+\rangle_{anom} = -7\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta,$$
$$\langle\gamma\pi^0\pi^+| : O_2 : |K^+\rangle_{anom} = -3\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta,$$
$$\langle\gamma\pi^0\pi^+| : O_5 : |K^+\rangle_{anom} = -\frac{16\bar{v}^2}{\Lambda_1^2}\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta. \quad (26)$$

As a result the contribution of chiral anomalous parts of the diagrams in Fig.2 is given by the expression

$$A(K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{Fig.2,\,anom} =$$
$$= \frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\left(7C_1 + 3C_2 + C_5\frac{16\bar{v}^2}{\Lambda_1^2}\right)\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta. \quad (27)$$

By summing up (20) and (27) we can obtain the total amplitude of the $K^+ \to \pi^+\pi^0\gamma$ decay, caused by chiral anomaly contributions, i.e.:

$$A(K^+(p) \to \pi^+(p_+)\pi^0(p_0)\gamma(q))_{anom} =$$
$$\frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}(9C_1 + 5C_2)\frac{e}{4\pi^2 F_0}\epsilon_{\mu\nu\alpha\beta}e^\mu(q)p_+^\nu p_0^\alpha q^\beta. \quad (28)$$

Since the C_5-coefficient is cancelled out, so the penguin operator does not contribute to the anomalous part of the $K^+ \to \pi^+\pi^0\gamma$ decay amplitude at all!

It means that the mechanism of the $\Delta I = 1/2$ transition enhancement in the $K^0 \to \pi^+\pi^-$ decay does not relate to the mechanism, governing the anomalous part of the $K^+ \to \pi^+\pi^0\gamma$ decay with the direct emission of a photon.

Let us estimate the contribution of chiral anomalies to the probability of the $K^+ \to \pi^+\pi^0\gamma$ decay with the direct emission of a photon

$$B(K^+ \to \pi^+\pi^0\gamma(DE))_{anom} = \tau_{K^+}|G_F V^*_{us}V_{ud}(9C_1 + 5C_2)|^2 \times$$
$$\times\frac{\alpha}{3.2^{14}\pi^6 F_0^2 m_K^3}\int_{4m_\pi^2}^{m_K^2} ds\, s(1 - 4m_\pi^2/s)^{3/2}(m_K^2 - s)^3 = (1.1 \pm 0.20) \times 10^{-5}, \quad (29)$$

where $\tau^{-1}_{K^+} = \Gamma(K^+ \to all) = (5.32 \pm 0.03) \times 10^{-17}$ GeV [4], $|G_F V^*_{us}V_{ud}| = 2.5 \times 10^{-6}\ \mathrm{GeV}^{-2}$, $m_K = 0.50$ GeV and $m_\pi = 0.14$ GeV, $C_1 = 1.262$ and $C_2 = 0.063$. The uncertainty $(\pm 0.2) \times 10^{-5}$ is due to the theoretical uncertainty of $(\mathrm{CHPT})_q$, being equal to 20% approximately [20]. The Wilson coefficients C_1 and C_2 depend on the renormalization scale μ, and the numerical values $C_1 = 1.262$ and $C_2 = 0.063$ are obtained $\mu = \Lambda_\chi = 0.94$ GeV [21]. It should be noted that the numerical values of C_1 and C_2 are found at the complete neglection of the penguin diagrams contribution, i.e. $C_5 = 0$, which gives $C_1 = C_-/2$ and $C_2 = C_+/10$ such that $C_-C_+^2 = 1$ [7,20]. The account for the penguin diagrams contribution to the (V-A)×(V-A) four quark operators yields to the change of the coefficients C_1 and C_2 by following way [7,20]: $C_1 = C_-/2 \to C_-/2 + C_5$ and $C_2 = C_+/10 \to C_+/10 + C_5$. Since $C_5 < 0$, so we have obtained the upper bound of the chiral anomaly contributions to the $K^+ \to \pi^+\pi^0\gamma$ decay with the direct emission of a photon.

Now let us compare the theoretical result with experimental data. In Particle Data Group [7] there are quoted three experimental results

$$10^5\, B(K^+ \to \pi^+\pi^0\gamma(DE))_{exp} = \begin{cases} 2.05 \pm 0.46^{+0.39}_{-0.23}\,, & \text{(Bolotov, 1987)} \\ 2.3 \pm 3.2\,, & \text{(Smith, 1976)} \\ 1.56 \pm 0.35 \pm 0.5\,, & \text{(Abrams, 1972)} \end{cases}$$

The world-average value is [4]

$$B(K^+ \to \pi^+\pi^0\gamma(DE))_{\rm exp.w.av.} = (1.8 \pm 0.4) \times 10^{-5}.$$

Our result is in agreement with those obtained by Abrams et al. and Smith et al. With the result, obtained by Bolotov et al., there is the agreement within two standard deviations only.

By using the results obtained both in the present paper and in ref. [22] one can conclude that chiral anomalies play a dominant role for the description of the $K^+ \to \pi^+\pi^0\gamma$(DE) decay. To our point of view, the present experimental data on this decay are not established good enough to be compared with theoretical results and must be revised. Of course, this proposal can be said out within the theory not containing low-energy free and ill-determined parameters like g_8, a_i, etc.

In the conclusion we would like to repeat that all shortcomings of the paper [3] are connected with the misrepresentation of transformation and dynamical properties of the penguin operator at the hadronic level, i.e. the use of the Cronin's operator for the bosonization of the penguin operator within non-linear realization of chiral SU(3)× SU(3) symmetry, i.e. within $(\rm CHPT)_h$. The incorrect bosonization of local four-quark operators [15], exploited in ref. [3], has led to the misrepresentation of the mechanism of the $K^+ \to \pi^+\pi^0\gamma$ decay with the direct emission of a photon. In fact as has been shown above the penguin operator does not contribute to the $K^+ \to \pi^+\pi^0\gamma$(DE) decay amplitude, caused by chiral anomalies. It means that the mechanism of the enhancement of $\Delta I = 1/2$ transition in the decays $K^0 \to \pi^+\pi^-$ and $K^+ \to \pi^+\pi^0\gamma$(DE) are different. The continuation of the use of the Cronin's operator for the description of the enhancement of the $\Delta I = 1/2$ transitions in non-leptonic weak decays will lead to new delusions.

It is necessary to draw attention to the attempt of the bosonization of the penguin operator, undertaken in ref. [23] within Chiral theory with non-linear realization of chiral SU(3)×SU(3) symmetry. The result of this version of the penguin operator at the hadronic level differs from the Cronin's operator, and presents the direct substitution of current quark densities $\bar{q}(1 \pm \gamma^5)q'$ in the formula (13) into hadronic densities, constructed in terms of U-fields and its derivatives. Unfortunately, this attempt cannot be appraised as successful. Indeed the matrix elements of this hadronic version

of the penguin operator satisfy the low-energy theorem

$$\lim_{p_+\to 0}\langle \pi^+(p_+)\pi^-(p_-)|O_5|K^0(p)\rangle = \frac{i}{F_0}\langle \pi^0(p)|O_5|K^0(p)\rangle,$$

being incorrect for the matrix elements of the penguin operator obtained at the quark level within current algebra approach [5,6].

So we must conclude that the results, obtained on the basis of the bosonization procedure, proposed in [23] and leading to the incorrect hadronic version of the penguin operator, are not good established and demand revision.

Acknowledgement

Two of us (A.N.I. and N.I.T.) have a pleasure to thank to Dr. A. Nogová and Dr. J. Boháčik for a very kindful hospitality during the stay at the Institute of Physics in Bratislava, where the part of this work was done.

References

[1] S.L.Adler: *Phys. Rev* **177** (1969), 2426; J.S.Bell, R.Jackiw: *Nuovo Cim.* **60A** (1969), 47; W.A.Bardeen: *Phys. Rev.* **184** (1969), 1848; S.L.Glashow: *Nucl. Phys.* **22** (1961), 579; S.Weinberg: *Phys. Rev. Lett.* **19** (1967), 1264; A.Salam: in *Proceedings of the 8th Nobel Symposium* Edit. N. Svartholm (Almqvist and Wiksells, Stockholm, 1968) p. 367.

[3] J.Bijnens, G.Ecker, A.Pich: *Phys. Lett. B* **286** (1992), 341;

[4] Particle Data Group:: *Phys. Lett. B* **239** (1990), 1;

[5] A.N.Ivanov, M.Nagy, N.I.Troitskaya: *Phys. Lett. B* **275** (1992), 441; *Mod. Phys. Lett.* **A7** (1992) 427;

[6] A.N.Ivanov, M.Nagy, N.I.Troitskaya: *Int. J. Mod. Phys.* **A8** (1993), 3425;

[7] a)M.A.Shifman, A.I.Vainshtain, V.I.Zakharov: *Nucl. Phys. B* **120** (1977), 315;
b)F.J.Gilman, M.B.Wise: *Phys. Rev. D* **20** (1979), 1216; ibid. **27** (1983) 1128;
c)A.Buras, W.Slominsky: *Nucl. Phys. B* **235** (1985), 231;

[8] J.D.Bjorken, S.D.Drell*Relativistic quantum fields* (Mc Graw-Hill, 1965); F.J.Yndurain: *Quantum chromodynamics*, (Springer, 1987).

[9] S.L.Adler and R.Dashen: in *Current Algebras and Applications to Particle Physics* (Benjamin, 1968) and references therein.

[10] A.N.Ivanov, N.I.Troitskaya, M.Nagy: *Phys. Lett. B* **311** (1993), 291;

[11] J.A.Cronin: *Phys. Rev. 161* **1967** (1483), G; .Ecker: *Non-leptonic weak interactions in chiral perturbation theory*, Proc. of Hadron Structure-87, Smolenice, Czechoslovakia, Nov. 1987 and references therein.

[12]A.N.Ivanov, M.Nagy, N.I.Troitskaya: *Czech. J. Phys. B* **42** (1992), 21;

[13]J.F.Donoghue et al.: *Phys. Rev. D* **21** (1986), 186;

[14] M.K.Volkov, A.N.Ivanov, N.I.Troitskaya: *Yad. Fiz.* **47** (1988), 1152;

[15] A.Pich, E.de Rafael: *Nucl. Phys. B* **358** (1991), 311;

[16] A.N.Ivanov: to appear in *Int. J. Mod. Phys. A*

[17] A.N.Ivanov, M.Nagy, N.I.Troitskaya: to appear in *Int. J. Mod. Phys. A*; A.N.Ivanov: *Int. J. Mod. Phys.* **A6** (1991), 4055; and references therein.

[18] N.I.Troitskaya: *PhD.Thesis*, Univ. of St. Petersburg, 1990;

[19] A.N.Ivanov, M.Nagy, N.I.Troitskaya: *Int. J. Mod. Phys.* **A7** (1992), 7305;

[20] A.N.Ivanov: *Phys. Lett. B* **275** (1992), 450;

[21] C.O.Dib, I.Dunietz, F.J.Gilman: *Phys. Rev. D* **39** (1989), 2639;

[22] A.N.Ivanov,M.Nagy, N.I.Troitskaya: *Mod. Phys. Lett.* **A8** (1993), 1599;

[23] D.Ebert, A.A.Bel'kov, A.V.Lanyov, A.Schaale: Preprint PHE-91-08, Zeuthen, 1991; A.A.Bel'kov, A.V.Lanyov, G.Böhm, A.Schaale, D.Ebert: Preprint DESY-92-106, July 1992.